14 Spur and Helical Gears

Chapter Outline

This chapter is devoted primarily to analysis and design of spur and helical gears to resist bending failure of the teeth as well as pitting failure of tooth surfaces. Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength. A surface failure occurs when the significant contact stress equals or exceeds the surface endurance strength. The first two sections present a little of the history of the analyses from which current methodology developed.

The American Gear Manufacturers Association[1] (AGMA) has for many years been the responsible authority for the dissemination of knowledge pertaining to the design and analysis of gearing. The methods this organization presents are in general use in the United States when strength and wear are primary considerations. In view of this fact it is important that the AGMA approach to the subject be presented here.

The general AGMA approach requires a great many charts and graphs—too many for a single chapter in this book. We have omitted many of these here by choosing a single pressure angle and by using only full-depth teeth. This simplification reduces the complexity but does not prevent the development of a basic understanding of the approach. Furthermore, the simplification makes possible a better development of the fundamentals and hence should constitute an ideal introduction to the use of the general AGMA method.[2] Sections 14–1 and 14–2 are elementary and serve as an examination of the foundations of the AGMA method. Table 14–1 is largely AGMA nomenclature.

14–1 The Lewis Bending Equation

Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth in which the tooth form entered into the formulation. The equation, announced in 1892, still remains the basis for most gear design today.

To derive the basic Lewis equation, refer to Fig. 14–1*a*, which shows a cantilever of cross-sectional dimensions F and t, having a length l and a load W^t, uniformly distributed across the face width F. The section modulus I/c is $Ft^2/6$, and therefore the bending stress is

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2} \tag{a}$$

Gear designers denote the components of gear-tooth forces as W_t, W_r, W_a or W^t, W^r, W^a interchangeably. The latter notation leaves room for post-subscripts essential to free-body diagrams. For instance, for gears 2 and 3 in mesh, W^t_{23} is the transmitted force of

[1]500 Montgomery Street, Suite 350, Alexandria, VA 22314-1560.

[2]The standards ANSI/AGMA 2001-D04 (revised AGMA 2001-C95) and ANSI/AGMA 2101-D04 (metric edition of ANSI/AGMA 2001-D04), *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* are used in this chapter. The use of American National Standards is completely voluntary; their existence does not in any respect preclude people, whether they have approved the standards or not, from manufacturing, marketing, purchasing, or using products, processes, or procedures not conforming to the standards.

The American National Standards Institute does not develop standards and will in no circumstances give an interpretation of any American National Standard. Requests for interpretation of these standards should be addressed to the American Gear Manufacturers Association. [Tables or other self-supporting sections may be quoted or extracted in their entirety. Credit line should read: "Extracted from ANSI/AGMA Standard 2001-D04 or 2101-D04 *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*" with the permission of the publisher, American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314-1560.] The foregoing is adapted in part from the ANSI foreword to these standards.

Table 14–1

Symbols, Their Names, and Locations*

Symbol	Name	Where Found
b	Net width of face of narrowest member	Eq. (14–16)
C_e	Mesh alignment correction factor	Eq. (14–35)
C_f	Surface condition factor	Eq. (14–16)
C_H	Hardness-ratio factor	Eq. (14–18)
C_{ma}	Mesh alignment factor	Eq. (14–34)
C_{mc}	Load correction factor	Eq. (14–31)
C_{mf}	Face load-distribution factor	Eq. (14–30)
C_p	Elastic coefficient	Eq. (14–13)
C_{pf}	Pinion proportion factor	Eq. (14–32)
C_{pm}	Pinion proportion modifier	Eq. (14–33)
d	Operating pitch diameter of pinion	Ex. (14–1)
d_P	Pitch diameter, pinion	Eq. (14–22)
d_G	Pitch diameter, gear	Eq. (14–22)
E	Modulus of elasticity	Eq. (14–10)
F	Net face width of narrowest member	Eq. (14–15)
f_P	Pinion surface finish	Fig. 14–13
H	Power	Fig. 14–17
H_B	Brinell hardness	Ex. 14–3
H_{BG}	Brinell hardness of gear	Sec. 14–12
H_{BP}	Brinell hardness of pinion	Sec. 14–12
hp	Horsepower	Ex. 14–1
h_t	Gear-tooth whole depth	Sec. 14–16
I	Geometry factor of pitting resistance	Eq. (14–16)
J	Geometry factor for bending strength	Eq. (14–15)
K	Contact load factor for pitting resistance	Eq. (6–65)
K_B	Rim-thickness factor	Eq. (14–40)
K_f	Fatigue stress-concentration factor	Eq. (14–9)
K_m	Load-distribution factor	Eq. (14–30)
K_o	Overload factor	Eq. (14–15)
K_R	Reliability factor	Eq. (14–17)
K_s	Size factor	Sec. 14–10
K_T	Temperature factor	Eq. (14–17)
K_v	Dynamic factor	Eq. (14–27)
m	Metric module	Eq. (14–15)
m_B	Backup ratio	Eq. (14–39)
m_G	Gear ratio (never less than 1)	Eq. (14–22)
m_N	Load-sharing ratio	Eq. (14–21)
N	Number of stress cycles	Fig. 14–14
N_G	Number of teeth on gear	Eq. (14–22)
N_P	Number of teeth on pinion	Eq. (14–22)
n	Speed	Ex. 14–1

(*Continued*)

Table 14–1

Symbols, Their Names, and Locations* (*Continued*)

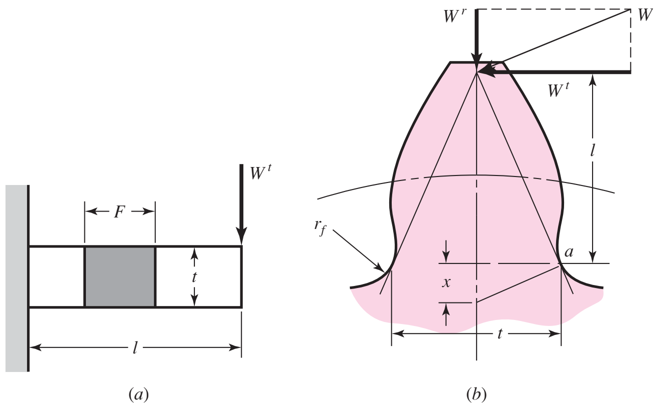
Symbol	Name	Where Found
n_P	Pinion speed	Ex. 14–4
P	Diametral pitch	Eq. (14–2)
P_d	Diametral pitch of pinion	Eq. (14–15)
p_N	Normal base pitch	Eq. (14–24)
p_n	Normal circular pitch	Eq. (14–24)
p_x	Axial pitch	Eq. (14–19)
Q_v	Transmission accuracy level number	Eq. (14–29)
R	Reliability	Eq. (14–38)
R_a	Root-mean-squared roughness	Fig. 14–13
r_f	Tooth fillet radius	Fig. 14–1
r_G	Pitch-circle radius, gear	In standard
r_P	Pitch-circle radius, pinion	In standard
r_{bP}	Pinion base-circle radius	Eq. (14–25)
r_{bG}	Gear base-circle radius	Eq. (14–25)
S_C	Buckingham surface endurance strength	Ex. 14–3
S_c	AGMA surface endurance strength	Eq. (14–18)
S_t	AGMA bending strength	Eq. (14–17)
S	Bearing span	Fig. 14–10
S_1	Pinion offset from center span	Fig. 14–10
S_F	Safety factor—bending	Eq. (14–41)
S_H	Safety factor—pitting	Eq. (14–42)
W^t or $W_t^{\dagger}$	Transmitted load	Fig. 14–1
Y_N	Stress cycle factor for bending strength	Fig. 14–14
Z_N	Stress cycle factor for pitting resistance	Fig. 14–15
β	Exponent	Eq. (14–44)
σ	Bending stress	Eq. (14–2)
σ_C	Contact stress from Hertzian relationships	Eq. (14–14)
σ_c	Contact stress from AGMA relationships	Eq. (14–16)
σ_{all}	Allowable bending stress	Eq. (14–17)
$\sigma_{c,all}$	Allowable contact stress, AGMA	Eq. (14–18)
ϕ	Pressure angle	Eq. (14–12)
ϕ_t	Transverse pressure angle	Eq. (14–23)
ψ	Helix angle at standard pitch diameter	Ex. 14–5

*Because ANSI/AGMA 2001-C95 introduced a significant amount of new nomenclature, and continued in ANSI/AGMA 2001-D04, this summary and references are provided for use until the reader's vocabulary has grown.

†See preference rationale following Eq. (*a*), Sec. 14–1.

body 2 on body 3, and W_{32}^t is the transmitted force of body 3 on body 2. When working with double- or triple-reduction speed reducers, this notation is compact and essential to clear thinking. Since gear-force components rarely take exponents, this causes no complication. Pythagorean combinations, if necessary, can be treated with parentheses or avoided by expressing the relations trigonometrically.

Figure 14–1



Referring now to Fig. 14–1*b*, we assume that the maximum stress in a gear tooth occurs at point *a*. By similar triangles, you can write

$$\frac{t/2}{x} = \frac{l}{t/2} \qquad \text{or} \qquad x = \frac{t^2}{4l} \tag{b}$$

By rearranging Eq. (*a*),

$$\sigma = \frac{6W^t l}{Ft^2} = \frac{W^t}{F}\frac{1}{t^2/6l} = \frac{W^t}{F}\frac{1}{t^2/4l}\frac{1}{\frac{4}{6}} \tag{c}$$

If we now substitute the value of x from Eq. (*b*) in Eq. (*c*) and multiply the numerator and denominator by the circular pitch p, we find

$$\sigma = \frac{W^t p}{F\left(\frac{2}{3}\right)xp} \tag{d}$$

Letting $y = 2x/3p$, we have

$$\sigma = \frac{W^t}{Fpy} \tag{14–1}$$

This completes the development of the original Lewis equation. The factor y is called the *Lewis form factor,* and it may be obtained by a graphical layout of the gear tooth or by digital computation.

In using this equation, most engineers prefer to employ the diametral pitch in determining the stresses. This is done by substituting $P = \pi/p$ and $Y = \pi y$ in Eq. (14–1). This gives

$$\sigma = \frac{W^t P}{FY} \tag{14–2}$$

where

$$Y = \frac{2xP}{3} \tag{14–3}$$

The use of this equation for Y means that only the bending of the tooth is considered and that the compression due to the radial component of the force is neglected. Values of Y obtained from this equation are tabulated in Table 14–2.

Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

The use of Eq. (14–3) also implies that the teeth do not share the load and that the greatest force is exerted at the tip of the tooth. But we have already learned that the contact ratio should be somewhat greater than unity, say about 1.5, to achieve a quality gearset. If, in fact, the gears are cut with sufficient accuracy, the tip-load condition is not the worst, because another pair of teeth will be in contact when this condition occurs. Examination of run-in teeth will show that the heaviest loads occur near the middle of the tooth. Therefore the maximum stress probably occurs while a single pair of teeth is carrying the full load, at a point where another pair of teeth is just on the verge of coming into contact.

Dynamic Effects

When a pair of gears is driven at moderate or high speed and noise is generated, it is certain that dynamic effects are present. One of the earliest efforts to account for an increase in the load due to velocity employed a number of gears of the same size, material, and strength. Several of these gears were tested to destruction by meshing and loading them at zero velocity. The remaining gears were tested to destruction at various pitch-line velocities. For example, if a pair of gears failed at 500 lbf tangential load at zero velocity and at 250 lbf at velocity V_1, then a *velocity factor,* designated K_v, of 2 was specified for the gears at velocity V_1. Then another, identical, pair of gears running at a pitch-line velocity V_1 could be assumed to have a load equal to twice the tangential or transmitted load.

Note that the definition of dynamic factor K_v has been altered. AGMA standards ANSI/AGMA 2001-D04 and 2101-D04 contain this caution:

> Dynamic factor K_v has been redefined as the reciprocal of that used in previous AGMA standards. It is now greater than 1.0. In earlier AGMA standards it was less than 1.0.

Care must be taken in referring to work done prior to this change in the standards.

In the nineteenth century, Carl G. Barth first expressed the velocity factor, and in terms of the current AGMA standards, they are represented as

$$K_v = \frac{600 + V}{600} \qquad \text{(cast iron, cast profile)} \tag{14–4a}$$

$$K_v = \frac{1200 + V}{1200} \qquad \text{(cut or milled profile)} \tag{14–4b}$$

where V is the pitch-line velocity in feet per minute. It is also quite probable, because of the date that the tests were made, that the tests were conducted on teeth having a cycloidal profile instead of an involute profile. Cycloidal teeth were in general use in the nineteenth century because they were easier to cast than involute teeth. Equation (14–4*a*) is called the *Barth equation*. The Barth equation is often modified into Eq. (14–4*b*), for cut or milled teeth. Later AGMA added

$$K_v = \frac{50 + \sqrt{V}}{50} \qquad \text{(hobbed or shaped profile)} \tag{14–5a}$$

$$K_v = \sqrt{\frac{78 + \sqrt{V}}{78}} \qquad \text{(shaved or ground profile)} \tag{14–5b}$$

In SI units, Eqs. (14–4*a*) through (14–5*b*) become

$$K_v = \frac{3.05 + V}{3.05} \qquad \text{(cast iron, cast profile)} \tag{14–6a}$$

$$K_v = \frac{6.1 + V}{6.1} \qquad \text{(cut or milled profile)} \tag{14–6b}$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \qquad \text{(hobbed or shaped profile)} \tag{14–6c}$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \qquad \text{(shaved or ground profile)} \tag{14–6d}$$

where V is in meters per second (m/s).

