and the dilatation is

$$e = -\frac{p}{K} = -\frac{160}{175(10^3)} = -9.14 \times 10^{-4}$$

Since the initial volume of the block (Fig. 2.6) is $V_o = 2 \times 1.5 \times 1 = 3\ \text{m}^3$, Eq. (2.24) yields

$$\Delta V = eV_o = (-9.14 \times 10^{-4})(3 \times 10^9) = -2.74 \times 10^6\ \text{mm}^3$$

where a minus sign means that the block experiences a decrease in the volume, as expected intuitively.

2.9 MEASUREMENT OF STRAIN: BONDED STRAIN GAGES

A wide variety of mechanical, electrical, and optical systems has been developed for measuring the average strain at a point on a free surface of a member [Ref. 2.5]. The method in widest use employs the bonded electric wire or foil resistance strain gages. The *bonded wire gage* consists of a grid of fine wire filament cemented between two sheets of treated paper or plastic backing (Fig. 2.12a). The backing insulates the grid from the metal surface on which it is to be bonded and functions also as a carrier so that the filament may be conveniently handled. Generally, 0.025-mm diameter wire is used. The grid in the case of *bonded foil gages* is constructed of very thin metal foil (approximately 0.0025 mm), rather than wire. Because the filament cross section of a foil gage is rectangular, the ratio of surface area to cross-sectional area is higher than that of a round wire. This results in increased heat dissipation and improved adhesion between the grid and the backing material. Foil gages are readily manufactured in a variety of configurations. In general, the selection of a particular bonded gage depends on the specific service application.

The ratio of the unit change in the resistance of the gage to the unit change in length (strain) of the gage is called the *gage factor*. The metal of which the filament element is made is the principal factor determining the

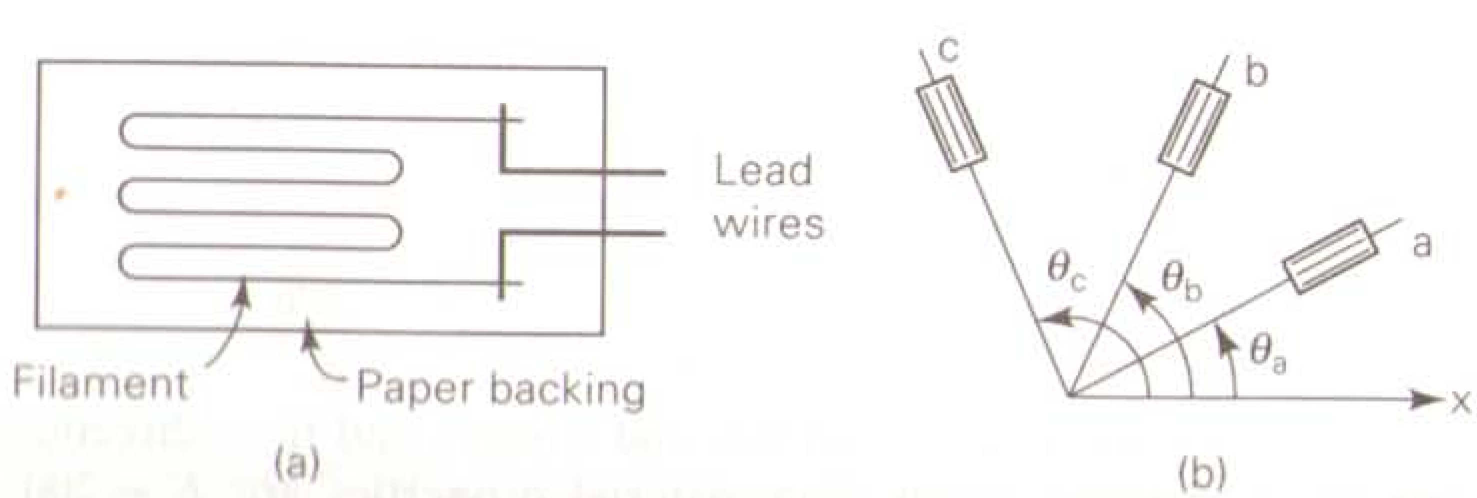

Figure 2.12 (a) Strain gage; (b) strain rosette.

magnitude of this factor. *Constantan*, an alloy composed of 60% copper and 40% nickel, produces wire or foil gages with a gage factor of approximately 2.

