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MAK506 THEORY OF ELASTICITY
SPRING 2009
Due date: March 30, 2009
HOMEWORK 6

1. (Pr. 4.23, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Given the displacement field

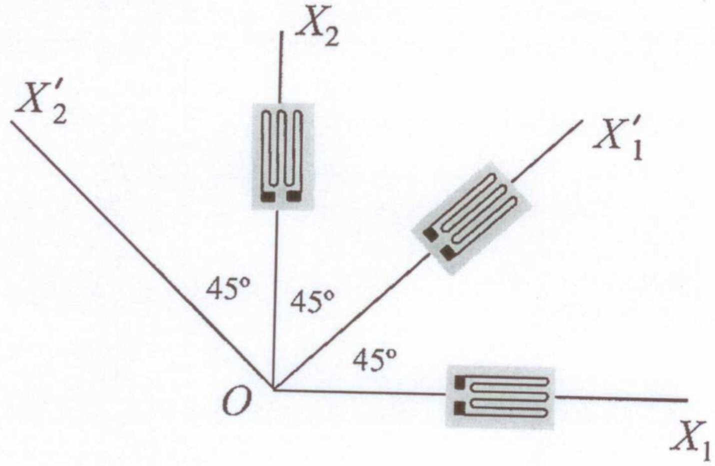
$$u_1 = AX_2X_3, \qquad u_2 = AX_3^2, \qquad u_3 = AX_1^2$$

where A is a very small constant, determine
   (a) The components of the infinitesimal strain tensor $\boldsymbol{\varepsilon}$, and the infinitesimal rotation tensor $\boldsymbol{\omega}$.
   (b) The principal values of $\boldsymbol{\varepsilon}$, at the point (1,1,0).

2. (Pr. 4.24, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) A 45° strain rosette measures longitudinal strains along the X_1, X_2 and X_1' axes shown in the sketch . At point P the strains recorded are

$$\varepsilon_{11} = 6\times10^{-4}, \qquad \varepsilon_{22} = 4\times10^{-4}, \qquad \varepsilon_{11}' = 8\times10^{-4}$$

Determine the shear strain γ_{12} at O, together with ε_{22}', and verify that $\varepsilon_{11} + \varepsilon_{22} = \varepsilon_{11}' + \varepsilon_{22}'$



1.) $u_1 = AX_2X_3$

$u_2 = AX_3^2$

$u_3 = AX_1^2$

a) $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$; $\omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i})$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = 0$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) = \frac{1}{2}\left(AX_3\right)$$

$$\varepsilon_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) = \frac{1}{2}\left(AX_2 + 2AX_1\right)$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = 0$$

$$\varepsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) = \frac{1}{2}\left(2AX_3\right)$$

$$\varepsilon_{33} = \frac{\partial u_3}{\partial x_3} = 0$$

similarly

$$\omega_{11} = 0 \quad ; \quad \omega_{22} = 0 \quad ; \quad \omega_{33} = 0$$

$$\omega_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) = \frac{1}{2}(AX_3) = -\omega_{21}$$

$$\omega_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = \frac{1}{2}\left(AX_2 - 2AX_1\right) = -\omega_{31}$$

$$\omega_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) = \frac{1}{2}\left(2AX_3\right) = -\omega_{32}$$

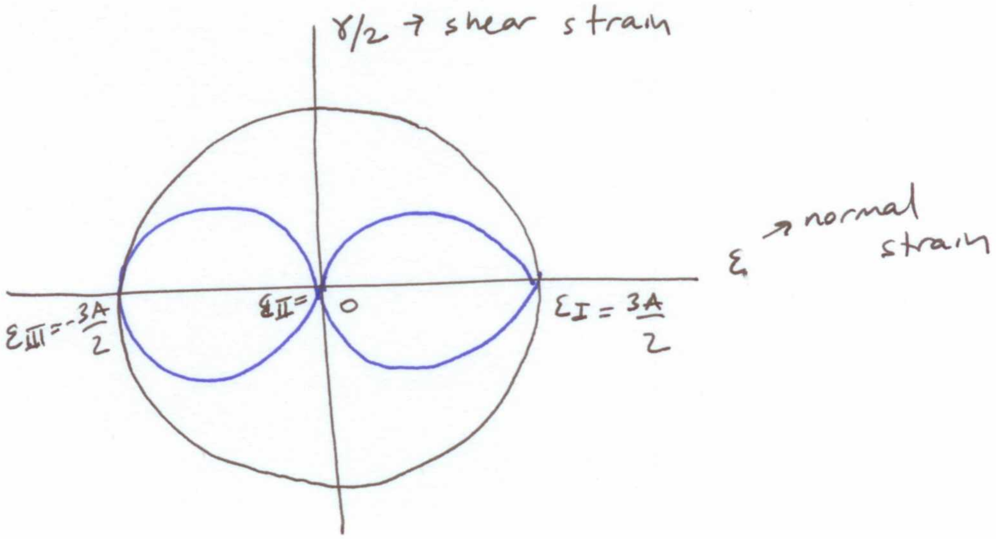
(b) strain tensor at the point (1, 1, 0)

