TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

Makina Mühendisliği Bölümü

MAK506 ELASTİSİTE TEORİSİ

Güz Dönemi 2010

Ara Sınav

Dr. Mehmet Ali Güler — Ad, Soyad ______________________

24 Kasım 2010 Çarşamba — Öğrenci No ______________________

Verilen Zaman: **2 saat** (13:30-15:30)

Kitap ve Notlar Kapalı

- *Her soruyu dikkatle okuyun.*
- *Yaptığınız işlemleri gösterin.*
- *Sonuçları verilen kutular içine yazın.*
- *Temiz çalışın.*
- *Sınavı salonunda cep telefonu kullanmak*

Soru No	Maksimum Puan	Puan
1	25	
2	25	
3	25	
4	25	
5 yada 6	25	
Toplam	**125**	

Ön sayfa dahil, bu sınav kağıdında toplam **(7)** sayfa vardır.

1. (35 puan) Aşağıda verilen ikinci ve birinci dereceden tensörler için

$$a_{ij} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \qquad b_i = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

aşağıdaki değerleri hesaplayınız:

(a) $c_{ij} = \frac{1}{2}(a_{ij} + a_{ji})$, $(i, j = 1,2,3)$ (7 puan)

(b) $d_{ij} = \frac{1}{2}(a_{ij} - a_{ji})$, $(i, j = 1,2,3)$ (7 puan)

(c) $a_{ij}a_{kj}$, $(i, j, k = 1,2,3)$ (7 puan)

(d) $a_{ij}b_j$, $(i, j = 1,2,3)$ (7 puan)

(e) $c_{ij}d_{ij}$, $(i, j = 1,2,3)$ (7 puan)

Not: c_{ij} (a) şıkkından, d_{ij} (b) şıkkından hesaplanacaktır.

(a) $c_{ij} = \frac{1}{2}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 8 & 3 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 4 & 1.5 \\ 0.5 & 1.5 & 1 \end{bmatrix}$ second order tensor

(b) $d_{ij} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ -0.5 & 0 & 0.5 \\ -0.5 & -0.5 & 0 \end{bmatrix}$ second order tensor

(c) $a_{ij} \cdot a_{kj} = a_{i1}a_{k1} + a_{i2}a_{k2} + a_{i3}a_{k3} = A_{ik} = A_{ki}$ $(i, k = 1, 2, 3)$

$A_{11} = a_{11}a_{11} + a_{12}a_{12} + a_{13}a_{13} = 1 + 1 + 1 = 3$

$A_{12} = a_{11}a_{21} + a_{12}a_{22} + a_{13}a_{23} = 0 + 4 + 2 = 6$

$A_{13} = a_{11}a_{31} + a_{12}a_{32} + a_{13}a_{33} = 0 + 1 + 1 = 2$

$A_{21} = a_{21}a_{11} + a_{22}a_{12} + a_{23}a_{13} = A_{12} = 6$

$A_{22} = a_{21}a_{21} + a_{22}a_{22} + a_{23}a_{23} = 0 + 16 + 4 = 20$

$A_{23} = a_{21}a_{31} + a_{22}a_{32} + a_{23}a_{33} = 0 + 4 + 2 = 6$

$A_{33} = a_{31}a_{31} + a_{32}a_{32} + a_{33}a_{33} = 0 + 1 + 1 = 2$

$$A_{ik} = \begin{bmatrix} 3 & 6 & 2 \\ 6 & 20 & 6 \\ 2 & 6 & 2 \end{bmatrix}$$ — second order tensor

(d) $a_{ij} b_j = a_{i1} b_1 + a_{i2} b_2 + a_{i3} b_3$

$$= \begin{Bmatrix} a_{11} b_1 + a_{12} b_2 + a_{13} b_3 \\ a_{21} b_1 + a_{22} b_2 + a_{23} b_3 \\ a_{31} b_1 + a_{32} b_2 + a_{33} b_3 \end{Bmatrix} = \begin{Bmatrix} 1+0+2 \\ 0+0+4 \\ 0+0+2 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 4 \\ 2 \end{Bmatrix}$$

