**Sample Questions**

**Fatigue Failure Resulting from Variable Loading**

**Mechanical Engineering and Design, Shingley, Seventh Edition, Questions 7.8, 7.9, 7.26, 7.29**

**7.8.** A solid round bar, 25 mm in diameter, has a groove 2.5 mm deep with a 2.5 mm radius machined into it. The bar is made of AISI 1018 CD steel and is subjected to a purely reversing torque of 200Nm. For the S-N curve of this material, let f = 0.9.

a) Estimate the number of cycles to failure.
b) If the bar is also placed in an environment with a temperature of $450^0$C, estimate the number of cycles to failure.

**7.9** A solid square rod cantilevered at one end. The rod is 0.8 m long and supports a completely reversing transverse load at the other hand ± 1 kN. The material is AISI 1045 hot-rolled steel. If the rod must support this load for $10^4$ cycles with a factor of safety of 1.5, what dimensions should the square cross section have? Neglect any stress concentrations at the support end and assume that f=0.9.

**7.26** In the figure shown, shaft A, made of AISI 1010 hot-rolled steel, is welded to a fixed support and is subjected to loading by equal and opposite forces F via shaft B. A theoretical stress concentration $K_{ts}$ of 1.6 is induced by the 3 mm fillet. The length of shaft A from the fixed support to the connection at shaft B is 1 m. The load F cycles from 0.5 to 2 kN.

a) For shaft A, find the factor of safety for infinite life using the modified Goodman fatigue failure criterion.
b) Repeat part(a) using Gerber fatigue failure criterion.



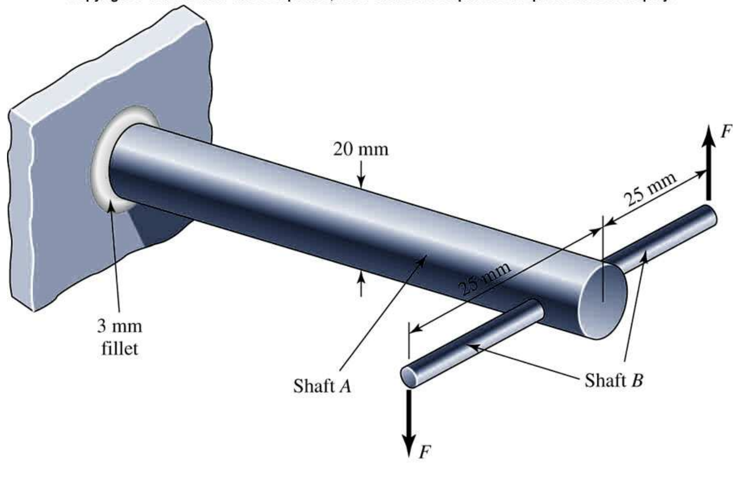
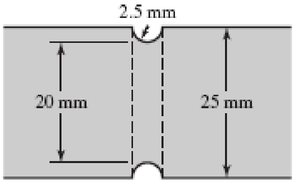



Fig 7.26

**7.29** A flat leaf spring has fluctuating stress of $\sigma_{max}$= 420 MPa and $\sigma_{min}$= 140 MPa applied for $5(10^4)$ cycles. If the load changes to $\sigma_{max}$= 350 MPa and $\sigma_{min}$= -200 MPa, how many cycles should the spring survive? The material is AISI 1040 CD and has a fully corrected endurance strength of $S_e$ = 200 MPa. Assume that f = 0.9

a) Use Miner's method.
b) Use Manson's method.

**7-8**




**(a)** For an AISI 1018 CD-machined steel, the strengths are

Eq. (3-17): $S_{ut} = 440 \text{ MPa} \quad \Rightarrow \quad H_B = \dfrac{440}{3.41} = 129$

$S_y = 370 \text{ MPa}$

$S_{su} = 0.67(440) = 295 \text{ MPa}$

Fig. A-15-15: $\dfrac{r}{d} = \dfrac{2.5}{20} = 0.125, \quad \dfrac{D}{d} = \dfrac{25}{20} = 1.25, \quad K_{ts} = 1.4$