$$\varepsilon_{ij} = \begin{bmatrix} 0 & 0 & \frac{3A}{2} \\ 0 & 0 & 0 \\ \frac{3A}{2} & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} 0-\lambda & 0 & \frac{3A}{2} \\ 0 & -\lambda & 0 \\ \frac{3A}{2} & 0 & -\lambda \end{vmatrix} = -\lambda\left(\lambda^2\right) + \frac{3A}{2}\left(0 + \frac{3A}{2}\lambda\right) = 0$$

$$\lambda\left(-\lambda^2 + \left(\frac{3A}{2}\right)^2\right) = 0$$

$$\left.\begin{aligned} \lambda_1 &= 0 = \\ \lambda_2 &= \frac{3A}{2} = \\ \lambda_3 &= -\frac{3A}{2} \end{aligned}\right\} \quad \begin{aligned} \varepsilon_{I} &= \frac{3A}{2} \\ \varepsilon_{II} &= 0 \\ \varepsilon_{III} &= -\frac{3A}{2} \end{aligned}$$

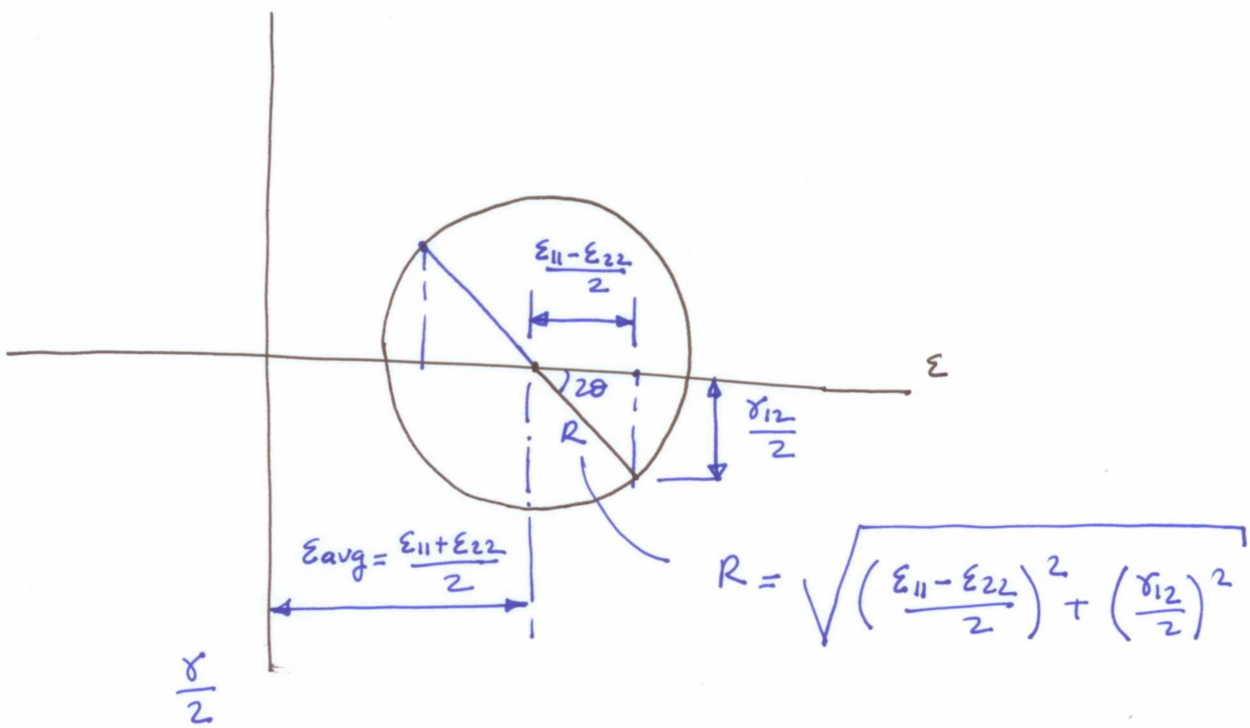


(2) strain rosette

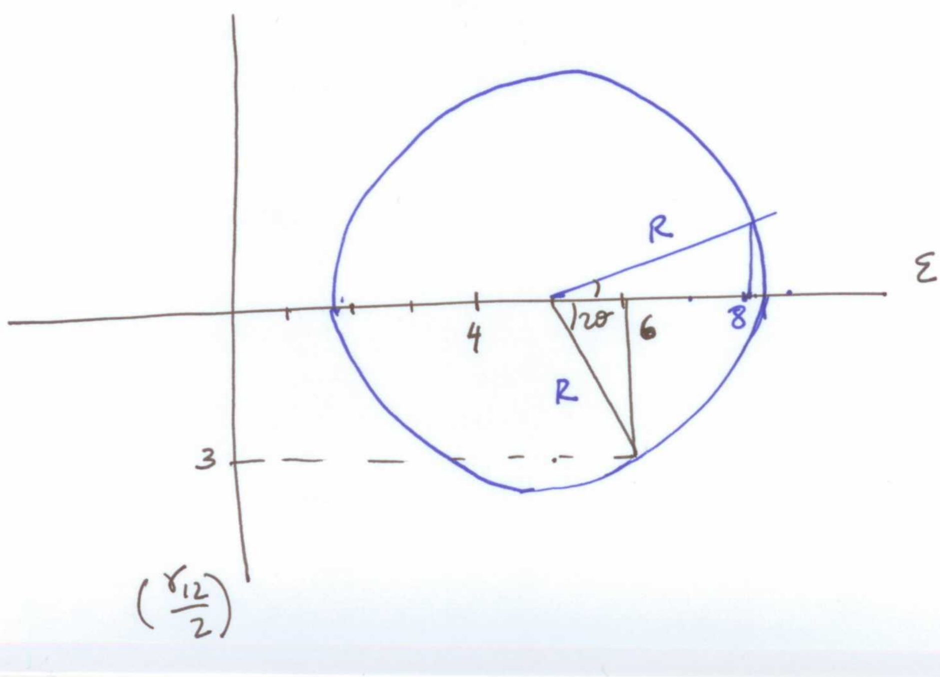
$$\varepsilon_{11}' = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \frac{\varepsilon_{11}-\varepsilon_{22}}{2}\cos 2\theta + \frac{\gamma_{12}}{2}\sin 2\theta$$

$$\varepsilon_{22}' = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} - \frac{\varepsilon_{11}-\varepsilon_{22}}{2}\cos 2\theta - \frac{\gamma_{12}}{2}\sin 2\theta$$

$$\frac{\gamma_{12}'}{2} = -\left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)\sin 2\theta + \frac{\gamma_{12}}{2}\cos 2\theta$$