Introducing the velocity factor into Eq. (14–2) gives

$$\sigma = \frac{K_v W^t P}{FY} \tag{14–7}$$

The metric version of this equation is

$$\sigma = \frac{K_v W^t}{FmY} \tag{14–8}$$

where the face width F and the module m are both in millimeters (mm). Expressing the tangential component of load W^t in newtons (N) then results in stress units of megapascals (MPa).

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p.

Equations (14–7) and (14–8) are important because they form the basis for the AGMA approach to the bending strength of gear teeth. They are in general use for estimating the capacity of gear drives when life and reliability are not important considerations. The equations can be useful in obtaining a preliminary estimate of gear sizes needed for various applications.

EXAMPLE 14–1

A stock spur gear is available having a diametral pitch of 8 teeth/in, a $1\frac{1}{2}$-in face, 16 teeth, and a pressure angle of 20° with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of $n_d = 3$ to rate the horsepower output of the gear corresponding to a speed of 1200 rev/m and moderate applications.

Solution

The term *moderate applications* seems to imply that the gear can be rated by using the yield strength as a criterion of failure. From Table A–20, we find $S_{ut} = 55$ kpsi and $S_y = 30$ kpsi. A design factor of 3 means that the allowable bending stress is $30/3 = 10$ kpsi. The pitch diameter is $N/P = 16/8 = 2$ in, so the pitch-line velocity is

$$V = \frac{\pi d n}{12} = \frac{\pi(2)1200}{12} = 628 \text{ ft/min}$$

The velocity factor from Eq. (14–4*b*) is found to be

$$K_v = \frac{1200 + V}{1200} = \frac{1200 + 628}{1200} = 1.52$$

Table 14–2 gives the form factor as $Y = 0.296$ for 16 teeth. We now arrange and substitute in Eq. (14–7) as follows:

$$W^t = \frac{FY\sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)10\,000}{1.52(8)} = 365 \text{ lbf}$$

The horsepower that can be transmitted is

Answer

$$hp = \frac{W^t V}{33\,000} = \frac{365(628)}{33\,000} = 6.95 \text{ hp}$$

It is important to emphasize that this is a rough estimate, and that this approach must not be used for important applications. The example is intended to help you understand some of the fundamentals that will be involved in the AGMA approach.

EXAMPLE 14–2

Estimate the horsepower rating of the gear in the previous example based on obtaining an infinite life in bending.

Solution

The rotating-beam endurance limit is estimated from Eq. (6–8)

$$S'_e = 0.5S_{ut} = 0.5(55) = 27.5 \text{ kpsi}$$

To obtain the surface finish Marin factor k_a we refer to Table 6–3 for machined surface, finding $a = 2.70$ and $b = -0.265$. Then Eq. (6–19) gives the surface finish Marin factor k_a as

$$k_a = aS_{ut}^b = 2.70(55)^{-0.265} = 0.934$$

The next step is to estimate the size factor k_b. From Table 13–1, the sum of the addendum and dedendum is

$$l = \frac{1}{P} + \frac{1.25}{P} = \frac{1}{8} + \frac{1.25}{8} = 0.281 \text{ in}$$

The tooth thickness t in Fig. 14–1b is given in Sec. 14–1 [Eq. (b)] as $t = (4lx)^{1/2}$ when $x = 3Y/(2P)$ from Eq. (14–3). Therefore, since from Ex. 14–1 $Y = 0.296$ and $P = 8$,

$$x = \frac{3Y}{2P} = \frac{3(0.296)}{2(8)} = 0.0555 \text{ in}$$

then

$$t = (4lx)^{1/2} = [4(0.281)0.0555]^{1/2} = 0.250 \text{ in}$$

We have recognized the tooth as a cantilever beam of rectangular cross section, so the equivalent rotating-beam diameter must be obtained from Eq. (6–25):

$$d_e = 0.808(hb)^{1/2} = 0.808(Ft)^{1/2} = 0.808[1.5(0.250)]^{1/2} = 0.495 \text{ in}$$

Then, Eq. (6–20) gives k_b as

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.495}{0.30}\right)^{-0.107} = 0.948$$

The load factor k_c from Eq. (6–26) is unity. With no information given concerning temperature and reliability we will set $k_d = k_e = 1$.

Two effects are used to evaluate the miscellaneous-effects Marin factor k_f. The first of these is the effect of one-way bending. In general, a gear tooth is subjected only to one-way bending. Exceptions include idler gears and gears used in reversing mechanisms.

For one-way bending the steady and alternating stress components are $\sigma_a = \sigma_m = \sigma/2$ where σ is the largest repeatedly applied bending stress as given in Eq. (14–7). If a material exhibited a Goodman failure locus,

$$\frac{S_a}{S'_e} + \frac{S_m}{S_{ut}} = 1$$

Since S_a and S_m are equal for one-way bending, we substitute S_a for S_m and solve the preceding equation for S_a, giving

$$S_a = \frac{S'_e S_{ut}}{S'_e + S_{ut}}$$

Now replace S_a with $\sigma/2$, and in the denominator replace S'_e with $0.5S_{ut}$ to obtain

$$\sigma = \frac{2S'_e S_{ut}}{0.5S_{ut} + S_{ut}} = \frac{2S'_e}{0.5 + 1} = 1.33S'_e$$

Now $k_f = \sigma/S'_e = 1.33S'_e/S'_e = 1.33$. However, a Gerber fatigue locus gives mean values of

$$\frac{S_a}{S'_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Setting $S_a = S_m$ and solving the quadratic in S_a gives

$$S_a = \frac{S_{ut}^2}{2S_e'}\left(-1 + \sqrt{1 + \frac{4S_e'^2}{S_{ut}^2}}\right)$$

Setting $S_a = \sigma/2$, $S_{ut} = S_e'/0.5$ gives

$$\sigma = \frac{S_e'}{0.5^2}\left[-1 + \sqrt{1 + 4(0.5)^2}\right] = 1.66S_e'$$

and $k_f = \sigma/S_e' = 1.66$. Since a Gerber locus runs in and among fatigue data and Goodman does not, we will use $k_f = 1.66$.

The second effect to be accounted for in using the miscellaneous-effects Marin factor k_f is stress concentration, for which we will use our fundamentals from Chap. 6. For a 20° full-depth tooth the radius of the root fillet is denoted r_f, where

$$r_f = \frac{0.300}{P} = \frac{0.300}{8} = 0.0375 \text{ in}$$

From Fig. A–15–6

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.0375}{0.250} = 0.15$$

Since $D/d = \infty$, we approximate with $D/d = 3$, giving $K_t = 1.68$. From Fig. 6–20, $q = 0.62$. From Eq. (6–32)

$$K_f = 1 + (0.62)(1.68 - 1) = 1.42$$

The miscellaneous-effects Marin factor for stress concentration can be expressed as

$$k_f = \frac{1}{K_f} = \frac{1}{1.42} = 0.704$$

The final value of k_f is the product of the two k_f factors, that is, 1.66(0.704) = 1.17. The Marin equation for the fully corrected endurance strength is

$$S_e = k_a k_b k_c k_d k_e k_f S_e'$$
$$= 0.934(0.948)(1)(1)(1)1.17(27.5) = 28.5 \text{ kpsi}$$

For a design factor of $n_d = 3$, as used in Ex. 14–1, applied to the load or strength, the allowable bending stress is

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{28.5}{3} = 9.5 \text{ kpsi}$$

The transmitted load W^t is

$$W^t = \frac{FY\sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)9\,500}{1.52(8)} = 347 \text{ lbf}$$

and the power is, with $V = 628$ ft/min from Ex. 14–1,

$$hp = \frac{W^t V}{33\,000} = \frac{347(628)}{33\,000} = 6.6 \text{ hp}$$

Again, it should be emphasized that these results should be accepted *only* as preliminary estimates to alert you to the nature of bending in gear teeth.

In Ex. 14–2 our resources (Fig. A–15–6) did not directly address stress concentration in gear teeth. A photoelastic investigation by Dolan and Broghamer reported in 1942 constitutes a primary source of information on stress concentration.[3] Mitchiner and Mabie[4] interpret the results in term of fatigue stress-concentration factor K_f as

$$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M \qquad (14\text{–}9)$$

where
$$H = 0.34 - 0.458\,366\,2\phi$$
$$L = 0.316 - 0.458\,366\,2\phi$$
$$M = 0.290 + 0.458\,366\,2\phi$$
$$r = \frac{(b - r_f)^2}{(d/2) + b - r_f}$$

In these equations l and t are from the layout in Fig. 14–1, ϕ is the pressure angle, r_f is the fillet radius, b is the dedendum, and d is the pitch diameter. It is left as an exercise for the reader to compare K_f from Eq. (14–9) with the results of using the approximation of Fig. A–15–6 in Ex. 14–2.

14–2 Surface Durability

In this section we are interested in the failure of the surfaces of gear teeth, which is generally called *wear*. *Pitting,* as explained in Sec. 6–16, is a surface fatigue failure due to many repetitions of high contact stresses. Other surface failures are *scoring,* which is a lubrication failure, and *abrasion,* which is wear due to the presence of foreign material.

To obtain an expression for the surface-contact stress, we shall employ the Hertz theory. In Eq. (3–74) it was shown that the contact stress between two cylinders may be computed from the equation

$$p_{max} = \frac{2F}{\pi b l} \qquad (a)$$

where
p_{max} = largest surface pressure
F = force pressing the two cylinders together
l = length of cylinders

and half-width b is obtained from Eq. (3–73):

$$b = \left\{ \frac{2F}{\pi l} \frac{\left[(1-\nu_1^2)/E_1\right] + \left[(1-\nu_2^2)/E_2\right]}{(1/d_1) + (1/d_2)} \right\}^{1/2} \qquad (14\text{–}10)$$

where ν_1, ν_2, E_1, and E_2 are the elastic constants and d_1 and d_2 are the diameters, respectively, of the two contacting cylinders.

To adapt these relations to the notation used in gearing, we replace F by $W^t/\cos\phi$, d by $2r$, and l by the face width F. With these changes, we can substitute the value of b

[3]T. J. Dolan and E. I. Broghamer, *A Photoelastic Study of the Stresses in Gear Tooth Fillets,* Bulletin 335, Univ. Ill. Exp. Sta., March 1942, See also W. D. Pilkey, *Peterson's Stress Concentration Factors,* 2nd ed., John Wiley & Sons, New York, 1997, pp. 383–385, 412–415.

[4]R. G. Mitchiner and H. H. Mabie, "Determination of the Lewis Form Factor and the AGMA Geometry Factor J of External Spur Gear Teeth," *J. Mech. Des.,* Vol. 104, No. 1, Jan. 1982, pp. 148–158.

as given by Eq. (14–10) in Eq. (*a*). Replacing $p_{\max}$ by σ_C, the *surface compressive stress (Hertzian stress)* is found from the equation

$$\sigma_C^2 = \frac{W^t}{\pi F \cos\phi} \frac{(1/r_1) + (1/r_2)}{\left[\left(1-\nu_1^2\right)/E_1\right] + \left[\left(1-\nu_2^2\right)/E_2\right]} \tag{14–11}$$

where r_1 and r_2 are the instantaneous values of the radii of curvature on the pinion- and gear-tooth profiles, respectively, at the point of contact. By accounting for load sharing in the value of W^t used, Eq. (14–11) can be solved for the Hertzian stress for any or all points from the beginning to the end of tooth contact. Of course, pure rolling exists only at the pitch point. Elsewhere the motion is a mixture of rolling and sliding. Equation (14–11) does not account for any sliding action in the evaluation of stress. We note that AGMA uses μ for Poisson's ratio instead of ν as is used here.

We have already noted that the first evidence of wear occurs near the pitch line. The radii of curvature of the tooth profiles at the pitch point are

$$r_1 = \frac{d_P \sin\phi}{2} \qquad r_2 = \frac{d_G \sin\phi}{2} \tag{14–12}$$

where ϕ is the pressure angle and d_P and d_G are the pitch diameters of the pinion and gear, respectively.

Note, in Eq. (14–11), that the denominator of the second group of terms contains four elastic constants, two for the pinion and two for the gear. As a simple means of combining and tabulating the results for various combinations of pinion and gear materials, AGMA defines an *elastic coefficient* C_p by the equation

$$C_p = \left[\frac{1}{\pi\left(\dfrac{1-\nu_P^2}{E_P} + \dfrac{1-\nu_G^2}{E_G}\right)}\right]^{1/2} \tag{14–13}$$

With this simplification, and the addition of a velocity factor K_v, Eq. (14–11) can be written as

$$\sigma_C = -C_p\left[\frac{K_v W^t}{F\cos\phi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2} \tag{14–14}$$

where the sign is negative because σ_C is a compressive stress.

EXAMPLE 14–3

The pinion of Examples 14–1 and 14–2 is to be mated with a 50-tooth gear manufactured of ASTM No. 50 cast iron. Using the tangential load of 382 lbf, estimate the factor of safety of the drive based on the possibility of a surface fatigue failure.

