Sample Questions

Failures Resulting from Static Loading

Mechanical Engineering and Design, Shingley, Seventh Edition, Questions 6.3, 6.5, 6.14 6.25 and 6.26

6.3 A bar of AISI 1020 cold-drawn steel, using the distortion-energy and maximum-shear stress theories determine the factors of safety for the following plane stress states.

a) $\sigma_x = 180MPa$, $\sigma_y = 100MPa$

b) $\sigma_x = 180MPa$, $\tau_{xy} = 100MPa$

c) $\sigma_x = -160MPa$, $\tau_{xy} = 100MPa$

d) $\tau_{xy} = 150MPa$

6.5 Repeat Prob. 6.3 by first plotting the failure loci in the σ_A, σ_B plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.

6.14 This problem illustrates that the factor of safety for a machine element depends on the particular point selected for analysis. Here you are to compute factors of safety, based upon the distortion energy theory, for stress elements at A and B of the member shown in the figure. This bar is made of AISI 1006 cold-drawn steel and is loaded by the forces F=0.55 kN, P=8.0 kN and T=30 Nm.

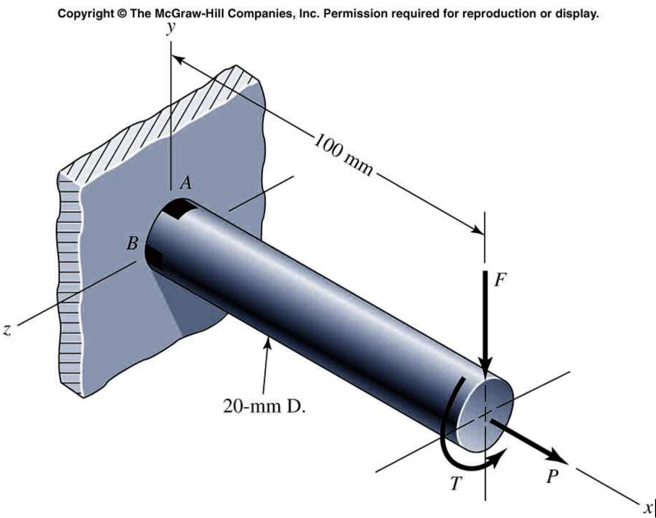


Fig. 6.14

6.25 The figure is a schematic drawing of a countershaft that supports two V-belt pulleys. For each pulley, the belt tensions are parallel. For pulley A consider the loose belt tension is 15 percent of the tension on the tight side. Cold-drawn UNS G10180 steel shaft of uniform diameter is to be selected for this application. For a static analysis with a factor of 3.0, determine the minimum preferred size diameter. Use the distortion-energy theory.

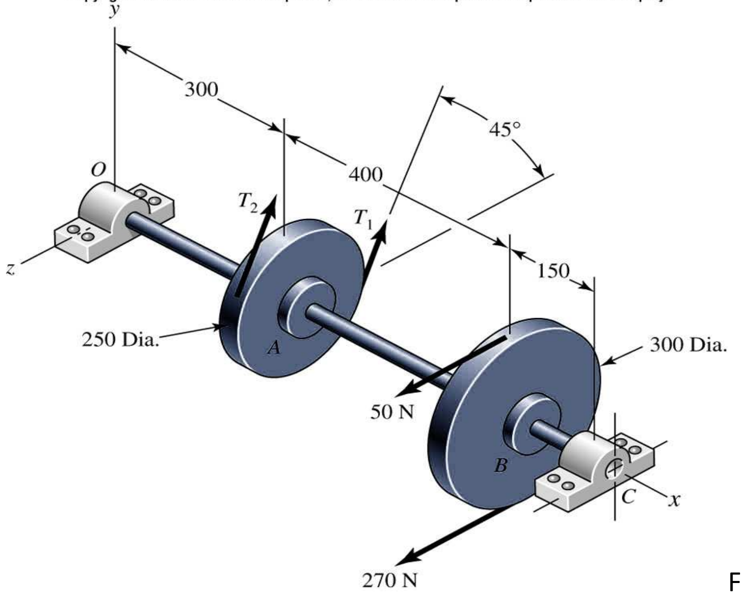


Fig. 6.25-6.26

6.26 Repeat problem 6-25 using maximum shear stress.

ANSWERS

6-3 $S_y = 390$ MPa

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$

(a) MSS: $\sigma_1 = 180$ MPa, $\sigma_3 = 0$, $n = \dfrac{390}{180} = 2.17$ *Ans.*

DE: $n = \dfrac{390}{[180^2 - 180(100) + 100^2]^{1/2}} = 2.50$ *Ans.*

(b) $\sigma_A, \sigma_B = \dfrac{180}{2} \pm \sqrt{\left(\dfrac{180}{2}\right)^2 + 100^2} = 224.5, -44.5 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{224.5 - (-44.5)} = 1.45$ *Ans.*

DE: $n = \dfrac{390}{[180^2 + 3(100^2)]^{1/2}} = 1.56$ *Ans.*

(c) $\sigma_A, \sigma_B = -\dfrac{160}{2} \pm \sqrt{\left(-\dfrac{160}{2}\right)^2 + 100^2} = 48.06, -208.06 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{48.06 - (-208.06)} = 1.52$ *Ans.*

DE: $n = \dfrac{390}{[-160^2 + 3(100^2)]^{1/2}} = 1.65$ *Ans.*

(d) $\sigma_A, \sigma_B = 150, -150 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{380}{150 - (-150)} = 1.27$ *Ans.*