$\varepsilon_{11} = 6\times10^{-4}$, $\varepsilon_{22} = 4\times10^{-4}$, $\varepsilon_{11}' = 8\times10^{-4}$

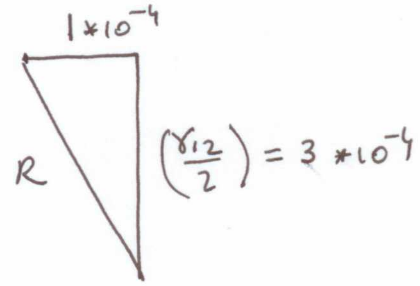


$\theta = 45^\circ \qquad 2\theta = 90^\circ$

$$\varepsilon_{11}' = \left(\frac{6+4}{2} + \frac{6-4}{2} \cos 90 + \frac{\gamma_{12}}{2} \sin 90 \right) * 10^{-4} = 8 * 10^{-4}$$

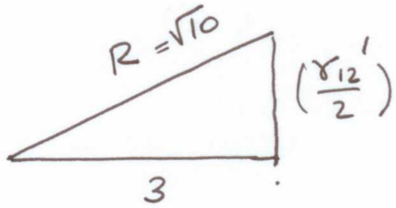
$$\frac{\gamma_{12}}{2} = 3 * 10^{-4}$$

From Mohr's circle



$$\Rightarrow \quad R = \sqrt{3^2+1}\ 10^{-4}$$

$$R = \sqrt{10} * 10^{-4}$$



$$\left(\frac{\gamma_{12}}{2}\right)' = \sqrt{(\sqrt{10})^2 - 3^2} * 10^{-4}$$

$$\left(\frac{\gamma_{12}}{2}\right)' = 1 * 10^{-4}$$

$$\varepsilon_{22}' = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} - \frac{\varepsilon_{11}-\varepsilon_{22}}{2} \cos 2\theta - \frac{\gamma_{12}}{2} \sin 2\theta$$

$$= \left(\frac{6+4}{2} - \frac{6-4}{2} \cos 90 - (3) \sin 90 \right) * 10^{-4}$$

$$\varepsilon_{22}' = 2 * 10^{-4}$$

$$\varepsilon_{11} + \varepsilon_{22} = (6+4) * 10^{-4} = 10 * 10^{-4}$$

$$\varepsilon_{11}' + \varepsilon_{22}' = (8+2) * 10^{-4} = 10 * 10^{-4}$$

$$\therefore \quad \varepsilon_{11} + \varepsilon_{22} = \varepsilon_{11}' + \varepsilon_{22}'$$