Solution

From Table A–5 we find the elastic constants to be $E_P = 30$ Mpsi, $\nu_P = 0.292$, $E_G = 14.5$ Mpsi, $\nu_G = 0.211$. We substitute these in Eq. (14–13) to get the elastic coefficient as

$$C_p = \left\{\frac{1}{\pi\left[\dfrac{1-(0.292)^2}{30(10^6)} + \dfrac{1-(0.211)^2}{14.5(10^6)}\right]}\right\}^{1/2} = 1817$$

From Example 14–1, the pinion pitch diameter is $d_P = 2$ in. The value for the gear is $d_G = 50/8 = 6.25$ in. Then Eq. (14–12) is used to obtain the radii of curvature at the pitch points. Thus

$$r_1 = \frac{2\sin 20^\circ}{2} = 0.342 \text{ in} \qquad r_2 = \frac{6.25\sin 20^\circ}{2} = 1.069 \text{ in}$$

The face width is given as $F = 1.5$ in. Use $K_v = 1.52$ from Example 14–1. Substituting all these values in Eq. (14–14) with $\phi = 20^\circ$ gives the contact stress as

$$\sigma_C = -1817\left[\frac{1.52(380)}{1.5\cos 20^\circ}\left(\frac{1}{0.342}+\frac{1}{1.069}\right)\right]^{1/2} = -72\,400 \text{ psi}$$

The surface endurance strength of cast iron can be estimated from

$$S_C = 0.32 H_B \text{ kpsi}$$

for 10^8 cycles, where S_C is in kpsi. Table A–24 gives $H_B = 262$ for ASTM No. 50 cast iron. Therefore $S_C = 0.32(262) = 83.8$ kpsi. Contact stress is not linear with transmitted load [see Eq. (14–14)]. If the factor of safety is defined as the loss-of-function load divided by the imposed load, then the ratio of loads is the ratio of stresses squared. In other words,

$$n = \frac{\text{loss-of-function load}}{\text{imposed load}} = \frac{S_C^2}{\sigma_C^2} = \left(\frac{83.8}{72.4}\right)^2 = 1.34$$

One is free to define factor of safety as S_C/σ_C. Awkwardness comes when one compares the factor of safety in bending fatigue with the factor of safety in surface fatigue for a particular gear. Suppose the factor of safety of this gear in bending fatigue is 1.20 and the factor of safety in surface fatigue is 1.34 as above. The threat, since 1.34 is greater than 1.20, is in bending fatigue since both numbers are based on load ratios. If the factor of safety in surface fatigue is based on $S_C/\sigma_C = \sqrt{1.34} = 1.16$, then 1.20 is greater than 1.16, but the threat is not from surface fatigue. The surface fatigue factor of safety can be defined either way. One way has the burden of requiring a squared number before numbers that instinctively seem comparable can be compared.

In addition to the dynamic factor K_v already introduced, there are transmitted load excursions, nonuniform distribution of the transmitted load over the tooth contact, and the influence of rim thickness on bending stress. Tabulated strength values can be means, ASTM minimums, or of unknown heritage. In surface fatigue there are no endurance limits. Endurance strengths have to be qualified as to corresponding cycle count, and the slope of the *S-N* curve needs to be known. In bending fatigue there is a definite change in slope of the *S-N* curve near 10^6 cycles, but some evidence indicates that an endurance limit does not exist. Gearing experience leads to cycle counts of 10^{11} or more. Evidence of diminishing endurance strengths in bending have been included in AGMA methodology.

14–3 AGMA Stress Equations

Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress). In AGMA terminology, these are called *stress numbers,* as contrasted with actual applied stresses, and are designated by a lowercase letter s instead of the Greek lower case σ we have used in this book (and shall continue to use). The fundamental equations are

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \tag{14–15}$$

where for U.S. customary units (SI units),

W^t is the tangential transmitted load, lbf (N)
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
P_d is the transverse diametral pitch
F (b) is the face width of the narrower member, in (mm)
K_m (K_H) is the load-distribution factor
K_B is the rim-thickness factor
J (Y_J) is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
(m_t) is the transverse metric module

Before you try to digest the meaning of all these terms in Eq. (14–15), view them as advice concerning items the designer should consider *whether he or she follows the voluntary standard or not*. These items include issues such as

- Transmitted load magnitude
- Overload
- Dynamic augmentation of transmitted load
- Size
- Geometry: pitch and face width
- Distribution of load across the teeth
- Rim support of the tooth
- Lewis form factor and root fillet stress concentration

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \tag{14–16}$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion,* in (mm)
I (Z_I) is the geometry factor for pitting resistance

The evaluation of all these factors is explained in the sections that follow. The development of Eq. (14–16) is clarified in the second part of Sec. 14–5.

14–4 AGMA Strength Equations

Instead of using the term *strength,* AGMA uses data termed *allowable stress numbers* and designates these by the symbols s_{at} and s_{ac}. It will be less confusing here if we continue the practice in this book of using the uppercase letter S to designate strength and the lowercase Greek letters σ and τ for stress. To make it perfectly clear we shall use the term *gear strength* as a replacement for the phrase *allowable stress numbers* as used by AGMA.

Following this convention, values for *gear bending strength,* designated here as S_t, are to be found in Figs. 14–2, 14–3, and 14–4, and in Tables 14–3 and 14–4. Since gear strengths are not identified with other strengths such as S_{ut}, S_e, or S_y as used elsewhere in this book, their use should be restricted to gear problems.

In this approach the strengths are modified by various factors that produce limiting values of the bending stress and the contact stress.

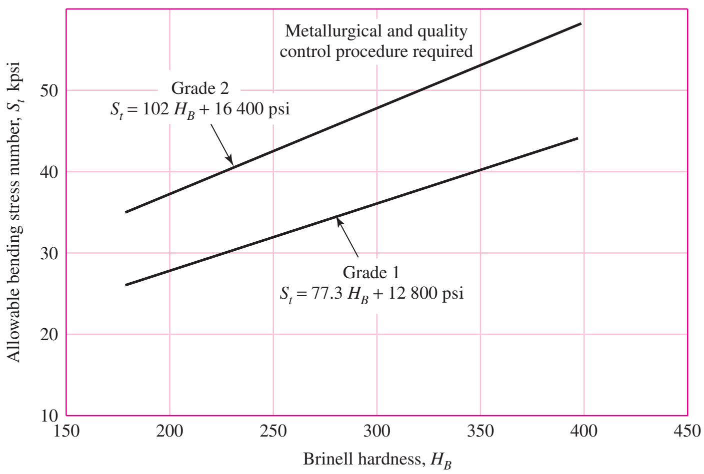


Figure 14–2

Allowable bending stress number for through-hardened steels. The SI equations are $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

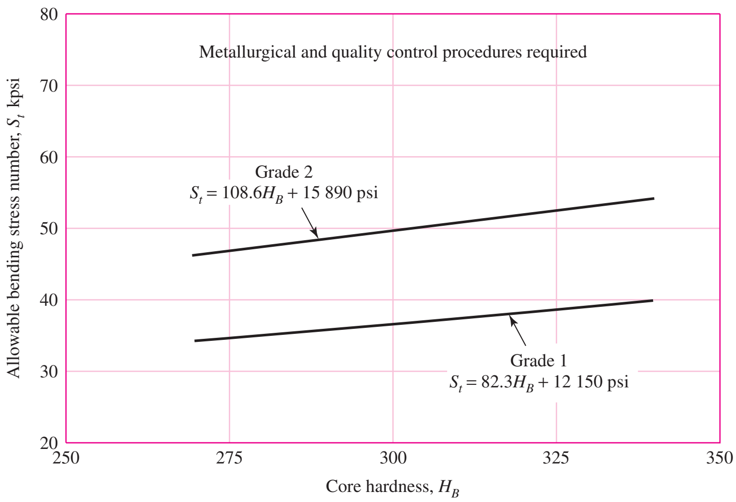


Figure 14–3

Allowable bending stress number for nitrided through-hardened steel gears (i.e., AISI 4140, 4340), S_t. The SI equations are $S_t = 0.568H_B + 83.8$ MPa, grade 1, and $S_t = 0.749H_B + 110$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

Figure 14–4

Allowable bending stress numbers for nitriding steel gears S_t. The SI equations are $S_t = 0.594H_B + 87.76$ MPa Nitralloy grade 1 $S_t = 0.784H_B + 114.81$ MPa Nitralloy grade 2 $S_t = 0.7255H_B + 63.89$ MPa 2.5% chrome, grade 1 $S_t = 0.7255H_B + 153.63$ MPa 2.5% chrome, grade 2 $S_t = 0.7255H_B + 201.91$ MPa 2.5% chrome, grade 3 *(Source: ANSI/AGMA 2001-D04, 2101-D04.)*

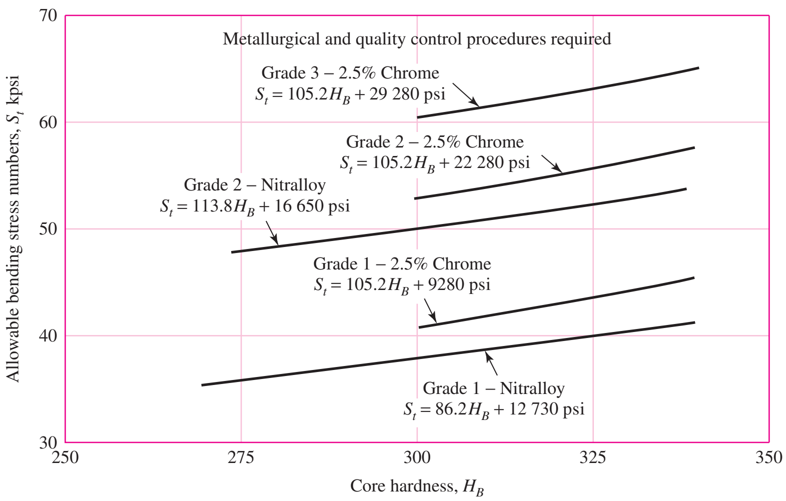


Table 14–3

Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears
Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000	—
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]	75 000
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1–7.
[1]Hardness to be equivalent to that at the root diameter in the center of the tooth space and face width.
[2]See tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.
[3]The steel selected must be compatible with the heat treatment process selected and hardness required.
[4]The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.
[5]See figure 12 for type A and type B hardness patterns.
[6]If bainite and microcracks are limited to grade 3 levels, 70,000 psi may be used.
[7]The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design. [7]
*Tables 8 and 9 of ANSI/AGMA 2001-D04 are comprehensive tabulations of the major metallurgical factors affecting S_t and S_c of flame-hardened and induction-hardened (Table 8) and carburized and hardened (Table 9) steel gears.

Table 14–4

Repeatedly Applied Bending Strength S_t for Iron and Bronze Gears at 10^7 Cycles and 0.99 Reliability

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Bending Stress Number, S_t,[3] psi
ASTM A48 gray cast iron	Class 20	As cast	—	5000
	Class 30	As cast	174 HB	8500
	Class 40	As cast	201 HB	13 000
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000
	Grade 120–90–02	Quenched and tempered	269 HB	31 000–44 000
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700
	ASTM B–148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600

Notes:

[1]See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual.*

[2]Measured hardness to be equivalent to that which would be measured at the root diameter in the center of the tooth space and face width.

[3]The lower values should be used for general design purposes. The upper values may be used when:
High quality material is used.
Section size and design allow maximum response to heat treatment.
Proper quality control is effected by adequate inspection.
Operating experience justifies their use.

The equation for the allowable bending stress is

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–17}$$

where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

The equation for the allowable contact stress $\sigma_{c,\text{all}}$ is

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–18}$$

where the upper equation is in U.S. customary units and the lower equation is in SI units, Also,

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
Z_N is the stress cycle life factor
C_H (Z_W) are the hardness ratio factors for pitting resistance
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_H is the AGMA factor of safety, a stress ratio

The values for the allowable contact stress, designated here as S_c, are to be found in Fig. 14–5 and Tables 14–5, 14–6, and 14–7.

AGMA allowable stress numbers (strengths) for bending and contact stress are for

- Unidirectional loading
- 10 million stress cycles
- 99 percent reliability

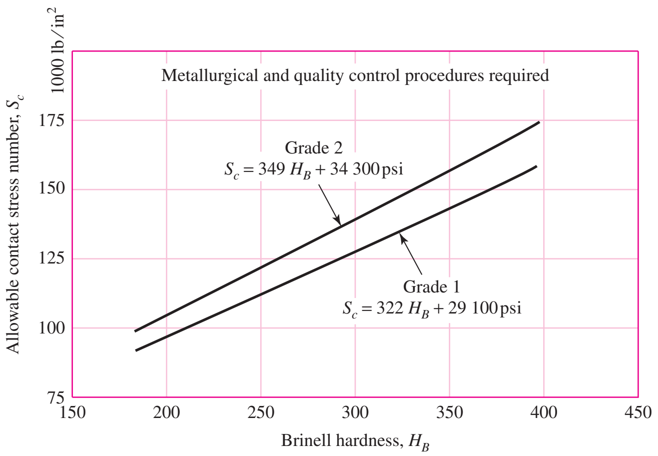


Figure 14–5

Contact-fatigue strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are $S_c = 2.22H_B + 200$ MPa, grade 1, and $S_c = 2.41H_B + 237$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

Table 14–5

Nominal Temperature Used in Nitriding and Hardnesses Obtained

Source: Darle W. Dudley, *Handbook of Practical Gear Design,* rev. ed., McGraw-Hill, New York, 1984.

