

Chapter 7, Problem 11

A cantilever *WF* aluminum alloy beam of yield strength S_y is loaded as shown in Figure P7.11. Using a factor of safety of n , determine whether failure occurs according to the maximum shear stress criterion.

Given: $S_y = 320 \text{ MPa}$, $n = 2$, $I_z = 13.4 \cdot 10^6 \text{ mm}^4$

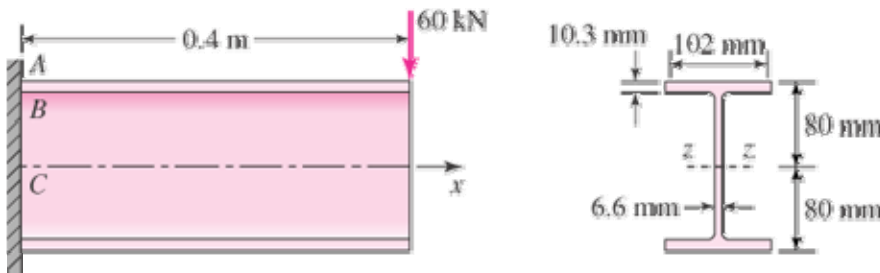


Figure P7.11

Chapter 7, Solution 11

$$Q_B = 102(10.3)(74.85) = 78.6 \times 10^3 \text{ mm}^3$$

$$Q_C = Q_B + 6.6(69.7)(34.85) = 94.6 \times 10^3 \text{ mm}^3$$

We have

$$\sigma_A = \frac{M_C}{I} = \frac{24(10^3)(0.08)}{13.4(10^{-6})} = 143.3 \text{ MPa}, \quad \sigma_B = \frac{69.7}{80}(143.3) = 124.9 \text{ MPa}$$

and

$$\tau_B = \frac{VQ_B}{Ib} = \frac{60(10^3)(78.6 \times 10^{-6})}{13.4(10^{-6})(0.0066)} = 53.3 \text{ MPa}, \quad \tau_c = \frac{VQ_c}{Ib} = 64.2 \text{ MPa}$$

Thus

$$(\sigma_{1,2})_B = \frac{124.9}{2} \pm \left[\left(\frac{124.9}{2} \right)^2 + 53.3^2 \right]^{\frac{1}{2}} = 62.5 \pm 82.1$$

or $\sigma_{1B} = 144.6 \text{ MPa} \quad \sigma_{2B} = -19.65 \text{ MPa}$

$$(\tau_{\max})_B = 82.1 \text{ MPa}$$

Hence

$$(\tau_{\max})_B = \frac{S_y}{2(2)} = \frac{320}{4}; \quad 82.1 > 80 \quad \therefore \text{Fails}$$

Alternatively, using Eq.(7.11), we have

$$[124.9^2 + 4(53.3)^2]^{\frac{1}{2}} = \frac{320}{2}; \quad 164.2 > 160 \quad \therefore \text{Fails}$$