(e) $c_{ij} \cdot d_{ij} = c_{i1} \cdot d_{i1} + c_{i2} d_{i2} + c_{i3} d_{i3} \quad , \quad i = 1,2,3$

$$= c_{11} d_{11} + c_{12} d_{12} + c_{13} d_{13}$$
$$+ c_{21} d_{21} + c_{22} d_{22} + c_{23} d_{23}$$
$$+ c_{31} d_{31} + c_{32} d_{32} + c_{33} d_{33}$$

$$= 0 + \frac{1}{4} + \frac{1}{4} - \frac{1}{4} + 0 + \frac{3}{4} - \frac{1}{4} - \frac{3}{4} + 0$$

$$= 0$$

2. (40 puan) Aşağıda (x_1, x_2, x_3) koordinatlarında verilen B tensörü için,

$$[B_{ij}] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix}$$

(a) Özdeğerleri bulunuz (10 puan)

(b) Özvektörleri bulunuz. (10 puan)

(c) Özvektörler birbirine diktir. Özvektörlerin oluşturduğu (x_1', x_2', x_3') sisteme gitmek için gerekli olan koordinat dönüşüm matrisini (a_{ij}) bulunuz. (10 puan)

$\{X'\} = [A]^T \{X\}$

	x_1	x_2	x_3
x_1'	a_{11}	a_{21}	a_{31}
x_2'	a_{12}	a_{22}	a_{32}
x_3'	a_{13}	a_{23}	a_{33}

$a_{ij} = \cos(x_i, x_j') = \hat{e}_i \cdot \hat{e}_j'$

(d) Bulduğunuz koordinat dönüşüm matrisini kullanarak $[B_{ij}]$ tensörünü (x_1', x_2', x_3') koordinat sisteminde yazınız. (10 puan)

(a) Invariants of B matrix

$\theta_1 = 2+3-3 = 2$

$$\theta_2 = \begin{vmatrix} 3 & 4 \\ 4 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = -25 - 6 + 6 = -25$$

$$\theta_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{vmatrix} = 2 \cdot (-25) = -50$$

$$-\lambda^3 + \theta_1 \lambda^2 - \theta_2 \lambda + \theta_3 = 0$$

$$-\lambda^3 + 2\lambda^2 + 25\lambda - 50 = 0$$

$\boxed{\lambda_1 = 5}$, $\boxed{\lambda_2 = 2}$, $\boxed{\lambda_3 = -5}$

$\boxed{\lambda_1 > \lambda_2 > \lambda_3}$

(b) for $\lambda_1 = 5$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & 4 & -8 \end{bmatrix} \begin{Bmatrix} n_1^{(1)} \\ n_2^{(1)} \\ n_3^{(1)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$-3 n_1^{(1)} = 0 \Rightarrow \boxed{n_1^{(1)} = 0}$$

$$-2 n_2^{(1)} + 4 n_3^{(1)} = 0 \Rightarrow n_2^{(1)} = 2 n_3^{(1)}$$

$$\cancel{[n_1^{(1)}]^2}^{\,0} + [n_2^{(1)}]^2 + [n_3^{(1)}]^2 = 1$$

$$4[n_3^{(1)}]^2 + [n_3^{(1)}]^2 = 1 \Rightarrow \boxed{n_3^{(1)} = \mp \frac{1}{\sqrt{5}}} \Rightarrow \boxed{n_2^{(1)} = \mp \frac{2}{\sqrt{5}}}$$

$$\hat{n}_1 = 0\hat{e}_1 \mp \frac{2}{\sqrt{5}} \hat{e}_2 \mp \frac{1}{\sqrt{5}} \hat{e}_3$$

$$\boxed{\hat{n}_1 = \frac{1}{\sqrt{5}} \left(0\hat{e}_1 + 2\hat{e}_2 + 1\hat{e}_3 \right)}$$

for $\lambda_1 = 2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 4 & -5 \end{bmatrix} \begin{Bmatrix} n_1^{(2)} \\ n_2^{(2)} \\ n_3^{(2)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$n_2^{(2)} + 4 n_3^{(2)} = 0 \Rightarrow n_2^{(2)} = -4 n_3^{(2)}$$

$$4 n_2^{(2)} - 5 n_3^{(2)} = 0 \Rightarrow -16 n_3^{(2)} - 5 n_3^{(2)} = 0$$

$$-21 n_3^{(2)} = 0 \Rightarrow \boxed{n_3^{(2)} = 0}$$

$$\boxed{n_2^{(1)} = 0}$$

$$[n_1^{(2)}]^2 + [n_2^{(2)}]^2 + [n_3^{(2)}]^2 = 1 \Rightarrow \boxed{n_1^{(2)} = 1}$$