DE: $n = \dfrac{390}{[3(150)^2]^{1/2}} = 1.50$ *Ans.*

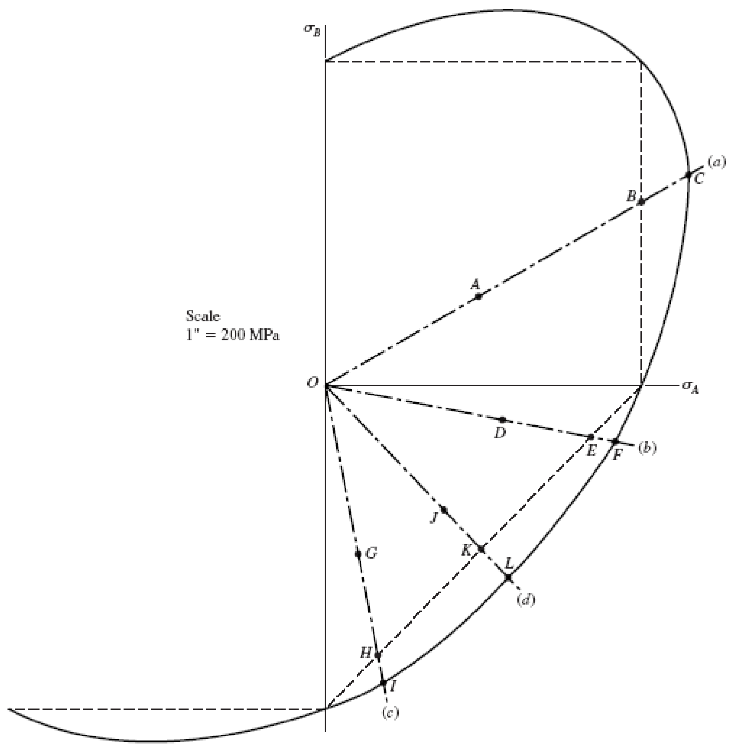
6-5

(a) MSS: $n = \dfrac{OB}{OA} = \dfrac{2.23}{1.08} = 2.1$

DE: $n = \dfrac{OC}{OA} = \dfrac{2.56}{1.08} = 2.4$

(b) MSS: $n = \dfrac{OE}{OD} = \dfrac{1.65}{1.10} = 1.5$

DE: $n = \dfrac{OF}{OD} = \dfrac{1.8}{1.1} = 1.6$



(c) MSS: $n = \dfrac{OH}{OG} = \dfrac{1.68}{1.05} = 1.6$

DE: $n = \dfrac{OI}{OG} = \dfrac{1.85}{1.05} = 1.8$

(d) MSS: $n = \dfrac{OK}{OJ} = \dfrac{1.38}{1.05} = 1.3$

DE: $n = \dfrac{OL}{OJ} = \dfrac{1.62}{1.05} = 1.5$

6-14 Given: AISI 1006 CD steel, $F = 0.55$ N, $P = 8.0$ kN, and $T = 30$ N · m, applying the DE theory to stress elements A and B with $S_y = 280$ MPa

A: $$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)(10^3)(0.1)}{\pi(0.020^3)} + \frac{4(8)(10^3)}{\pi(0.020^2)}$$

$$= 95.49(10^6)\text{ Pa} = 95.49\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020^3)} = 19.10(10^6)\text{ Pa} = 19.10\text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = [95.49^2 + 3(19.1)^2]^{1/2} = 101.1\text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{101.1} = 2.77 \quad \textit{Ans.}$$

B:

$$\sigma_x = \frac{4P}{\pi d^3} = \frac{4(8)(10^3)}{\pi(0.020^2)} = 25.47(10^6)\text{ Pa} = 25.47\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(30)}{\pi(0.020^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)(0.020^2)}\right]$$

$$= 21.43(10^6)\text{ Pa} = 21.43\text{ MPa}$$

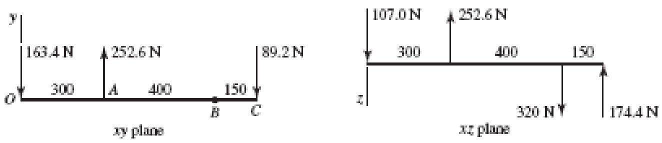
$$\sigma' = [25.47^2 + 3(21.43^2)]^{1/2} = 45.02\text{ MPa}$$

$$n = \frac{280}{45.02} = 6.22 \quad \textit{Ans.}$$

6-25 $T = (270 - 50)(0.150) = 33\text{ N}\cdot\text{m}$, $S_y = 370\text{ MPa}$

$$(T_1 - 0.15T_1)(0.125) = 33 \quad \Rightarrow \quad T_1 = 310.6\text{ N}, \quad T_2 = 0.15(310.6) = 46.6\text{ N}$$

$$(T_1 + T_2)\cos 45 = 252.6\text{ N}$$



$$M_A = 0.3\sqrt{163.4^2 + 107^2} = 58.59\text{ N}\cdot\text{m} \qquad \text{(maximum)}$$

$$M_B = 0.15\sqrt{89.2^2 + 174.4^2} = 29.38\text{ N}\cdot\text{m}$$

$$\sigma_x = \frac{32(58.59)}{\pi d^3} = \frac{596.8}{d^3}$$

$$\tau_{xy} = \frac{16(33)}{\pi d^3} = \frac{168.1}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[\left(\frac{596.8}{d^3}\right)^2 + 3\left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{664.0}{d^3} = \frac{370(10^6)}{3.0}$$

$d = 17.5(10^{-3})\text{ m} = 17.5\text{ mm}$, so use 18 mm *Ans.*

6-26 From Prob. 6-25,

$$\tau_{max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

$$\left[\left(\frac{596.8}{2d^3}\right)^2 + \left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{342.5}{d^3} = \frac{370(10^6)}{2(3.0)}$$

$d = 17.7(10^{-3})\text{ m} = 17.7\text{ mm}$, so use 18 mm *Ans.*