Fig. 7-21: $q_s = 0.94$

Eq. (7-31): $K_{fs} = 1 + 0.94(1.4 - 1) = 1.376$

For a purely reversing torque of 200 N · m

$$\tau_{\max} = \frac{K_{fs} 16T}{\pi d^3} = \frac{1.376(16)(200 \times 10^3 \text{ N} \cdot \text{mm})}{\pi (20 \text{ mm})^3}$$

$$\tau_{\max} = 175.2 \text{ MPa} = \tau_a$$

$$S'_e = 0.504(440) = 222 \text{ MPa}$$

The Marin factors are

$$k_a = 4.51(440)^{-0.265} = 0.899$$

$$k_b = \left(\frac{20}{7.62}\right)^{-0.107} = 0.902$$

$$k_c = 0.59, \quad k_d = 1, \quad k_e = 1$$

Eq. (7-17): $S_e = 0.899(0.902)(0.59)(222) = 106.2 \text{ MPa}$

Eq. (7-13): $a = \dfrac{[0.9(295)]^2}{106.2} = 664$

Eq. (7-14): $b = -\dfrac{1}{3} \log \dfrac{0.9(295)}{106.2} = -0.132\,65$

Eq. (7-15): $N = \left(\dfrac{175.2}{664}\right)^{1/-0.132\,65}$

$N = 23\,000$ cycles *Ans.*

**(b)** For an operating temperature of 450°C, the temperature modification factor, from Table 7-6, is

$$k_d = 0.843$$

Thus $S_e = 0.899(0.902)(0.59)(0.843)(222) = 89.5 \text{ MPa}$

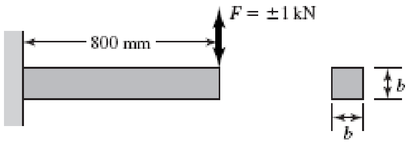
$$a = \frac{[0.9(295)]^2}{89.5} = 788$$

$$b = -\frac{1}{3} \log \frac{0.9(295)}{89.5} = -0.157\,41$$

$$N = \left(\frac{175.2}{788}\right)^{1/-0.157\,41}$$

$N = 14\,100$ cycles *Ans.*

**7-9**




$f = 0.9$
$n = 1.5$
$N = 10^4$ cycles

For AISI 1045 HR steel, $S_{ut} = 570$ MPa and $S_y = 310$ MPa

$$S'_e = 0.504(570\text{ MPa}) = 287.3\text{ MPa}$$

Find an initial guess based on yielding:

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3}$$

$$M_{\max} = (1\text{ kN})(800\text{ mm}) = 800\text{ N}\cdot\text{m}$$

$$\sigma_{\max} = \frac{S_y}{n} \quad \Rightarrow \quad \frac{6(800\times 10^3\text{ N}\cdot\text{mm})}{b^3} = \frac{310\text{ N/mm}^2}{1.5}$$

$$b = 28.5\text{ mm}$$

Eq. (7-24): $d_e = 0.808b$

Eq. (7-19): $k_b = \left(\dfrac{0.808b}{7.62}\right)^{-0.107} = 1.2714b^{-0.107}$

$$k_b = 0.888$$

The remaining Marin factors are

$$k_a = 57.7(570)^{-0.718} = 0.606$$

$$k_c = k_d = k_e = k_f = 1$$

Eq. (7-17): $S_e = 0.606(0.888)(287.3\text{ MPa}) = 154.6\text{ MPa}$

Eq. (7-13): $a = \dfrac{[0.9(570)]^2}{154.6} = 1702$

Eq. (7-14): $b = -\dfrac{1}{3}\log\dfrac{0.9(570)}{154.6} = -0.173\,64$

Eq. (7-12): $S_f = aN^b = 1702[(10^4)^{-0.173\,64}] = 343.9\text{ MPa}$

$$n = \frac{S_f}{\sigma_a} \quad \text{or} \quad \sigma_a = \frac{S_f}{n}$$

$$\frac{6(800\times 10^3)}{b^3} = \frac{343.9}{1.5} \quad \Rightarrow \quad b = 27.6\text{ mm}$$

Check values for $k_b$, $S_e$, etc.

$$k_b = 1.2714(27.6)^{-0.107} = 0.891$$
$$S_e = 0.606(0.891)(287.3) = 155.1 \text{ MPa}$$
$$a = \frac{[0.9(570)]^2}{155.1} = 1697$$
$$b = -\frac{1}{3}\log\frac{0.9(570)}{155.1} = -0.173\,17$$
$$S_f = 1697[(10^4)^{-0.173\,17}] = 344.4 \text{ MPa}$$
$$\frac{6(800 \times 10^3)}{b^3} = \frac{344.4}{1.5}$$
$$b = 27.5 \text{ mm} \quad Ans.$$