$$\boxed{\hat{n}_2 = 1\hat{e}_1 + 0\hat{e}_2 + 0\hat{e}_3}$$

for $\lambda_3 = -5$

$$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 4 & 2 \end{bmatrix} \begin{Bmatrix} n_1^{(3)} \\ n_2^{(3)} \\ n_3^{(3)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$7 n_1^{(3)} = 0 \Rightarrow \boxed{n_1^{(3)} = 0}$$

$$8 n_2^{(3)} + 4 n_3^{(3)} = 0 \Rightarrow n_3^{(3)} = -2 n_2^{(3)}$$

$$\overset{0}{\cancel{[n_1^{(3)}]^2}} + [n_2^{(3)}]^2 + [n_3^{(3)}]^2 = 1$$

$$[n_2^{(3)}]^2 + 4 [n_2^{(3)}]^2 = 1$$

$$\boxed{n_2^{(3)} = \mp \frac{1}{\sqrt{5}}} \Rightarrow \boxed{n_3^{(3)} = \pm \frac{2}{\sqrt{5}}}$$

$$\hat{n}_3 = 0\hat{e}_1 \mp \frac{1}{\sqrt{5}}\hat{e}_2 \pm \frac{2}{\sqrt{5}}\hat{e}_3$$

$$\boxed{\hat{n}_3 = \frac{1}{\sqrt{5}}\left(0\hat{e}_1 \mp 1\hat{e}_2 \pm 2\hat{e}_3\right)}$$

$\therefore$

$$\hat{n}_1 = 0\hat{e}_1 + \frac{2}{\sqrt{5}}\hat{e}_2 + \frac{1}{\sqrt{5}}\hat{e}_3$$

$$\hat{n}_2 = -1\hat{e}_1 + 0\hat{e}_2 + 0\hat{e}_3$$

$$\hat{n}_3 = 0\hat{e}_1 + \frac{1}{\sqrt{5}}\hat{e}_2 - \frac{2}{\sqrt{5}}\hat{e}_3$$

(c)

	x_1	x_2	x_3
x_1'	a_{11}	a_{21}	a_{31}
x_2'	a_{12}	a_{22}	a_{32}
x_3'	a_{13}	a_{23}	a_{33}

$$a_{ij} = \cos(x_i, x_j') = \hat{e}_i \cdot \hat{e}_j'$$

	x_1	x_2	x_3
x_1'	0	$2/\sqrt{5}$	$1/\sqrt{5}$
x_2'	1	0	0
x_3'	0	$1/\sqrt{5}$	$-2/\sqrt{5}$

$$\therefore [A]^T = \begin{bmatrix} 0 & 2/\sqrt{5} & 1/\sqrt{5} \\ 1 & 0 & 0 \\ 0 & 1/\sqrt{5} & -2/\sqrt{5} \end{bmatrix}$$

$$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 2/\sqrt{5} & 0 & 1/\sqrt{5} \\ 1/\sqrt{5} & 0 & -2/\sqrt{5} \end{bmatrix}$$

(d) Transformation rule

$$[B_{ij}'] = [A]^T [B_{ij}] [A]$$

$$= \begin{bmatrix} 0 & 2/\sqrt{5} & 1/\sqrt{5} \\ 1 & 0 & 0 \\ 0 & 1/\sqrt{5} & -2/\sqrt{5} \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2/\sqrt{5} & 0 & 1/\sqrt{5} \\ 1/\sqrt{5} & 0 & -2/\sqrt{5} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 10/\sqrt{5} & 5/\sqrt{5} \\ 2 & 0 & 0 \\ 0 & -5/\sqrt{5} & 10/\sqrt{5} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2/\sqrt{5} & 0 & 1/\sqrt{5} \\ 1/\sqrt{5} & 0 & -2/\sqrt{5} \end{bmatrix}$$

$$[B_{ij}'] = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

2. **(25 points)** (Pr. 4.7, Elasticity, M. H. Sadd) The displacements in an elastic material are given by

$$u = -\frac{M(1-\nu^2)}{EI}xy, \qquad v = \frac{M(1+\nu)\nu}{2EI}y^2 + \frac{M(1-\nu^2)}{2EI}\left(x^2 - \frac{l^2}{4}\right), \qquad w = 0,$$

where M, E, I, and l are constant parameters. Determine the corresponding strain and stress fields and show that this problem represents the pure bending of a rectangular beam in the x,y plane.

