



MAK506 THEORY OF ELASTICITY
SPRING 2009

Due date: March 12, 2009

HOMEWORK 3

1. (Pr. 2.12, Advanced Strength and Applied Stress Analysis by Budynas) The state of stress at a point within a structure relative to an xyz coordinate system is given by,

$$[\sigma] = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 1 & 2 \\ -2 & 2 & -1 \end{bmatrix} \text{ kpsi}$$

Determine the normal and shear stresses and their directional cosines on a surface intersecting the point and parallel to the plane given by the equation

$$x + 2y - 2z = -4$$

2. (Pr. 3.3, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) The stress tensor at P relative to axes $Px_1x_2x_3$ has components in MPa given by the matrix representation

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

where σ_{11} is unspecified. Determine a direction \hat{n} at P for which the plane perpendicular to \hat{n} will be stress-free, that is, for which $t^{(\hat{n})} = 0$ on that plane. What is the required value of σ_{11} for this condition?

3. (Pr. 3.8, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Relative to cartesian axes $Ox_1x_2x_3$ a stress field is given by the matrix

$$[\sigma_{ij}] = \begin{bmatrix} (1-x_1^2)x_2 + \frac{2}{3}x_2^3 & -(4-x_2^2)x_1 & 0 \\ -(4-x_2^2)x_1 & -\frac{1}{3}(x_2^3 - 12x_2) & 0 \\ 0 & 0 & (3-x_1^2)x_2 \end{bmatrix}$$

- (a) Show that the equilibrium equations are satisfied everywhere for zero body forces.
- (b) Determine the stress vector at the point $P(2, -1, 6)$ of the plane whose equation is $3x_1 + 6x_2 + 2x_3 = 12$.

4. (Pr. 3.14, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) When referred to principal axes at P, the stress matrix in ksi units is

$$[\sigma_{ij}^*] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

If the transformation matrix between the principal axes and axes $Px_1x_2x_3$ is

$$[a_{ij}] = \frac{1}{\sqrt{2}} \begin{bmatrix} -\frac{3}{5} & 1 & -\frac{4}{5} \\ a_{21} & a_{22} & a_{23} \\ -\frac{3}{5} & -1 & -\frac{4}{5} \end{bmatrix}$$

Where a_{21} , a_{22} , and a_{23} are to be determined, calculate $[\sigma_{ij}]$.