**7-26**

**(a)**
$$\tau_{max} = \frac{16K_{fs}T_{max}}{\pi d^3}$$

Fig. 7-21 for $H_B > 200$, $r = 3$ mm, $q_s \doteq 1$

$$K_{fs} = 1 + q_s(K_{ts} - 1)$$
$$K_{fs} = 1 + 1(1.6 - 1) = 1.6$$
$$T_{max} = 2000(0.05) = 100 \text{ N}\cdot\text{m}, \quad T_{min} = \frac{500}{2000}(100) = 25 \text{ N}\cdot\text{m}$$
$$\tau_{max} = \frac{16(1.6)(100)(10^{-6})}{\pi(0.02)^3} = 101.9 \text{ MPa}$$
$$\tau_{min} = \frac{500}{2000}(101.9) = 25.46 \text{ MPa}$$
$$\tau_m = \frac{1}{2}(101.9 + 25.46) = 63.68 \text{ MPa}$$
$$\tau_a = \frac{1}{2}(101.9 - 25.46) = 38.22 \text{ MPa}$$
$$S_{su} = 0.67S_{ut} = 0.67(320) = 214.4 \text{ MPa}$$
$$S_{sy} = 0.577S_y = 0.577(180) = 103.9 \text{ MPa}$$
$$S'_e = 0.504(320) = 161.3 \text{ MPa}$$
$$k_a = 57.7(320)^{-0.718} = 0.917$$
$$d_e = 0.370(20) = 7.4 \text{ mm}$$
$$k_b = \left(\frac{7.4}{7.62}\right)^{-0.107} = 1.003$$
$$k_c = 0.59$$
$$S_e = 0.917(1.003)(0.59)(161.3) = 87.5 \text{ MPa}$$

Modified Goodman, Table 7-9,

$$n_f = \frac{1}{(\tau_a/S_e) + (\tau_m/S_{su})} = \frac{1}{(38.22/87.5) + (63.68/214.4)} = 1.36 \quad Ans.$$

**(b)** Gerber, Table 7-10

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_e}{S_{su}\tau_a}\right)^2}\,\right]$$

$$= \frac{1}{2}\left(\frac{214.4}{63.68}\right)^2 \frac{38.22}{87.5}\left\{-1 + \sqrt{1 + \left[\frac{2(63.68)(87.5)}{214.4(38.22)}\right]^2}\,\right\} = 1.70 \quad Ans.$$

**7-29**

$$S_y = 490\ \text{MPa}, \quad S_{ut} = 590\ \text{MPa}, \quad S_e = 200\ \text{MPa}$$

$$\sigma_m = \frac{420 + 140}{2} = 280\ \text{MPa}, \quad \sigma_a = \frac{420 - 140}{2} = 140\ \text{MPa}$$

Goodman:

$$(\sigma_a)_e = \frac{\sigma_a}{1 - \sigma_m/S_{ut}} = \frac{140}{1 - (280/590)} = 266.5\ \text{MPa} > S_e$$

$$a = \frac{[0.9(590)]^2}{200} = 1409.8\ \text{MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(590)}{200} = -0.141\,355$$

$$N = \left(\frac{266.5}{1409.8}\right)^{-1/0.143\,55} = 131\,200 \text{ cycles}$$

$$N_{\text{remaining}} = 131\,200 - 50\,000 = 81\,200 \text{ cycles}$$

Second loading:

$$(\sigma_m)_2 = \frac{350 + (-200)}{2} = 75 \text{ MPa}$$

$$(\sigma_a)_2 = \frac{350 - (-200)}{2} = 275 \text{ MPa}$$

$$(\sigma_a)_{e2} = \frac{275}{1 - (75/590)} = 315.0 \text{ MPa}$$

**(a)** Miner's method

$$N_2 = \left(\frac{315}{1409.8}\right)^{-1/0.141\,355} = 40\,200 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \quad \frac{50\,000}{131\,200} + \frac{n_2}{40\,200} = 1$$

$$n_2 = 24\,880 \text{ cycles} \quad \textit{Ans.}$$

**(b)** Manson's method

Two data points: 0.9(590 MPa), $10^3$ cycles
266.5 MPa, 81 200 cycles

$$\frac{0.9(590)}{266.5} = \frac{a_2(10^3)^{b_2}}{a_2(81\,200)^{b_2}}$$

$$1.9925 = (0.012\,315)^{b_2}$$

$$b_2 = \frac{\log 1.9925}{\log 0.012\,315} = -0.156\,789$$

$$a_2 = \frac{266.5}{(81\,200)^{-0.156\,789}} = 1568.4 \text{ MPa}$$

$$n_2 = \left(\frac{315}{1568.4}\right)^{1/-0.156\,789} = 27\,950 \text{ cycles} \quad \textit{Ans.}$$