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right] \quad \text{——— (1)}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} = 0 \quad \text{——— (2)}$$

From (1) and (2)

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \quad \text{——— (3)}$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] \quad \text{——— (4)}$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right] \quad \text{——— (5)}$$

Substituting (3) into (4) & (5) we have

$$\varepsilon_{xx} = \frac{1+\nu}{E}\left[(1-\nu)\sigma_{xx} - \nu\sigma_{yy}\right] \quad \text{——— (6)}$$

$$\varepsilon_{yy} = \frac{1+\nu}{E}\left[(1-\nu)\sigma_{yy} - \nu\sigma_{xx}\right] \quad \text{——— (7)}$$

From (6) & (7) we can solve for σ_{xx} & σ_{yy}

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu\varepsilon_{yy}\right] \quad \text{——— (8)}$$

$$\sigma_{yy} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{yy} + \nu\varepsilon_{xx}\right] \quad \text{——— (9)}$$

Also

$$\sigma_{xy} = \frac{1}{2\mu}\varepsilon_{xy} = \frac{1+\nu}{E}\varepsilon_{xy} \quad \text{——— (10)}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -\frac{M(1-\nu^2)}{EI}y \quad \text{———} (11)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{M(1+\nu)\nu}{EI}y \quad \text{———} (12)$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\varepsilon_{xy} = \frac{1}{2}\left[-\frac{M(1-\nu^2)}{EI}x + \frac{M(1-\nu^2)}{EI}x\right]$$

$$\varepsilon_{xy} = 0 \quad \text{———} (13)$$

Substituting (11) and (12) into (9)

$$\sigma_{yy} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{M(1+\nu)\nu}{EI}y + \nu\left(-\frac{M(1-\nu^2)}{EI}y\right)\right]$$

$$\sigma_{yy} = 0 \quad \text{———} (14)$$

Substituting (11) and (12) into (8)

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\left(-\frac{M(1-\nu^2)}{EI}y\right) + \nu\frac{M(1+\nu)\nu}{EI}y\right]$$

$$= \frac{E}{\cancel{(1+\nu)}(1-2\nu)} \cdot \frac{\cancel{(1+\nu)}My}{EI}\left[-(1-\nu)^2 + \nu^2\right]$$

$$= -\frac{\cancel{E}}{\cancel{(1-2\nu)}} \cdot \frac{My}{\cancel{E}I}\cancel{(1-2\nu)}$$

$$\sigma_{xx} = -\frac{My}{I} \quad \text{———} (15)$$

Pure bending of a rectangular beam in the x,y plane

3. **(25 points)** (Pr. 3.7, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) With respect to $Ox_1x_2x_3$ the stress state is given in terms of the coordinates by the matrix

$$[\sigma_{ij}] = \begin{bmatrix} x_1x_2 & x_2^2 & 0 \\ x_2^2 & x_2x_3 & x_3^2 \\ 0 & x_3^2 & x_3x_1 \end{bmatrix}$$

Determine

(a) The body force components as functions of the coordinates if the equilibrium equations are to be satisfied everywhere

(b) The stress vector at point P(1,2,3) on the plane whose outward unit normal makes equal angles with the positive coordinate axes.

(a) $\sigma_{ij,j} + F_i = 0$

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} + F_1 = 0 \quad \text{———} (1)$$

$$\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + F_2 = 0 \quad \text{———} (2)$$

$$\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + F_3 = 0 \quad \text{———} (3)$$