The operation of the bonded strain gage is based on the change in electrical resistance of the filament that accompanies a change in the strain. Deformation of the surface on which the gage is bonded results in a deformation of the backing and the grid as well. Thus, with straining, a variation in the resistance of the grid will manifest itself as a change in the voltage across the grid. An electrical bridge circuit, attached to the gage by means of lead wires, is then used to translate electrical changes into strains. The *Wheatstone bridge*, one of the most accurate and convenient systems of this type employed, is capable of measuring strains as small as 1 μ.

Strain Rosette. Special combination gages are available for the measurement of the state of strain at a point on a surface simultaneously in three or more directions. Generally, these consist of three gages whose axes are either 45° or 60° apart. Consider three strain gages located at angles θ_a, θ_b, and θ_c with respect to reference axis x (Fig. 2.12b). The a-, b-, and c-directed normal strains are, from Eq. (2.10a),

$$
\begin{aligned}
\varepsilon_a &= \varepsilon_x \cos^2 \theta_a + \varepsilon_y \sin^2 \theta_a + \gamma_{xy} \sin \theta_a \cos \theta_a \\
\varepsilon_b &= \varepsilon_x \cos^2 \theta_b + \varepsilon_y \sin^2 \theta_b + \gamma_{xy} \sin \theta_b \cos \theta_b \\
\varepsilon_c &= \varepsilon_x \cos^2 \theta_c + \varepsilon_y \sin^2 \theta_c + \gamma_{xy} \sin \theta_c \cos \theta_c
\end{aligned}
\tag{2.34}
$$

When the values of ε_a, ε_b, and ε_c are measured for given *gage orientations* θ_a, θ_b, and θ_c, the values of ε_x, ε_y, and γ_{xy} can be obtained by simultaneous solution of Eqs. (2.34). The arrangement of gages employed for this kind of measurement is called a *strain rosette*.

Once plane strain components are known, we can apply Eq. (3.11b) of Sec. 3.3 to determine the *out-of-plane principal strain* ε_z. The in-plane principal strains and their orientations may be obtained readily using Eqs. (2.12) and (2.13), as illustrated next, or Mohr's circle for strain.

Example 2.6

Strain rosette readings are made at a critical point on the free surface in a structural steel member. The 60° rosette contains three wire gages positioned at 0°, 60°, and 120°. The readings are

$$
\varepsilon_a = 190\ \mu, \qquad \varepsilon_b = 200\ \mu, \qquad \varepsilon_c = -300\ \mu \tag{a}
$$

Determine (a) the in-plane principal strains and stresses and their directions, and (b) the true maximum shearing strain. The material properties are $E = 200$ GPa and $\nu = 0.3$.

Solution For the situation described, Eq. (2.34) provides three simultaneous expressions:

$$\varepsilon_a = \varepsilon_x$$

$$\varepsilon_b = \tfrac{1}{4}\varepsilon_x + \tfrac{3}{4}\varepsilon_y + \frac{\sqrt{3}}{4}\gamma_{xy}$$

$$\varepsilon_c = \tfrac{1}{4}\varepsilon_x + \tfrac{3}{4}\varepsilon_y - \frac{\sqrt{3}}{4}\gamma_{xy}$$

From these,

$$\varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \tfrac{1}{3}[2(\varepsilon_b + \varepsilon_c) - \varepsilon_a]$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c) \qquad \text{(b)}$$

Note that the relationships between ε_a, ε_b, and ε_c may be observed from a Mohr's circle construction corresponding to the state of strain ε_x, ε_y, and γ_{xy} at the point under consideration.

(a) Upon substituting numerical values, we obtain $\varepsilon_x = 190\ \mu$, $\varepsilon_y = -130\ \mu$, and $\gamma_{xy} = 577\ \mu$. Then, from Eq. (2.13), the principal strains are

$$\varepsilon_{1,2} = \frac{190 - 130}{2}\mu \pm \mu\left[\left(\frac{190 + 130}{2}\right)^2 + \left(\frac{577}{2}\right)^2\right]^{1/2}$$

$$= 30\ \mu \pm 330\ \mu$$

or

$$\varepsilon_1 = 360\ \mu, \qquad \varepsilon_2 = -300\ \mu \qquad \text{(c)}$$

The maximum shear strain is found from

$$\gamma_{\max} = \pm(\varepsilon_1 - \varepsilon_2) = \pm[360 - (-300)]\ \mu = \pm 660\ \mu$$

The orientations of the principal axes are given by Eq. (2.12):

$$2\theta_p = \tan^{-1}\frac{577}{320} = 61^\circ \quad \text{or} \quad \theta_p' = 30.5^\circ, \qquad \theta_p'' = 120.5^\circ \qquad \text{(d)}$$