Steel	Temperature before nitriding, °F	Nitriding, °F	Hardness, Rockwell C Scale Case	Hardness, Rockwell C Scale Core
Nitralloy 135*	1150	975	62–65	30–35
Nitralloy 135M	1150	975	62–65	32–36
Nitralloy N	1000	975	62–65	40–44
AISI 4340	1100	975	48–53	27–35
AISI 4140	1100	975	49–54	27–35
31 Cr Mo V 9	1100	975	58–62	27–33

*Nitralloy is a trademark of the Nitralloy Corp., New York.

Table 14–6

Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi		
			Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1–5.
[1]Hardness to be equivalent to that at the start of active profile in the center of the face width.
[2]See Tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.
[3]The steel selected must be compatible with the heat treatment process selected and hardness required.
[4]These materials must be annealed or normalized as a minimum.
[5]The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.
*Table 9 of ANSI/AGMA 2001-D04 is a comprehensive tabulation of the major metallurgical factors affecting S_t and S_c of carburized and hardened steel gears.

The factors in this section, too, will be evaluated in subsequent sections.

When two-way (reversed) loading occurs, as with idler gears, AGMA recommends using 70 percent of S_t values. This is equivalent to $1/0.70 = 1.43$ as a value of k_e in Ex. 14–2. The recommendation falls between the value of $k_e = 1.33$ for a Goodman failure locus and $k_e = 1.66$ for a Gerber failure locus.

14–5 Geometry Factors I and J (Z_I and Y_J)

We have seen how the factor Y is used in the Lewis equation to introduce the effect of tooth form into the stress equation. The AGMA factors[5] I and J are intended to accomplish the same purpose in a more involved manner.

The determination of I and J depends upon the *face-contact ratio* m_F. This is defined as

$$m_F = \frac{F}{p_x} \tag{14–19}$$

where p_x is the axial pitch and F is the face width. For spur gears, $m_F = 0$.

[5]A useful reference is AGMA 908-B89, *Geometry Factors for Determining Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth.*

Table 14–7

Repeatedly Applied Contact Strength S_c 10^7 Cycles and 0.99 Reliability for Iron and Bronze Gears

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Contact Stress Number,[3] S_c, psi
ASTM A48 gray cast iron	Class 20	As cast	—	50 000–60 000
	Class 30	As cast	174 HB	65 000–75 000
	Class 40	As cast	201 HB	75 000–85 000
ASTM A536 ductile (nodular) iron	Grade 60–40–18	Annealed	140 HB	77 000–92 000
	Grade 80–55–06	Quenched and tempered	179 HB	77 000–92 000
	Grade 100–70–03	Quenched and tempered	229 HB	92 000–112 000
	Grade 120–90–02	Quenched and tempered	269 HB	103 000–126 000
Bronze	—	Sand cast	Minimum tensile strength 40 000 psi	30 000
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	65 000

Notes:

[1]See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual.*

[2]Hardness to be equivalent to that at the start of active profile in the center of the face width.

[3]The lower values should be used for general design purposes. The upper values may be used when:
High-quality material is used.
Section size and design allow maximum response to heat treatment.
Proper quality control is effected by adequate inspection.
Operating experience justifies their use.

Low-contact-ratio (LCR) helical gears having a small helix angle or a thin face width, or both, have face-contact ratios less than unity ($m_F \le 1$), and will not be considered here. Such gears have a noise level not too different from that for spur gears. Consequently we shall consider here only spur gears with $m_F = 0$ and conventional helical gears with $m_F > 1$.

Bending-Strength Geometry Factor J (Y_J)

The AGMA factor J employs a modified value of the Lewis form factor, also denoted by Y; a *fatigue stress-concentration factor* K_f; and a tooth *load-sharing ratio* m_N. The resulting equation for J for spur and helical gears is

$$J = \frac{Y}{K_f m_N} \tag{14–20}$$

It is important to note that the form factor Y in Eq. (14–20) is *not* the Lewis factor at all. The value of Y here is obtained from calculations within AGMA 908-B89, and is often based on the highest point of single-tooth contact.

The factor K_f in Eq. (14–20) is called a *stress correction factor* by AGMA. It is based on a formula deduced from a photoelastic investigation of stress concentration in gear teeth over 50 years ago.

The load-sharing ratio m_N is equal to the face width divided by the minimum total length of the lines of contact. This factor depends on the transverse contact ratio m_p, the face-contact ratio m_F, the effects of any profile modifications, and the tooth deflection. For spur gears, $m_N = 1.0$. For helical gears having a face-contact ratio $m_F > 2.0$, a conservative approximation is given by the equation

$$m_N = \frac{p_N}{0.95Z} \tag{14–21}$$

where p_N is the normal base pitch and Z is the length of the line of action in the transverse plane (distance L_{ab} in Fig. 13–15).

Use Fig. 14–6 to obtain the geometry factor J for spur gears having a 20° pressure angle and full-depth teeth. Use Figs. 14–7 and 14–8 for helical gears having a 20° normal pressure angle and face-contact ratios of $m_F = 2$ or greater. For other gears, consult the AGMA standard.

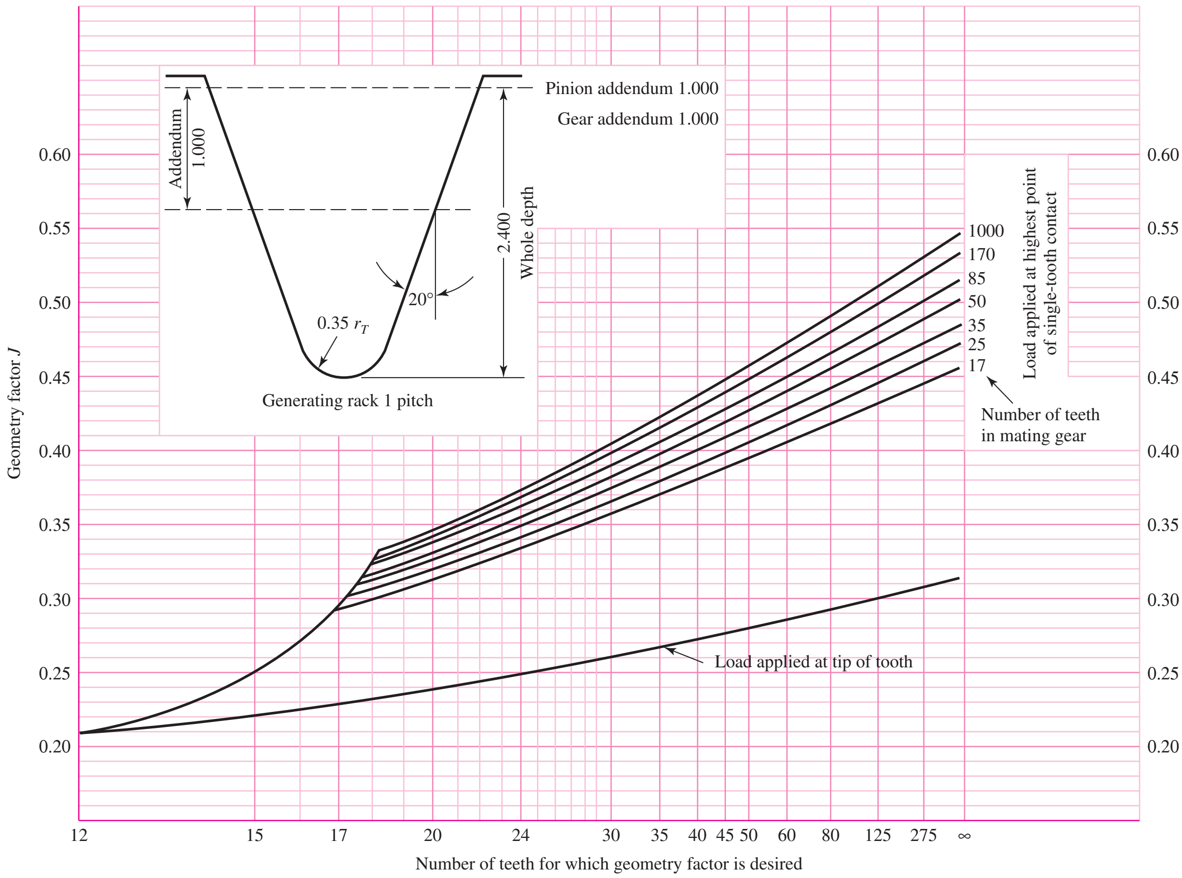
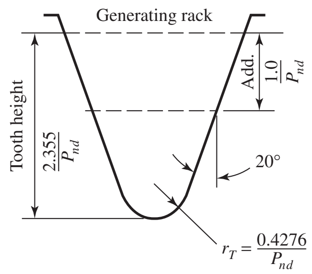
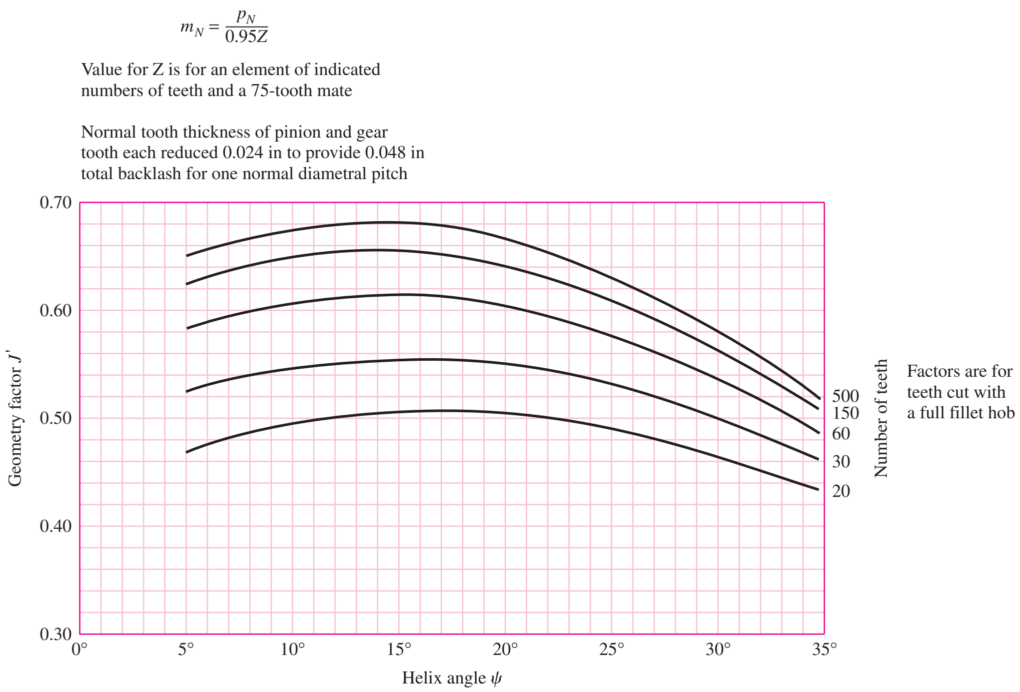


Figure 14–6

Spur-gear geometry factors *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



(a)



(b)

Figure 14–7

Helical-gear geometry factors J'. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

Surface-Strength Geometry Factor I (Z_I)

The factor I is also called the *pitting-resistance geometry factor* by AGMA. We will develop an expression for I by noting that the sum of the reciprocals of Eq. (14–14), from Eq. (14–12), can be expressed as

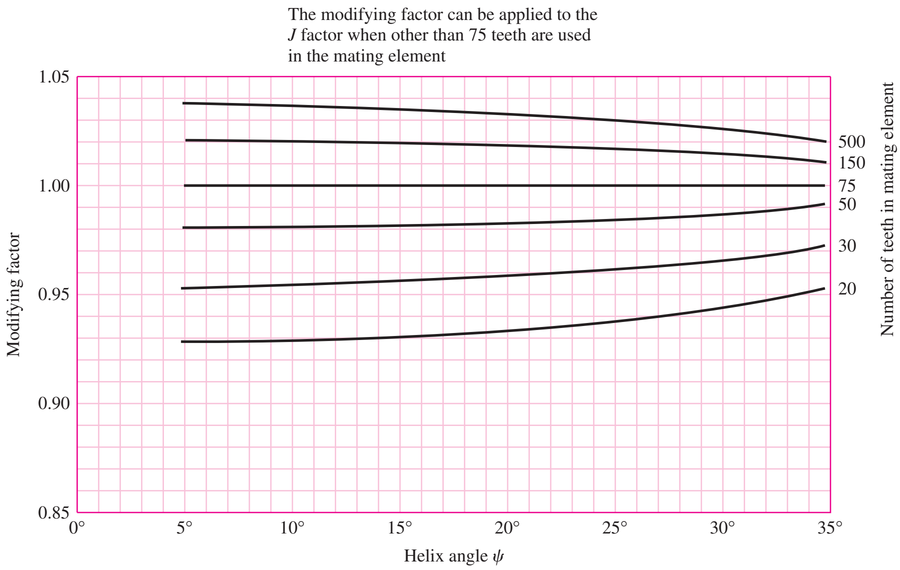
$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{\sin\phi_t}\left(\frac{1}{d_P} + \frac{1}{d_G}\right) \tag{a}$$

where we have replaced ϕ by ϕ_t, the transverse pressure angle, so that the relation will apply to helical gears too. Now define *speed ratio* m_G as