From (1)

$$x_2 + 2x_2 + F_1 = 0 \quad \Rightarrow \quad F_1 = -3x_2$$

From (2)

$$x_3 + 2x_3 + F_2 = 0 \quad \Rightarrow \quad F_2 = -3x_3$$

From (3)

$$x_1 + F_3 = 0 \quad \Rightarrow \quad F_3 = -x_1$$

(b) stress vector at point $P(1,2,3)$

$$[\sigma_{ij}] = \begin{bmatrix} (1)(2) & 2^2 & 0 \\ 2^2 & (2)(3) & (3)^2 \\ 0 & (3)^2 & (3)(1) \end{bmatrix} = \begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 9 \\ 0 & 9 & 3 \end{bmatrix}$$

$$\hat{n} = \frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\vec{T}_i^{\,n} = \sigma_{ij} \cdot n_j$$

$$= \begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 9 \\ 0 & 9 & 3 \end{bmatrix} \begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} 6/\sqrt{3} \\ 19/\sqrt{3} \\ 12/\sqrt{3} \end{Bmatrix}$$

$$\vec{T}_i^{\,n} = \frac{1}{\sqrt{3}}\left(6\hat{i} + 19\hat{j} + 12\hat{k}\right)$$

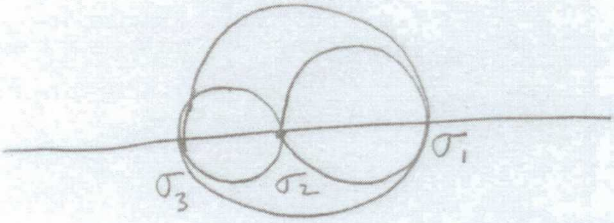
4. **(25 Mase)** (Pr. 3.16, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) At point P, the stress matrix relative to axes $Px_1x_2x_3$ is given in MPa by

$$[\sigma_{ij}] = \begin{bmatrix} 5 & a & -a \\ a & 0 & b \\ -a & b & 0 \end{bmatrix}$$

Where a and b are unspecified. At the same point relative to axes $Px_1^*x_2^*x_3^*$ the matrix is

$$[\sigma_{ij}^*] = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix}$$

If the magnitude of the maximum shear stress at P is 5.5 Mpa, determine σ_1 and σ_{III}



$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 5.5 \text{ MPa}}$$

$$\sigma_1 - \sigma_3 = 11 \text{ MPa} \quad \text{——— (1)}$$

we know that $\sigma_2 = 2$ MPa

Principal stresses may be found by

$$\begin{vmatrix} 5-\lambda & a & -a \\ a & 0-\lambda & b \\ -a & b & 0-\lambda \end{vmatrix} = (5-\lambda)(\lambda^2 - b^2) - a(-a\lambda + ab) - a(ab - \lambda a)$$

$$= 5\lambda^2 - 5b^2 - \lambda^3 + \lambda b^2 + a^2\lambda - a^2 b - a^2 b + a^2\lambda$$

$$= -\lambda^3 + 5\lambda^2 + (b^2 + 2a^2)\lambda - 5b^2 - 2a^2 b = 0$$

characteristic equation is

$$-\lambda^3 + 5\lambda^2 + (b^2 + 2a^2)\lambda - 5b^2 - 2a^2 b = 0 \quad \text{——— (2)}$$

since we know one of the roots of this equation we can write

$$(\lambda-2)(\lambda-\sigma_1)(\lambda-\sigma_3) = (\lambda-2)\left[\lambda^2-(\sigma_3+\sigma_1)\lambda+\sigma_1\sigma_3\right]$$

$$= \lambda^3-(\sigma_3+\sigma_1)\lambda^2+\sigma_1\sigma_3\lambda$$
$$-2\lambda^2+2(\sigma_3+\sigma_1)\lambda-2\sigma_1\sigma_3$$

$$= \lambda^3-(2+\sigma_3+\sigma_1)\lambda^2+\left[\sigma_1\sigma_3+2(\sigma_3+\sigma_1)\right]\lambda$$
$$-2\sigma_1\sigma_3$$

or multibly by -1

characteristic equation becomes

$$-\lambda^3+(2+\sigma_3+\sigma_1)\lambda^2-\left[\sigma_1\sigma_3+2(\sigma_3+\sigma_1)\right]\lambda+2\sigma_1\sigma_3=0 \quad \text{—(3)}$$

equating (2) and (3) we have

$$2+\sigma_3+\sigma_1 = 5$$

$$\therefore \quad \sigma_1+\sigma_3 = 3 \quad \text{——— (4)}$$

using (4) and (1)

$$\sigma_1-\sigma_3 = 11$$
$$+ \quad \sigma_1+\sigma_3 = 3$$

$$2\sigma_1 = 14 \quad \Rightarrow \quad \sigma_1 = 7 \text{ MPa}$$

using (4)

$$7+\sigma_3 = 3 \quad \Rightarrow \quad \sigma_3 = -4 \text{ MPa}$$

5. **(25 puan)** (Pr. 3.22, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) The state of stress referred to axes $Px_1x_2x_3$ is given in MPa by the matrix

$$[\sigma_{ij}] = \begin{bmatrix} 9 & 12 & 0 \\ 12 & -9 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Determine