When θ_p' is substituted into Eq. (2.11) together with Eq. (b), we obtain 360 μ. Therefore, 30.5° and 120.5° are the respective directions of ε_1 and ε_2, measured from the horizontal axis in a counterclockwise direction. The principal stresses may now be found from the generalized Hooke's law. Thus, the first two equations of (2.28) for plane stress, letting $\sigma_z = 0$, $\sigma_x = \sigma_1$, and $\sigma_y = \sigma_2$, together with Eqs. (c), yield

$$\sigma_1 = \frac{200 \times 10^9}{1 - 0.09}[360 + 0.3(-300)](10^{-6}) = 59.34 \text{ MPa}$$

$$\sigma_2 = \frac{200 \times 10^3}{091}[-300 + 0.3(360)] = -42.2 \text{ MPa}$$

The directions of σ_1 and σ_2 are given by Eq. (d). From Eq. (2.30), the maximum shear stress is

$$\tau_{\max} = \frac{200 \times 10^9}{2(1 + 0.3)} 660 \times 10^{-6} = 50.77 \text{ MPa}$$

Note as a check that $(\sigma_1 - \sigma_2)/2$ yields the same result.

(b) Applying Eq. (3.11b), the out-of-plane principal strain is

$$\varepsilon_z = -\frac{\nu}{1 - \nu}(\varepsilon_x + \varepsilon_y) = -\frac{0.3}{1 - 0.3}(190 - 130)\ \mu = -26\ \mu$$

The principal strain ε_2 found in part (a) is redesignated $\varepsilon_3 = -300\ \mu$ so that algebraically $\varepsilon_2 > \varepsilon_3$, where $\varepsilon_2 = -26\ \mu$. The *true maximum shearing strain*

$$(\gamma_{\max})_t = \pm(\varepsilon_1 - \varepsilon_3) \tag{2.35}$$

is therefore $\pm 660\ \mu$, as already calculated in part (a).

Employing a procedure similar to that used in the preceding numerical example, it is possible to develop expressions relating three-element gage outputs of various rosettes to principal strains and stresses. Table 2.1 provides

TABLE 2.1 Strain Rosette Equations

1. *Rectangular rosette or 45° strain rosette*

Principal strains:

$$\varepsilon_{1,2} = \frac{1}{2}\left[\varepsilon_a + \varepsilon_c \pm \sqrt{(\varepsilon_a - \varepsilon_c)^2 + (2\varepsilon_b - \varepsilon_a - \varepsilon_c)^2}\right] \tag{2.36a}$$

Principal stresses:

$$\sigma_{1,2} = \frac{E}{2}\left[\frac{\varepsilon_a + \varepsilon_c}{1 - \nu} \pm \frac{1}{1 + \nu}\sqrt{(\varepsilon_a - \varepsilon_c)^2 + (2\varepsilon_b - \varepsilon_a - \varepsilon_c)^2}\right] \tag{2.36b}$$

Directions of principal planes:

$$\tan 2\theta_p = \frac{2\varepsilon_b - \varepsilon_a - \varepsilon_c}{\varepsilon_a - \varepsilon_c} \tag{2.36c}$$

2. *Delta rosette or 60° strain rosette*

Principal strains:

$$\varepsilon_{1,2} = \frac{1}{3}\left[\varepsilon_a + \varepsilon_b + \varepsilon_c \pm \sqrt{2}\sqrt{(\varepsilon_a - \varepsilon_b)^2 + (\varepsilon_b - \varepsilon_c)^2 + (\varepsilon_c - \varepsilon_a)^2}\right] \tag{2.37a}$$

Principal stresses:

$$\sigma_{1,2} = \frac{E}{3}\left[\frac{\varepsilon_a + \varepsilon_b + \varepsilon_c}{1 - \nu} \pm \frac{\sqrt{2}}{1 + \nu}\sqrt{(\varepsilon_a - \varepsilon_b)^2 + (\varepsilon_b - \varepsilon_c)^2 + (\varepsilon_c - \varepsilon_a)^2}\right] \tag{2.37b}$$

Directions of principal planes:

$$\tan 2\theta_p = \frac{\sqrt{3}(\varepsilon_b - \varepsilon_c)}{2\varepsilon_a - \varepsilon_b - \varepsilon_c} \tag{2.37c}$$

two typical cases: equations for the *rectangular rosette* ($\theta_a = 0°$, $\theta_b = 45°$, and $\theta_c = 90°$) and the *delta rosette* ($\theta_a = 0°$, $\theta_b = 60°$, and $\theta_c = 120°$). Experimental stress analysis is facilitated by this kind of compilation.