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \tag{14–22}$$

Figure 14–8

J′-factor multipliers for use with Fig. 14–7 to find *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



Equation (*a*) can now be written

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{d_P \sin\phi_t}\,\frac{m_G + 1}{m_G} \tag{b}$$

Now substitute Eq. (*b*) for the sum of the reciprocals in Eq. (14–14). The result is found to be

$$\sigma_c = -\sigma_C = C_p \left[\frac{K_V W^t}{d_P F}\,\frac{1}{\dfrac{\cos\phi_t \sin\phi_t}{2}\,\dfrac{m_G}{m_G + 1}} \right]^{1/2} \tag{c}$$

The geometry factor *I* for external spur and helical gears is the denominator of the second term in the brackets in Eq. (*c*). By adding the load-sharing ratio m_N, we obtain a factor valid for both spur and helical gears. The equation is then written as

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\,\dfrac{m_G}{m_G + 1} & \text{external gears} \\[2ex] \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\,\dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \tag{14–23}$$

where $m_N = 1$ for spur gears. In solving Eq. (14–21) for m_N, note that

$$p_N = p_n \cos\phi_n \tag{14–24}$$

where p_n is the normal circular pitch. The quantity *Z*, for use in Eq. (14–21), can be obtained from the equation

$$Z = \left[(r_P + a)^2 - r_{bP}^2\right]^{1/2} + \left[(r_G + a)^2 - r_{bG}^2\right]^{1/2} - (r_P + r_G)\sin\phi_t \tag{14–25}$$

where r_P and r_G are the pitch radii and r_{bP} and r_{bG} the base-circle radii of the pinion and gear, respectively.[6] Recall from Eq. (13–6), the radius of the base circle is

$$r_b = r\cos\phi_t \tag{14–26}$$

[6]For a development, see Joseph E. Shigley and John J. Uicker Jr., *Theory of Machines and Mechanisms,* McGraw-Hill, New York, 1980, p. 262.

Certain precautions must be taken in using Eq. (14–25). The tooth profiles are not conjugate below the base circle, and consequently, if either one or the other of the first two terms in brackets is larger than the third term, then it should be replaced by the third term. In addition, the effective outside radius is sometimes less than $r + a$, owing to removal of burrs or rounding of the tips of the teeth. When this is the case, always use the effective outside radius instead of $r + a$.

14–6 The Elastic Coefficient C_p (Z_E)

Values of C_p may be computed directly from Eq. (14–13) or obtained from Table 14–8.

14–7 Dynamic Factor K_v

As noted earlier, dynamic factors are used to account for inaccuracies in the manufacture and meshing of gear teeth in action. *Transmission error* is defined as the departure from uniform angular velocity of the gear pair. Some of the effects that produce transmission error are:

- Inaccuracies produced in the generation of the tooth profile; these include errors in tooth spacing, profile lead, and runout
- Vibration of the tooth during meshing due to the tooth stiffness
- Magnitude of the pitch-line velocity
- Dynamic unbalance of the rotating members
- Wear and permanent deformation of contacting portions of the teeth
- Gearshaft misalignment and the linear and angular deflection of the shaft
- Tooth friction

In an attempt to account for these effects, AGMA has defined a set of *quality numbers*.[7] These numbers define the tolerances for gears of various sizes manufactured to a specified accuracy. Quality numbers 3 to 7 will include most commercial-quality gears. Quality numbers 8 to 12 are of precision quality. The AGMA *transmission accuracy-level number* Q_v could be taken as the same as the quality number. The following equations for the dynamic factor are based on these Q_v numbers:

$$K_v = \begin{cases} \left(\dfrac{A+\sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A+\sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \tag{14–27}$$

where

$$A = 50 + 56(1 - B) \qquad B = 0.25(12 - Q_v)^{2/3} \tag{14–28}$$

and the maximum velocity, representing the end point of the Q_v curve, is given by

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \tag{14–29}$$

[7]AGMA 2000-A88. ANSI/AGMA 2001-D04, adopted in 2004, replaced Q_v with A_v and incorporated ANSI/AGMA 2015-1-A01. A_v ranges from 6 to 12, with lower numbers representing greater accuracy. The Q_v approach was maintained as an alternate approach, and resulting K_v values are comparable.

Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source:* AGMA 218.01

		Gear Material and Modulus of Elasticity E_G, lbf/in^2 (MPa)*					
Pinion Material	**Pinion Modulus of Elasticity E_p psi (MPa)***	**Steel 30×10^6 (2×10^5)**	**Malleable Iron 25×10^6 (1.7×10^5)**	**Nodular Iron 24×10^6 (1.7×10^5)**	**Cast Iron 22×10^6 (1.5×10^5)**	**Aluminum Bronze 17.5×10^6 (1.2×10^5)**	**Tin Bronze 16×10^6 (1.1×10^5)**
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

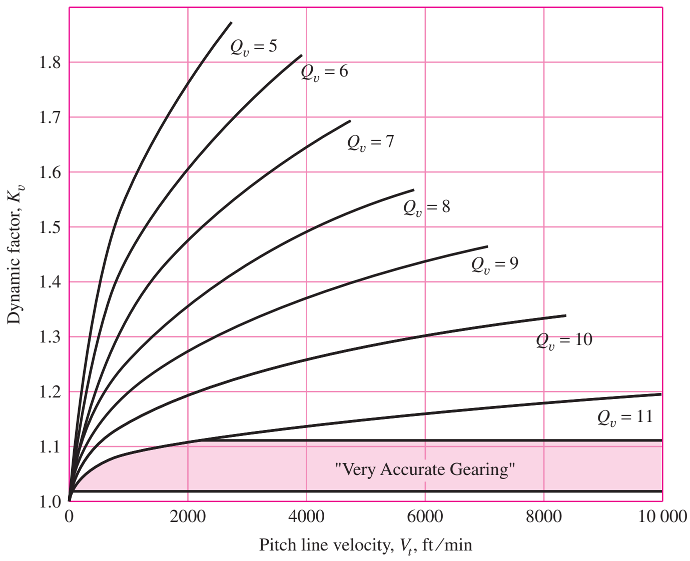


Figure 14–9

Dynamic factor K_v. The equations to these curves are given by Eq. (14–27) and the end points by Eq. (14–29). *(ANSI/AGMA 2001-D04, Annex A)*

Figure 14–9 is a graph of K_v, the dynamic factor, as a function of pitch-line speed for graphical estimates of K_v.

14–8 Overload Factor K_o

The overload factor K_o is intended to make allowance for all externally applied loads in excess of the nominal tangential load W^t in a particular application (see Figs. 14–17 and 14–18). Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive. There are other similar factors such as application factor or service factor. These factors are established after considerable field experience in a particular application.[8]

14–9 Surface Condition Factor C_f (Z_R)

The surface condition factor C_f or Z_R is used only in the pitting resistance equation, Eq. (14–16). It depends on

- Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shotpeening
- Residual stress
- Plastic effects (work hardening)

Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA specifies a value of C_f greater than unity.

[8]An extensive list of service factors appears in Howard B. Schwerdlin, "Couplings," Chap. 16 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004.

14–10 Size Factor K_s

The size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

Standard size factors for gear teeth have not yet been established for cases where there is a detrimental size effect. In such cases AGMA recommends a size factor greater than unity. If there is no detrimental size effect, use unity.

AGMA has identified and provided a symbol for size factor. Also, AGMA suggests $K_s = 1$, which makes K_s a placeholder in Eqs. (14–15) and (14–16) until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge. From Table 13–1, $l = a + b = 2.25/P$. The tooth thickness t in Fig. 14–6 is given in Sec. 14–1, Eq. (*b*), as $t = \sqrt{4lx}$ where $x = 3Y/(2P)$ from Eq. (14–3). From Eq. (6–25) the equivalent diameter d_e of a rectangular section in bending is $d_e = 0.808\sqrt{Ft}$. From Eq. (6–20) $k_b = (d_e/0.3)^{-0.107}$. Noting that K_s is the reciprocal of k_b, we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \tag{a}$$

K_s can be viewed as Lewis's geometry incorporated into the Marin size factor in fatigue. You may set $K_s = 1$, or you may elect to use the preceding Eq. (*a*). This is a point to discuss with your instructor. We will use Eq. (*a*) to remind you that you have a choice. If K_s in Eq. (*a*) is less than 1, use $K_s = 1$.

14–11 Load-Distribution Factor Km (KH)

The load-distribution factor modified the stress equations to reflect nonuniform distribution of load across the line of contact. The ideal is to locate the gear "midspan" between two bearings at the zero slope place when the load is applied. However, this is not always possible. The following procedure is applicable to

- Net face width to pinion pitch diameter ratio $F/d \le 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

The load-distribution factor under these conditions is currently given by the *face load distribution factor*, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \tag{14–30}$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

Note that for values of $F/(10d) < 0.05$, $F/(10d) = 0.05$ is used.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases} \qquad (14\text{–}33)$$

$$C_{ma} = A + BF + CF^2 \quad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad (14\text{–}34)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

See Fig. 14–10 for definitions of S and S_1 for use with Eq. (14–33), and see Fig. 14–11 for graph of C_{ma}.

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-*D04.*

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

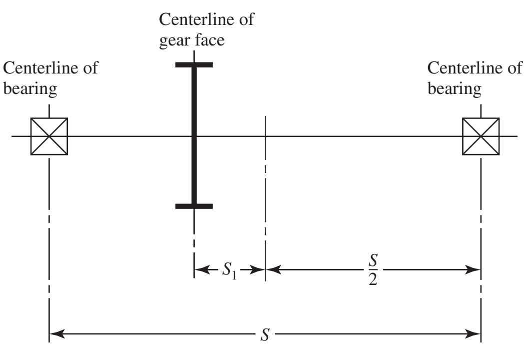


Figure 14–10

Definition of distances S and S_1 used in evaluating C_{pm}, Eq. (14–33). *(ANSI/AGMA 2001-D04.)*

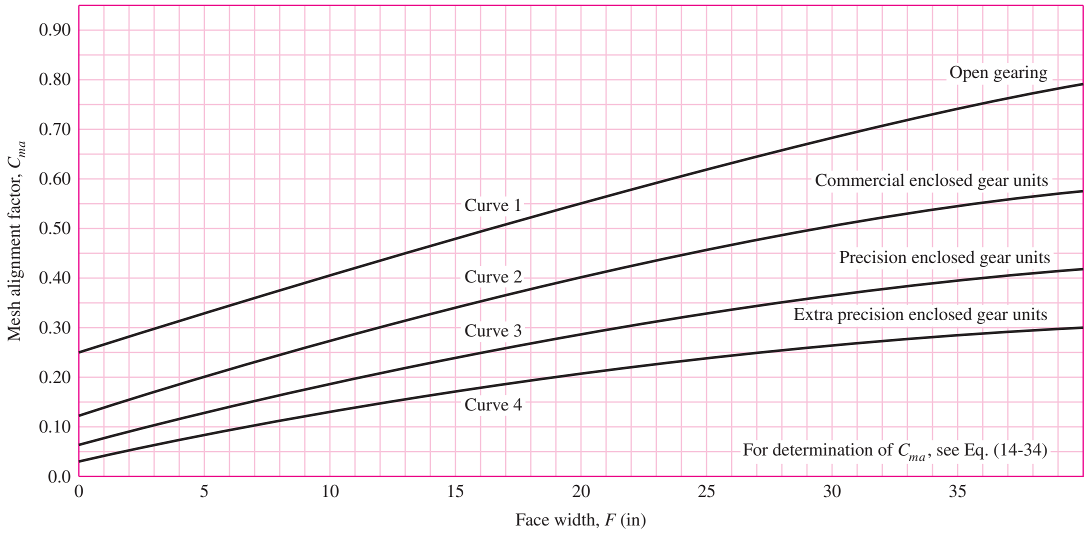


Figure 14–11

Mesh alignment factor C_{ma}. Curve-fit equations in Table 14–9. *(ANSI/AGMA 2001-D04.)*

14–12 Hardness-Ratio Factor C_H

The pinion generally has a smaller number of teeth than the gear and consequently is subjected to more cycles of contact stress. If both the pinion and the gear are through-hardened, then a uniform surface strength can be obtained by making the pinion harder than the gear. A similar effect can be obtained when a surface-hardened pinion is mated with a through-hardened gear. The hardness-ratio factor C_H is used *only for the gear*. Its purpose is to adjust the surface strengths for this effect. The values of C_H are obtained from the equation

$$C_H = 1.0 + A'(m_G - 1.0) \tag{14–36}$$

where

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3}) \quad 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