(a) The normal and shear components, σ_N and σ_S, respectively, on the plane at P whose unit normal is

$$\hat{n} = \frac{1}{5}(4\hat{e}_1 + 3\hat{e}_2)$$

(b) Verify the results determined in (a) by a Mohr's circle construction.

$$n = \frac{4}{5} e_1 + \frac{3}{5} e_2$$

$$\vec{T}_i^n = \sigma_{ij} \cdot n_j = \begin{bmatrix} 9 & 12 & 0 \\ 12 & -9 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{Bmatrix} 4/5 \\ 3/5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 72/5 \\ 21/5 \\ 0 \end{Bmatrix}$$

$$\vec{T}^n = \frac{72}{5}\hat{e}_1 + \frac{21}{5}\hat{e}_2$$

$$|\vec{T}^n| = \sqrt{\left(\frac{72}{5}\right)^2 + \left(\frac{21}{5}\right)^2} = 15 \text{ MPa}$$

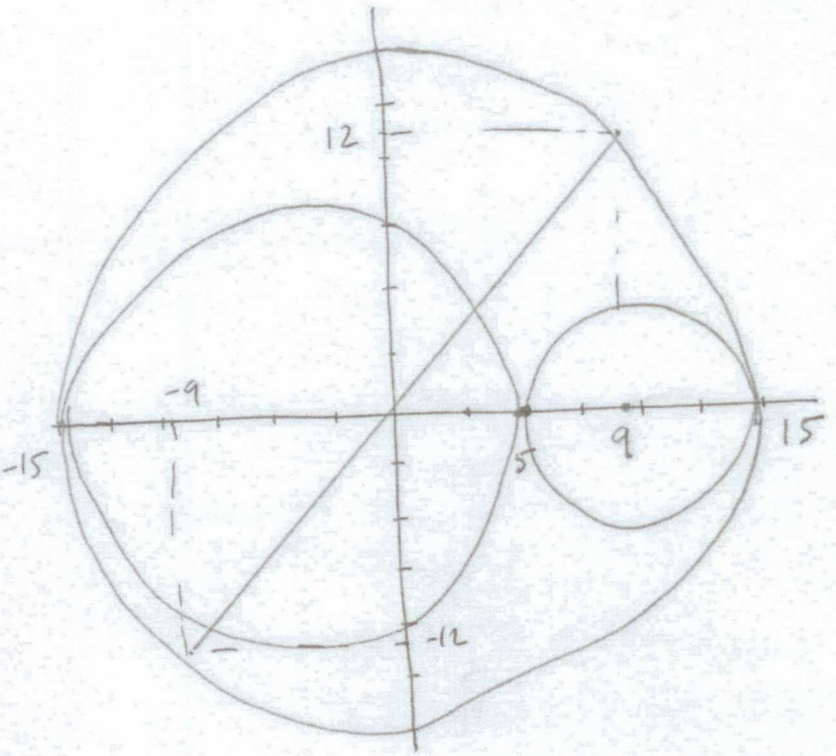
$$\sigma_N = \vec{T}^n \cdot \hat{n} = \left(\frac{72}{5}\hat{e}_1 + \frac{21}{5}\hat{e}_2\right) \cdot \left(\frac{4}{5}\hat{e}_1 + \frac{3}{5}\hat{e}_2\right)$$

$$= \frac{288}{25} + \frac{63}{25} = 14.04 \text{ MPa}$$

$$|\vec{T}^n|^2 = \sigma_N^2 + \sigma_S^2$$

$$\sqrt{\sigma_S^2} = \sqrt{|\vec{T}^n|^2 - \sigma_N^2}$$

$$\sigma_S = 5.28 \text{ MPa}$$



$\sigma_1 = 15$ MPa

$\sigma_2 = 5$ MPa

$\sigma_3 = -15$ MPa