TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK506 THEORY OF ELASTICITY
FALL 2010
HOMEWORK 4 SOLUTION

1. (Pr. 4.2., Elasticity, M. H. Sadd) Substituting the general isotropic fourth-order form (4.2.6) into (4.2.3), explicitly develop the stress-strain relation (4.2.7).

$$\sigma_{ij} = C_{ijkl} e_{kl} \qquad (4.2.3)$$
$$C_{ijkl} = \alpha\delta_{ij}\delta_{kl} + \beta\delta_{ik}\delta_{jl} + \gamma\delta_{il}\delta_{jk} \qquad (4.2.6)$$
$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} \qquad (4.2.7)$$

$$\sigma_{ij} = \left(\alpha\delta_{ij}\delta_{kl} + \beta\delta_{ik}\delta_{jl} + \gamma\delta_{il}\delta_{jk}\right)e_{kl}$$
$$\sigma_{ij} = \alpha\delta_{ij}\delta_{kl}e_{kl} + \beta\delta_{ik}\delta_{jl}e_{kl} + \gamma\delta_{il}\delta_{jk}e_{kl}$$
$$\sigma_{ij} = \alpha\delta_{ij}e_{kk} + \beta\delta_{ik}e_{kj} + \gamma\delta_{il}e_{lj}$$
$$\sigma_{ij} = \alpha\delta_{ij}e_{kk} + \beta e_{ij} + \gamma e_{ij}$$
$$\sigma_{ij} = \alpha\delta_{ij}e_{kk} + e_{ij}\left(\beta + \gamma\right) = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}$$
$$\alpha = \lambda, \;\; \beta + \gamma = 2\mu$$

2. (Pr. 4.3., Elasticity, M. H. Sadd) Following the steps outlined in the text, invert the form of Hooke's law given by (4.2.7) and develop form (4.2.10). Explicitly show that $E = \mu\left(3\lambda + 2\mu\right)/\left(\lambda + \mu\right)$ and $\nu = \lambda / \left[2\left(\lambda + \mu\right)\right]$

$$e_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \qquad (4.2.10)$$

for $i = j = k$ eq. (4.2.7) becomes

$$\sigma_{kk} = \lambda e_{kk}\delta_{kk} + 2\mu e_{kk}$$
$$\sigma_{kk} = e_{kk}\left(3\lambda + 2\mu\right)$$
$$e_{kk} = \frac{\sigma_{kk}}{3\lambda + 2\mu}$$

Substitute e_{kk} into $eq.4.2.7$

$$\sigma_{ij} = \lambda\delta_{ij}\left(\frac{\sigma_{kk}}{3\lambda+2\mu}\right)+2\mu e_{ij}$$

$$e_{ij} = \frac{1}{2\mu}\sigma_{ij} - \frac{\lambda}{2\mu\left(2\mu+3\lambda\right)}\delta_{ij}\sigma_{kk}.............(a)$$

For defining the elastic module, simple tensile specimen is considered,

$$\sigma_{11} = \sigma$$

$$\sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{13} = \sigma_{23} = 0$$

for $i = j = 1,\ k = 1, 2, 3$ eq (a) becomes

$$e_{11} = \frac{1}{2\mu}\sigma_{11} - \frac{\lambda}{2\mu\left(2\mu+3\lambda\right)}\delta_{11}\sigma_{11}$$

and

$$e_{11} = \frac{\sigma_{11}}{E}$$

$$E = \frac{\mu\left(2\mu+3\lambda\right)}{\mu+\lambda}$$

for $i = j = 2,\ k = 1, 2, 3$ eq (a) becomes

$$e_{22} = \frac{1}{2\mu}\sigma_{22} - \frac{\lambda}{2\mu\left(2\mu+3\lambda\right)}\delta_{22}\sigma_{11}\ \ and\ \ \sigma_{22} = 0$$

$$e_{22} = -\frac{\lambda}{2\mu\left(2\mu+3\lambda\right)}\delta_{22}\sigma_{11}$$

$$e_{22} = -\nu\frac{\sigma}{E}$$

$$\nu = \frac{\lambda}{2(\mu+\lambda)}$$

and eq(a) becomes,

$$e_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij}$$

3. (Pr. 4.4., Elasticity, M. H. Sadd)Using the results of Exercise 4-3,show that $\mu = E/\left[2\left(1+\nu\right)\right]$ and $\lambda = E\nu/\left[\left(1+\nu\right)\left(1-2\nu\right)\right]$

$$E = \frac{\mu\left(2\mu+3\lambda\right)}{\mu+\lambda}......................(b)$$

$$\nu = \frac{\lambda}{2(\mu+\lambda)}............................(c)$$

$$\lambda = \frac{2\mu\nu}{1-2\nu}................................(d)$$

Substituting (d) into (b)

$$E = 2\mu(1+\nu)$$

$$\mu = \frac{E}{2(1+\nu)}.............................(e)$$

and substituting (e) into (d)

$$\lambda = \frac{E\nu}{\left(1+\nu\right)\left(1-2\nu\right)}$$