The terms H_{BP} and H_{BG} are the Brinell hardness (10-mm ball at 3000-kg load) of the pinion and gear, respectively. The term m_G is the speed ratio and is given by Eq. (14–22). See Fig. 14–12 for a graph of Eq. (14–36). For

$$\frac{H_{BP}}{H_{BG}} < 1.2, \qquad A' = 0$$

$$\frac{H_{BP}}{H_{BG}} > 1.7, \qquad A' = 0.006\,98$$

When surface-hardened pinions with hardnesses of 48 Rockwell C scale (Rockwell C48) or harder are run with through-hardened gears (180–400 Brinell), a work hardening occurs. The C_H factor is a function of pinion surface finish f_P and the mating gear hardness. Figure 14–13 displays the relationships:

$$C_H = 1 + B'(450 - H_{BG}) \tag{14–37}$$

Figure 14–12

Hardness ratio factor C_H (through-hardened steel). *(ANSI/AGMA 2001-D04.)*

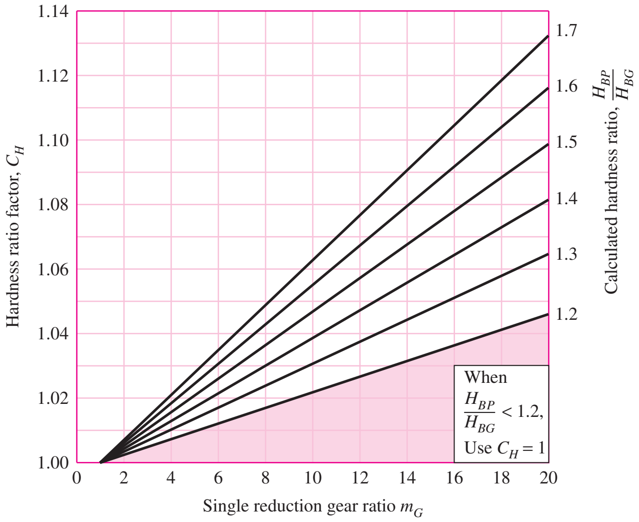
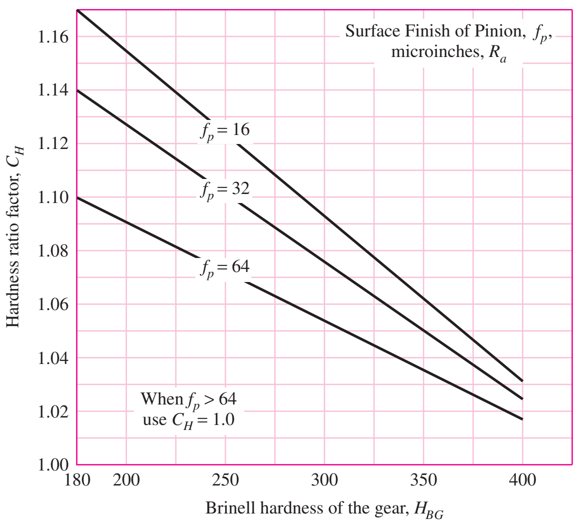


Figure 14–13

Hardness ratio factor C_H (surface-hardened steel pinion). *(ANSI/AGMA 2001-D04.)*



where $B' = 0.000\,75 \exp[-0.0112 f_P]$ and f_P is the surface finish of the pinion expressed as root-mean-square roughness R_a in μ in.

14–13 Stress Cycle Factors Y_N and Z_N

The AGMA strengths as given in Figs. 14–2 through 14–4, in Tables 14–3 and 14–4 for bending fatigue, and in Fig. 14–5 and Tables 14–5 and 14–6 for contact-stress fatigue are based on 10^7 load cycles applied. The purpose of the load cycle factors Y_N and Z_N is to modify the gear strength for lives other than 10^7 cycles. Values for these factors are given in Figs. 14–14 and 14–15. Note that for 10^7 cycles $Y_N = Z_N = 1$ on each graph. Note also that the equations for Y_N and Z_N change on either side of 10^7 cycles. For life goals slightly higher than 10^7 cycles, the mating gear may be experiencing fewer than 10^7 cycles and the equations for $(Y_N)_P$ and $(Y_N)_G$ can be different. The same comment applies to $(Z_N)_P$ and $(Z_N)_G$.

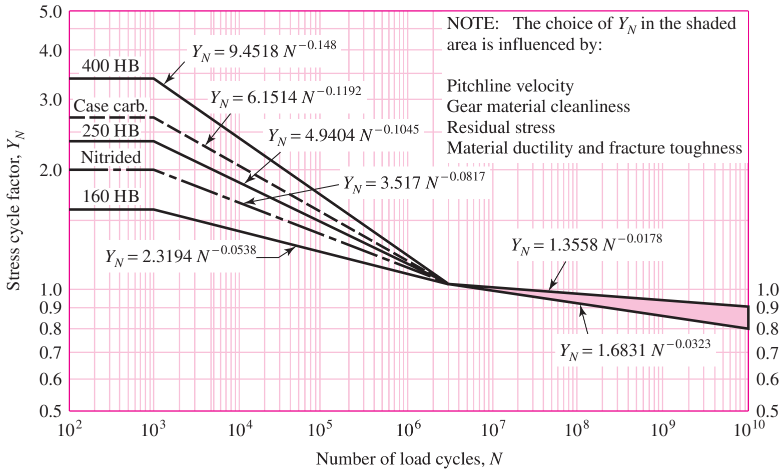


Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

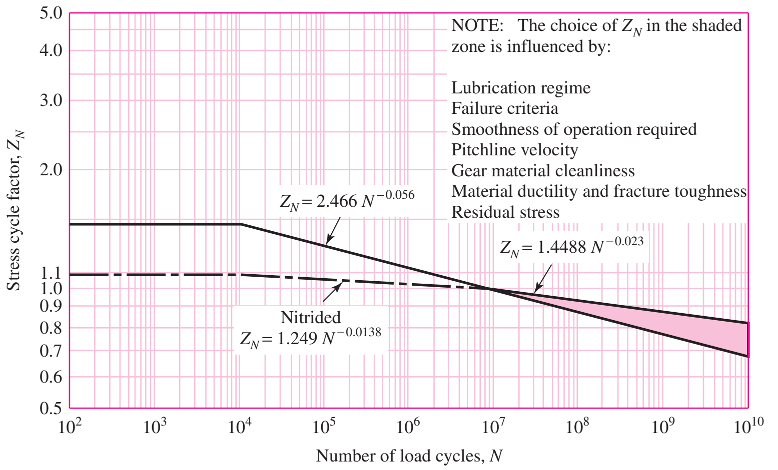


Figure 14–15

Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*

14–14 Reliability Factor K_R (Y_Z)

The reliability factor accounts for the effect of the statistical distributions of material fatigue failures. Load variation is not addressed here. The gear strengths S_t and S_c are based on a reliability of 99 percent. Table 14–10 is based on data developed by the U.S. Navy for bending and contact-stress fatigue failures.

The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is

$$K_R = \begin{cases} 0.658 - 0.0759\ln(1-R) & 0.5 < R < 0.99 \\ 0.50 - 0.109\ln(1-R) & 0.99 \le R \le 0.9999 \end{cases} \tag{14–38}$$

For cardinal values of R, take K_R from the table. Otherwise use the logarithmic interpolation afforded by Eqs. (14–38).

Table 14–10

Reliability Factors K_R (Y_Z)

Source: ANSI/AGMA 2001-D04.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

14–15 Temperature Factor K_T (Y_θ)

For oil or gear-blank temperatures up to 250°F (120°C), use $K_T = Y_\theta = 1.0$. For higher temperatures, the factor should be greater than unity. Heat exchangers may be used to ensure that operating temperatures are considerably below this value, as is desirable for the lubricant.

14–16 Rim-Thickness Factor K_B

When the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim rather than at the tooth fillet. In such cases, the use of a stress-modifying factor K_B or (t_R) is recommended. This factor, the *rim-thickness factor* K_B, adjusts the estimated bending stress for the thin-rimmed gear. It is a function of the backup ratio m_B,

$$m_B = \frac{t_R}{h_t} \tag{14–39}$$

where t_R = rim thickness below the tooth, in, and h_t = the tooth height. The geometry is depicted in Fig. 14–16. The rim-thickness factor K_B is given by

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \tag{14–40}$$

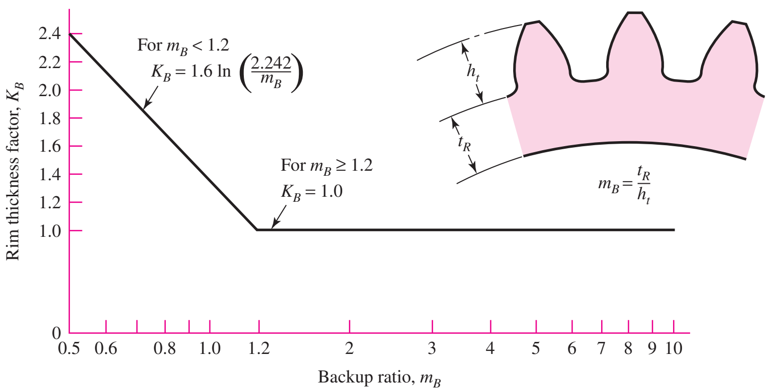


Figure 14–16

Rim thickness factor K_B. *(ANSI/AGMA 2001-D04.)*

Figure 14–16 also gives the value of K_B graphically. The rim-thickness factor K_B is applied in addition to the 0.70 reverse-loading factor when applicable.

14–17 Safety Factors S_F and S_H

The ANSI/AGMA standards 2001-D04 and 2101-D04 contain a safety factor S_F guarding against bending fatigue failure and safety factor S_H guarding against pitting failure.

The definition of S_F, from Eq. (14–17), is

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \tag{14–41}$$

where σ is estimated from Eq. (14–15). It is a strength-over-stress definition in a case where the stress is linear with the transmitted load.

The definition of S_H, from Eq. (14–18), is

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \tag{14–42}$$

when σ_c is estimated from Eq. (14–16). This, too, is a strength-over-stress definition but in a case where the stress is *not* linear with the transmitted load W^t.

While the definition of S_H does not interfere with its intended function, a caution is required when comparing S_F with S_H in an analysis in order to ascertain the nature and severity of the threat to loss of function. To render S_H linear with the transmitted load, W^t it could have been defined as

$$S_H = \left(\frac{\text{fully corrected contact strength}}{\text{contact stress imposed}}\right)^2 \tag{14–43}$$

with the exponent 2 for linear or helical contact, or an exponent of 3 for crowned teeth (spherical contact). With the definition, Eq. (14–42), compare S_F with S_H^2 (or S_H^3 for crowned teeth) when trying to identify the threat to loss of function with confidence.

The role of the overload factor K_o is to include predictable excursions of load beyond W^t based on experience. A safety factor is intended to account for unquantifiable elements in addition to K_o. When designing a gear mesh, the quantity S_F becomes a design factor $(S_F)_d$ within the meanings used in this book. The quantity S_F evaluated as part of a design assessment is a factor of safety. This applies equally well to the quantity S_H.

14–18 Analysis

Description of the procedure based on the AGMA standard is highly detailed. The best review is a "road map" for bending fatigue and contact-stress fatigue. Figure 14–17 identifies the bending stress equation, the endurance strength in bending equation, and the factor of safety S_F. Figure 14–18 displays the contact-stress equation, the contact fatigue endurance strength equation, and the factor of safety S_H. When analyzing a gear problem, this figure is a useful reference.

The following example of a gear mesh analysis is intended to make all the details presented concerning the AGMA method more familiar.

SPUR GEAR BENDING
BASED ON ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

- K_o: Table below
- K_v: Eq. (14–27); p. 736
- K_s: 1 [or Eq. (a), Sec. 14–10]; p. 739
- K_m: Eq. (14–30); p. 739
- K_B: Eq. (14–40); p. 744
- J: Fig. 14–6; p. 733

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

- S_t: ${}_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 728, 729
- Y_N: Fig. 14–14; p. 743
- K_T: 1 if $T < 250°$F
- K_R: Table 14–10, Eq. (14–38); pp. 744, 743

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–17

Roadmap of gear bending equations based on AGMA standards. *(ANSI/AGMA 2001-D04.)*

SPUR GEAR WEAR
BASED ON ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\text{ H}}{V}$$

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)^{1/2}$$

- C_p: Eq. (14–13), Table 14–8; pp. 724, 737
- K_o: Table below
- K_v: Eq. (14–27); p. 736
- K_s: 1 [or Eq. (a), Sec. 14–10]; p. 739
- K_m: Eq. (14–30); p. 739
- C_f: 1
- I: Eq. (14–23); p. 735

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

- S_c: ${}_{0.99}(S_c)_{10^7}$ Tables, 14–6, 14–7; pp. 731, 732
- Z_N: Fig. 14–15; p. 743
- C_H: Section 14–12, gear only; pp. 741, 742
- K_T: 1 if $T < 250°$ F
- K_R: Table 14–10, Eqs. (14–38); pp. 744, 743

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}$$

- C_H: Gear only

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–18

Roadmap of gear wear equations based on AGMA standards. *(ANSI/AGMA 2001-D04.)*

EXAMPLE 14–4

A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.
(*b*) Find the factor of safety of the gears in wear.
(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution

There will be many terms to obtain so use Figs. 14–17 and 14–18 as guides to what is needed.

$$d_P = N_P/P_d = 17/10 = 1.7 \text{ in} \qquad d_G = 52/10 = 5.2 \text{ in}$$

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.7)1800}{12} = 801.1 \text{ ft/min}$$

$$W^t = \frac{33\,000\,H}{V} = \frac{33\,000(4)}{801.1} = 164.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$. To evaluate K_v, from Eq. (14–28) with a quality number $Q_v = 6$,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14–27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

To determine the size factor, K_s, the Lewis form factor is needed. From Table 14–2, with $N_P = 17$ teeth, $Y_P = 0.303$. Interpolation for the gear with $N_G = 52$ teeth yields $Y_G = 0.412$. Thus from Eq. (*a*) of Sec. 14–10, with $F = 1.5$ in,

$$(K_s)_P = 1.192\left(\frac{1.5\sqrt{0.303}}{10}\right)^{0.0535} = 1.043$$

$$(K_s)_G = 1.192\left(\frac{1.5\sqrt{0.412}}{10}\right)^{0.0535} = 1.052$$

The load distribution factor K_m is determined from Eq. (14–30), where five terms are needed. They are, where $F = 1.5$ in when needed:

Uncrowned, Eq. (14–30): $C_{mc} = 1$,
Eq. (14–32): $C_{pf} = 1.5/[10(1.7)] - 0.0375 + 0.0125(1.5) = 0.0695$
Bearings immediately adjacent, Eq. (14–33): $C_{pm} = 1$
Commercial enclosed gear units (Fig. 14–11): $C_{ma} = 0.15$
Eq. (14–35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0695(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with $N(\text{pinion}) = 10^8$ cycles and $N(\text{gear}) = 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14.10, with a reliability of 0.9, $K_R = 0.85$. From Fig. 14–18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$. From Eq. (14–23), with $m_N = 1$ for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8, $C_p = 2300\sqrt{\text{psi}}$.

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with $H_{BP} = 240$ and $H_{BG} = 200$, we use Fig. 14–2, which gives

$$(S_t)_P = 77.3(240) + 12\,800 = 31\,350 \text{ psi}$$

$$(S_t)_G = 77.3(200) + 12\,800 = 28\,260 \text{ psi}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 322(240) + 29\,100 = 106\,400 \text{ psi}$$

$$(S_c)_G = 322(200) + 29\,100 = 93\,500 \text{ psi}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

For the hardness ratio factor C_H, the hardness ratio is $H_{BP}/H_{BG} = 240/200 = 1.2$. Then, from Sec. 14–12,

$$A' = 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3})$$
$$= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.002\,49$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.002\,49(3.059 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}\right)_P = 164.8(1)1.377(1.043)\frac{10}{1.5}\frac{1.22\,(1)}{0.30}$$

$$= 6417 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{6417} = 5.62$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 164.8(1)1.377(1.052)\frac{10}{1.5}\frac{1.22(1)}{0.40} = 4854 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{4854} = 6.82$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[164.8(1)1.377(1.043)\frac{1.22}{1.7(1.5)}\frac{1}{0.121}\right]^{1/2} = 70\,360 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer

$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \frac{106\,400(0.948)/[1(0.85)]}{70\,360} = 1.69$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2} (\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 70\,360 = 70\,660 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{70\,660} = 1.52$$

(*c*) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.73 with $1.69^2 = 2.86$, so the threat in the pinion is from wear. For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.96 with $1.52^2 = 2.31$, so the threat in the gear is also from wear.

There are perspectives to be gained from Ex. 14–4. First, the pinion is overly strong in bending compared to wear. The performance in wear can be improved by surface-hardening techniques, such as flame or induction hardening, nitriding, or carburizing and case hardening, as well as shot peening. This in turn permits the gearset to be made smaller. Second, in bending, the gear is stronger than the pinion, indicating that both the gear core hardness and tooth size could be reduced; that is, we may increase P and reduce diameter of the gears, or perhaps allow a cheaper material. Third, in wear, surface strength equations have the ratio $(Z_N)/K_R$. The values of $(Z_N)_P$ and $(Z_N)_G$ are affected by gear ratio m_G. The designer can control strength by specifying surface hardness. This point will be elaborated later.

Having followed a spur-gear analysis in detail in Ex. 14–4, it is timely to analyze a helical gearset under similar circumstances to observe similarities and differences.

EXAMPLE 14–5 A 17-tooth 20° normal pitch-angle helical pinion with a right-hand helix angle of 30° rotates at 1800 rev/min when transmitting 4 hp to a 52-tooth helical gear. The normal diametral pitch is 10 teeth/in, the face width is 1.5 in, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on the pinion and surface and core hardnesses of 200 Brinell on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of 10^8 cycles and a reliability of 0.9 and use the upper curves in Figs. 14–14 and 14–15.

(*a*) Find the factors of safety of the gears in bending.
(*b*) Find the factors of safety of the gears in wear.
(*c*) By examining the factors of safety identify the threat to each gear and to the mesh.

Solution All of the parameters in this example are the same as in Ex. 14–4 with the exception that we are using helical gears. Thus, several terms will be the same as Ex. 14–4. The reader should verify that the following terms remain unchanged: $K_o = 1$, $Y_P = 0.303$, $Y_G = 0.412$, $m_G = 3.059$, $(K_s)_P = 1.043$, $(K_s)_G = 1.052$, $(Y_N)_P = 0.977$, $(Y_N)_G = 0.996$, $K_R = 0.85$, $K_T = 1$, $C_f = 1$, $C_p = 2300\sqrt{\text{psi}}$, $(S_t)_P = 31\,350$ psi, $(S_t)_G = 28\,260$ psi, $(S_c)_P = 106\,380$ psi, $(S_c)_G = 93\,500$ psi, $(Z_N)_P = 0.948$, $(Z_N)_G = 0.973$, and $C_H = 1.005$.

For helical gears, the transverse diametral pitch, given by Eq. (13–18), is

$$P_t = P_n \cos\psi = 10 \cos 30^\circ = 8.660 \text{ teeth/in}$$

Thus, the pitch diameters are $d_P = N_P/P_t = 17/8.660 = 1.963$ in and $d_G = 52/8.660 = 6.005$ in. The pitch-line velocity and transmitted force are

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.963)1800}{12} = 925 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(4)}{925} = 142.7 \text{ lbf}$$

As in Ex. 14–4, for the dynamic factor, $B = 0.8255$ and $A = 59.77$. Thus, Eq. (14–27) gives

$$K_v = \left(\frac{59.77 + \sqrt{925}}{59.77}\right)^{0.8255} = 1.404$$

The geometry factor I for helical gears requires a little work. First, the transverse pressure angle is given by Eq. (13–19)

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^{\circ}}{\cos 30^{\circ}}\right) = 22.80^{\circ}$$

The radii of the pinion and gear are $r_P = 1.963/2 = 0.9815$ in and $r_G = 6.004/2 = 3.002$ in, respectively. The addendum is $a = 1/P_n = 1/10 = 0.1$, and the base-circle radii of the pinion and gear are given by Eq. (13–6) with $\phi = \phi_t$:

$$(r_b)_P = r_P \cos\phi_t = 0.9815 \cos 22.80^{\circ} = 0.9048 \text{ in}$$

$$(r_b)_G = 3.002 \cos 22.80^{\circ} = 2.767 \text{ in}$$

From Eq. (14–25), the surface strength geometry factor

$$\begin{aligned} Z &= \sqrt{(0.9815+0.1)^2 - 0.9048^2} + \sqrt{(3.004+0.1)^2 - 2.769^2} \\ &\quad - (0.9815+3.004)\sin 22.80^{\circ} \\ &= 0.5924 + 1.4027 - 1.5444 = 0.4507 \text{ in} \end{aligned}$$

Since the first two terms are less than 1.5444, the equation for Z stands. From Eq. (14–24) the normal circular pitch p_N is

$$p_N = p_n \cos\phi_n = \frac{\pi}{P_n}\cos 20^{\circ} = \frac{\pi}{10}\cos 20^{\circ} = 0.2952 \text{ in}$$

From Eq. (14–21), the load sharing ratio

$$m_N = \frac{p_N}{0.95Z} = \frac{0.2952}{0.95(0.4507)} = 0.6895$$

Substituting in Eq. (14–23), the geometry factor I is

$$I = \frac{\sin 22.80^{\circ}\cos 22.80^{\circ}}{2(0.6895)}\,\frac{3.06}{3.06+1} = 0.195$$

From Fig. 14–7, geometry factors $J'_P = 0.45$ and $J'_G = 0.54$. Also from Fig. 14–8 the J-factor multipliers are 0.94 and 0.98, correcting J'_P and J'_G to

$$J_P = 0.45(0.94) = 0.423$$

$$J_G = 0.54(0.98) = 0.529$$

The load-distribution factor K_m is estimated from Eq. (14–32):

$$C_{pf} = \frac{1.5}{10(1.963)} - 0.0375 + 0.0125(1.5) = 0.0577$$

with $C_{mc} = 1$, $C_{pm} = 1$, $C_{ma} = 0.15$ from Fig. 14–11, and $C_e = 1$. Therefore, from Eq. (14–30),

$$K_m = 1 + (1)[0.0577(1) + 0.15(1)] = 1.208$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms into Eq. (14–15) using P_t gives

$$\begin{aligned} (\sigma)_P &= \left(W^t K_o K_v K_s \frac{P_t}{F}\frac{K_m K_B}{J}\right)_P = 142.7(1)1.404(1.043)\frac{8.66}{1.5}\,\frac{1.208(1)}{0.423} \\ &= 3445 \text{ psi} \end{aligned}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{3445} = 10.5$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 142.7(1)1.404(1.052)\frac{8.66}{1.5}\frac{1.208(1)}{0.529} = 2779 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{2779} = 11.9$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[142.7(1)1.404(1.043)\frac{1.208}{1.963(1.5)}\frac{1}{0.195}\right]^{1/2} = 48\,230 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer

$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right)_P = \frac{106\,400(0.948)/[1(0.85)]}{48\,230} = 2.46$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 48\,230 = 48\,440 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{48\,440} = 2.22$$

(*c*) For the pinion we compare S_F with S_H^2, or 10.5 with $2.46^2 = 6.05$, so the threat in the pinion is from wear. For the gear we compare S_F with S_H^2, or 11.9 with $2.22^2 = 4.93$, so the threat is also from wear in the gear. For the meshing gearset wear controls.

It is worthwhile to compare Ex. 14–4 with Ex. 14–5. The spur and helical gearsets were placed in nearly identical circumstances. The helical gear teeth are of greater length because of the helix and identical face widths. The pitch diameters of the helical gears are larger. The J factors and the I factor are larger, thereby reducing stresses. The result is larger factors of safety. In the design phase the gearsets in Ex. 14–4 and Ex. 14–5 can be made smaller with control of materials and relative hardnesses.

Now that examples have given the AGMA parameters substance, it is time to examine some desirable (and necessary) relationships between material properties of spur gears in mesh. In bending, the AGMA equations are displayed side by side:

$$\sigma_P = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}\right)_P \qquad \sigma_G = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}\right)_G$$

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P \qquad (S_F)_G = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_G$$

Equating the factors of safety, substituting for stress and strength, canceling identical terms (K_s virtually equal or exactly equal), and solving for $(S_t)_G$ gives

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G} \frac{J_P}{J_G} \tag{a}$$

The stress-cycle factor Y_N comes from Fig. 14–14, where for a particular hardness, $Y_N = \alpha N^\beta$. For the pinion, $(Y_N)_P = \alpha N_P^\beta$, and for the gear, $(Y_N)_G = \alpha(N_P/m_G)^\beta$. Substituting these into Eq. (a) and simplifying gives

$$(S_t)_G = (S_t)_P m_G^\beta \frac{J_P}{J_G} \tag{14–44}$$

Normally, $m_G > 1$ and $J_G > J_P$, so equation (14–44) shows that the gear can be less strong (lower Brinell hardness) than the pinion for the same safety factor.

EXAMPLE 14–6

In a set of spur gears, a 300-Brinell 18-tooth 16-pitch 20° full-depth pinion meshes with a 64-tooth gear. Both gear and pinion are of grade 1 through-hardened steel. Using $\beta = -0.023$, what hardness can the gear have for the same factor of safety?

Solution

For through-hardened grade 1 steel the pinion strength $(S_t)_P$ is given in Fig. 14–2:

$$(S_t)_P = 77.3(300) + 12\,800 = 35\,990 \text{ psi}$$

From Fig. 14–6 the form factors are $J_P = 0.32$ and $J_G = 0.41$. Equation (14–44) gives

$$(S_t)_G = 35\,990 \left(\frac{64}{18}\right)^{-0.023} \frac{0.32}{0.41} = 27\,280 \text{ psi}$$

Use the equation in Fig. 14–2 again.

Answer

$$(H_B)_G = \frac{27\,280 - 12\,800}{77.3} = 187 \text{ Brinell}$$

The AGMA contact-stress equations also are displayed side by side:

$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I}\right)_P^{1/2} \qquad (\sigma_c)_G = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I}\right)_G^{1/2}$$

$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right)_P \qquad (S_H)_G = \left(\frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}\right)_G$$

Equating the factors of safety, substituting the stress relations, and canceling identical terms including K_s gives, after solving for $(S_c)_G$,

$$(S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G} \left(\frac{1}{C_H}\right)_G = (S_C)_P m_G^{\beta} \left(\frac{1}{C_H}\right)_G$$

where, as in the development of Eq. (14–44), $(Z_N)_P/(Z_N)_G = m_G^{\beta}$ and the value of β for wear comes from Fig. 14–15. Since C_H is so close to unity, it is usually neglected; therefore

$$(S_c)_G = (S_c)_P m_G^{\beta} \qquad (14\text{–}45)$$

EXAMPLE 14–7 For $\beta = -0.056$ for a through-hardened steel, grade 1, continue Ex. 14–6 for wear.

Solution From Fig. 14–5,

$$(S_c)_P = 322(300) + 29\,100 = 125\,700 \text{ psi}$$

From Eq. (14–45),

$$(S_c)_G = (S_c)_P \left(\frac{64}{18}\right)^{-0.056} = 125\,700 \left(\frac{64}{18}\right)^{-0.056} = 117\,100 \text{ psi}$$

Answer

$$(H_B)_G = \frac{117\,100 - 29\,200}{322} = 273 \text{ Brinell}$$

which is slightly less than the pinion hardness of 300 Brinell.

Equations (14–44) and (14–45) apply as well to helical gears.

14–19 Design of a Gear Mesh

A useful decision set for spur and helical gears includes

a priori decisions:

- Function: load, speed, reliability, life, K_o
- Unquantifiable risk: design factor n_d
- Tooth system: ϕ, ψ, addendum, dedendum, root fillet radius
- Gear ratio m_G, N_p, N_G
- Quality number Q_v

design decisions:

- Diametral pitch P_d
- Face width F
- Pinion material, core hardness, case hardness
- Gear material, core hardness, case hardness

The first item to notice is the dimensionality of the decision set. There are four design decision categories, eight different decisions if you count them separately. This is a larger number than we have encountered before. It is important to use a design strategy that is convenient in either longhand execution or computer implementation. The design decisions have been placed in order of importance (impact on the amount of work to be redone in iterations). The steps are, after the a priori decisions have been made,

- Choose a diametral pitch.
- Examine implications on face width, pitch diameters, and material properties. If not satisfactory, return to pitch decision for change.
- Choose a pinion material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.
- Choose a gear material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.

With these plan steps in mind, we can consider them in more detail.

First select a trial diametral pitch.

Pinion bending:

- Select a median face width for this pitch, $4\pi/P$
- Find the range of necessary ultimate strengths
- Choose a material and a core hardness
- Find face width to meet factor of safety in bending
- Choose face width
- Check factor of safety in bending

Gear bending:

- Find necessary companion core hardness
- Choose a material and core hardness
- Check factor of safety in bending

Pinion wear:

- Find necessary S_c and attendant case hardness
- Choose a case hardness
- Check factor of safety in wear

Gear wear:

- Find companion case hardness
- Choose a case hardness
- Check factor of safety in wear

Completing this set of steps will yield a satisfactory design. Additional designs with diametral pitches adjacent to the first satisfactory design will produce several among which to choose. A figure of merit is necessary in order to choose the best. Unfortunately, a figure of merit in gear design is complex in an academic environment because material and processing cost vary. The possibility of using a process depends on the manufacturing facility if gears are made in house.

After examining Ex. 14–4 and Ex. 14–5 and seeing the wide range of factors of safety, one might entertain the notion of setting all factors of safety equal.[9] In steel

[9]In designing gears it makes sense to define the factor of safety in wear as $(S)^2_H$ for uncrowned teeth, so that there is no mix-up. ANSI, in the preface to ANSI/AGMA 2001-D04 and 2101-D04, states "the use is completely voluntary. . . does not preclude anyone from using . . . procedures . . . not conforming to the standards."

gears, wear is usually controlling and $(S_H)_P$ and $(S_H)_G$ can be brought close to equality. The use of softer cores can bring down $(S_F)_P$ and $(S_F)_G$, but there is value in keeping them higher. A tooth broken by bending fatigue not only can destroy the gear set, but can bend shafts, damage bearings, and produce inertial stresses up- and downstream in the power train, causing damage elsewhere if the gear box locks.

EXAMPLE 14–8

Design a 4:1 spur-gear reduction for a 100-hp, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Solution

Make the a priori decisions:

- Function: 100 hp, 1120 rev/min, $R = 0.95$, $N = 10^9$ cycles, $K_o = 1$
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20°$
- Tooth count: $N_P = 18$ teeth, $N_G = 72$ teeth (no interference)
- Quality number: $Q_v = 6$, use grade 1 material
- Assume $m_B \geq 1.2$ in Eq. (14–40), $K_B = 1$

Pitch: Select a trial diametral pitch of $P_d = 4$ teeth/in. Thus, $d_P = 18/4 = 4.5$ in and $d_G = 72/4 = 18$ in. From Table 14–2, $Y_P = 0.309$, $Y_G = 0.4324$ (interpolated). From Fig. 14–6, $J_P = 0.32$, $J_G = 0.415$.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(4.5)1120}{12} = 1319 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(100)}{1319} = 2502 \text{ lbf}$$

From Eqs. (14–28) and (14–27),

$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left(\frac{59.77 + \sqrt{1319}}{59.77}\right)^{0.8255} = 1.480$$

From Eq. (14–38), $K_R = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$. From Fig. 14–14,

$$(Y_N)_P = 1.3558(10^9)^{-0.0178} = 0.938$$

$$(Y_N)_G = 1.3558(10^9/4)^{-0.0178} = 0.961$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^9)^{-0.023} = 0.900$$

$$(Z_N)_G = 1.4488(10^9/4)^{-0.023} = 0.929$$

From the recommendation after Eq. (14–8), $3p \le F \le 5p$. Try $F = 4p = 4\pi/P = 4\pi/4 = 3.14$ in. From Eq. (a), Sec. 14–10,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{3.14\sqrt{0.309}}{4}\right)^{0.0535} = 1.140$$

From Eqs. (14–31), (14–33), (14–35), $C_{mc} = C_{pm} = C_e = 1$. From Fig. 14–11, $C_{ma} = 0.175$ for commercial enclosed gear units. From Eq. (14–32), $F/(10d_P) = 3.14/[10(4.5)] = 0.0698$. Thus,

$$C_{pf} = 0.0698 - 0.0375 + 0.0125(3.14) = 0.0715$$

From Eq. (14–30),

$$K_m = 1 + (1)[0.0715(1) + 0.175(1)] = 1.247$$

From Table 14–8, for steel gears, $C_p = 2300\sqrt{\text{psi}}$. From Eq. (14–23), with $m_G = 4$ and $m_N = 1$,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\frac{4}{4+1} = 0.1286$$

Pinion tooth bending. With the above estimates of K_s and K_m from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Eqs. (14–15) and (14–17), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{bend}}$ necessary to resist bending fatigue, we obtain

$$(F)_{\text{bend}} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J_P}\frac{K_T K_R}{S_t Y_N} \tag{1}$$

Equating Eqs. (14–16) and (14–18), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{wear}}$ necessary to resist wear fatigue, we obtain

$$(F)_{\text{wear}} = \left(\frac{C_p Z_N}{S_c K_T K_R}\right)^2 n_d W^t K_o K_v K_s \frac{K_m C_f}{d_P I} \tag{2}$$

From Table 14–5 the hardness range of Nitralloy 135M is Rockwell C32–36 (302–335 Brinell). Choosing a midrange hardness as attainable, using 320 Brinell. From Fig. 14–4,

$$S_t = 86.2(320) + 12\,730 = 40\,310 \text{ psi}$$

Inserting the numerical value of S_t in Eq. (1) to estimate the face width gives

$$(F)_{\text{bend}} = 2(2502)(1)1.48(1.14)4\frac{1.247(1)(1)0.885}{0.32(40\,310)0.938} = 3.08 \text{ in}$$

From Table 14–6 for Nitralloy 135M, $S_c = 170\,000$ psi. Inserting this in Eq. (2), we find

$$(F)_{\text{wear}} = \left(\frac{2300(0.900)}{170\,000(1)0.885}\right)^2 2(2502)1(1.48)1.14\frac{1.247(1)}{4.5(0.1286)} = 3.44 \text{ in}$$

Decision Make face width 3.50 in. Correct K_s and K_m:

$$K_s = 1.192\left(\frac{3.50\sqrt{0.309}}{4}\right)^{0.0535} = 1.147$$

$$\frac{F}{10d_P} = \frac{3.50}{10(4.5)} = 0.0778$$

$$C_{pf} = 0.0778 - 0.0375 + 0.0125(3.50) = 0.0841$$

$$K_m = 1 + (1)[0.0841(1) + 0.175(1)] = 1.259$$

The bending stress induced by W^t in bending, from Eq. (14–15), is

$$(\sigma)_P = 2502(1)1.48(1.147)\frac{4}{3.50}\frac{1.259(1)}{0.32} = 19\ 100\ \text{psi}$$

The AGMA factor of safety in bending of the pinion, from Eq. (14–41), is

$$(S_F)_P = \frac{40\ 310(0.938)/[1(0.885)]}{19\ 100} = 2.24$$

Decision **Gear tooth bending.** Use cast gear blank because of the 18-in pitch diameter. Use the same material, heat treatment, and nitriding. The load-induced bending stress is in the ratio of J_P/J_G. Then

$$(\sigma)_G = 19\ 100\frac{0.32}{0.415} = 14\ 730\ \text{psi}$$

The factor of safety of the gear in bending is

$$(S_F)_G = \frac{40\ 310(0.961)/[1(0.885)]}{14\ 730} = 2.97$$

Pinion tooth wear. The contact stress, given by Eq. (14–16), is

$$(\sigma_c)_P = 2300\left[2502(1)1.48(1.147)\frac{1.259}{4.5(3.5)}\frac{1}{0.129}\right]^{1/2} = 118\ 000\ \text{psi}$$

The factor of safety from Eq. (14–42), is

$$(S_H)_P = \frac{170\ 000(0.900)/[1(0.885)]}{118\ 000} = 1.465$$

By our definition of factor of safety, pinion bending is $(S_F)_P = 2.24$, and wear is $(S_H)_P^2 = (1.465)^2 = 2.15$.

Gear tooth wear. The hardness of the gear and pinion are the same. Thus, from Fig. 14–12, $C_H = 1$, the contact stress on the gear is the same as the pinion, $(\sigma_c)_G =$ 118 000 psi. The wear strength is also the same, $S_c = 170\ 000$ psi. The factor of safety of the gear in wear is

$$(S_H)_G = \frac{170\ 000(0.929)/[1(0.885)]}{118\ 000} = 1.51$$

So, for the gear in bending, $(S_F)_G = 2.97$, and wear $(S_H)_G^2 = (1.51)^2 = 2.29$.

Rim. Keep $m_B \geq 1.2$. The whole depth is h_t = addendum + dedendum $= 1/P_d + 1.25/P_d = 2.25/P_d = 2.25/4 = 0.5625$ in. The rim thickness t_R is

$$t_R \geq m_B h_t = 1.2(0.5625) = 0.675 \text{ in}$$

In the design of the gear blank, be sure the rim thickness exceeds 0.675 in; if it does not, review and modify this mesh design.

This design example showed a satisfactory design for a four-pitch spur-gear mesh. Material could be changed, as could pitch. There are a number of other satisfactory designs, thus a figure of merit is needed to identify the best.

One can appreciate that gear design was one of the early applications of the digital computer to mechanical engineering. A design program should be interactive, presenting results of calculations, pausing for a decision by the designer, and showing the consequences of the decision, with a loop back to change a decision for the better. The program can be structured in totem-pole fashion, with the most influential decision at the top, then tumbling down, decision after decision, ending with the ability to change the current decision or to begin again. Such a program would make a fine class project. Troubleshooting the coding will reinforce your knowledge, adding flexibility as well as bells and whistles in subsequent terms.

Standard gears may not be the most economical design that meets the functional requirements, because no application is standard in all respects.[10] Methods of designing custom gears are well-understood and frequently used in mobile equipment to provide good weight-to-performance index. The required calculations including optimizations are within the capability of a personal computer.

PROBLEMS

Because gearing problems can be difficult, the problems are presented by section.

Section 14–1

14–1 A steel spur pinion has a pitch of 6 teeth/in, 22 full-depth teeth, and a 20° pressure angle. The pinion runs at a speed of 1200 rev/min and transmits 15 hp to a 60-tooth gear. If the face width is 2 in, estimate the bending stress.

14–2 A steel spur pinion has a diametral pitch of 12 teeth/in, 16 teeth cut full-depth with a 20° pressure angle, and a face width of $\frac{3}{4}$ in. This pinion is expected to transmit 1.5 hp at a speed of 700 rev/min. Determine the bending stress.

14–3 A steel spur pinion has a module of 1.25 mm, 18 teeth cut on the 20° full-depth system, and a face width of 12 mm. At a speed of 1800 rev/min, this pinion is expected to carry a steady load of 0.5 kW. Determine the resulting bending stress.

14–4 A steel spur pinion has 15 teeth cut on the 20° full-depth system with a module of 5 mm and a face width of 60 mm. The pinion rotates at 200 rev/min and transmits 5 kW to the mating steel gear. What is the resulting bending stress?

[10]See H. W. Van Gerpen, C. K. Reece, and J. K. Jensen, *Computer Aided Design of Custom Gears,* Van Gerpen–Reece Engineering, Cedar Falls, Iowa, 1996.