TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK506 THEORY OF ELASTICITY
FALL 2010
Due date: 22.11.2010
HOMEWORK 5

1. (Pr. 6.24, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) For the case of plane stress, let the stress components be defined in terms of the function $\phi = \phi(x_1, x_2)$, known as the Airy stress function, by the relation ships,

   $$\sigma_{11} = \phi_{,22}, \qquad \sigma_{22} = \phi_{,11}, \qquad \sigma_{12} = -\phi_{,12}$$

   Show that ϕ must satisfy the biharmonic equation $\nabla^4\phi = 0$ and that, in the absence of body forces, the equilibrium equations are satisfied identically by these stress components. If $\phi = Ax_1^3x_2^2 - Bx_1^5$ where A and B are constants, determine the relationship between A and B for this to be a valid stress function

2. (Pr. 6.26, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Show that $\phi = x_1^4x_2 + 4x_1^2x_2^3 - x_2^5$ is a valid Airy stress function, that is, that $\nabla^4\phi = 0$, and compute the stress tensor for this case assuming a state of plane strain with $\nu = 0.25$
3. (Pr. 6.29, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Consider the Airy stress function

   $$\phi_5 = D_5x_1^2x_2^3 + F_5x_2^5$$

   (a) Show that for this to be valid stress function, $F = -D_5/5$
   (b) Construct the composite stress function

   $$\phi = \phi_5 + \phi_3 + \phi_2$$

   where

   $$\phi = D_5\left(x_1^2x_2^3 - \frac{1}{5}x_2^5\right) + \frac{1}{2}B_3x_1^2x_2 + \frac{1}{2}A_2x_1^2x_2$$

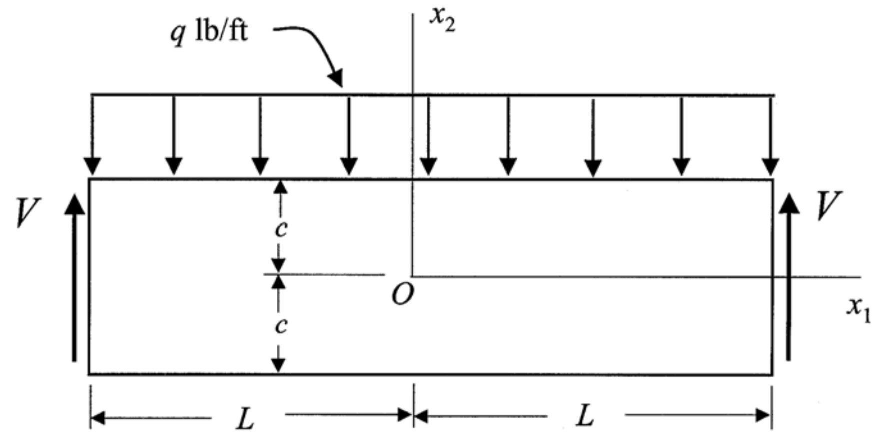
   For this stress function show that the stress components are

   $$\sigma_{11} = D_5\left(6x_1^2x_2 - 4x_2^3\right)$$

   $$\sigma_{22} = 2D_5x_2^3 + B_3x_2 + A_2$$

   $$\sigma_{12} = -6D_5x_1x_2^2 - B_3x_1$$

4. (Pr. 6.30, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) A rectangular beam of width unity and length $2L$ carries a uniformly distributed load of q lb/ft as shown below. Shear forces V support the beam at both ends. List the six boundary conditions for this beam the stresses must satisfy.



5. (Pr. 6.31, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Using boundary conditions 1, 2, and 3 listed in Problem 4 (Pr. 6.30, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase), show that the stresses in Problem 3 (Pr. 6.29, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) require that

$$A_2 = -\frac{q}{2}; \quad B_3 = -\frac{3q}{4c}; \quad D_5 = \frac{q}{8c^3}$$

Thus, for the beam shown the stresses are

$$\sigma_{11} = \frac{q}{2I}\left(x_1^2 x_2 - \frac{2}{3}x_2^3\right)$$

$$\sigma_{22} = \frac{q}{2I}\left(\frac{1}{3}x_2^3 - c^2 x_2 - \frac{2}{3}c^3\right)$$

$$\sigma_{12} = -\frac{q}{2I}\left(x_1^2 x_2^2 + c^2 x_1\right)$$

where $I = \frac{2}{3}c^3$ is the plane moment of inertia of the beam cross section

6. (Pr. 6.32, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Show that, using the stresses calculated in Problem 5 (Pr. 6.31, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase), the boundary conditions 4 and 5 are satisfied, but boundary condition 6 is not satisfied.

7. (Pr. 6.33, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Continuing Problems 6.31 and 6.32, in order for boundary condition 6 to be satisfied and additional term is added to the stress function, namely

$$\phi_3 = D_3 x_2^3$$

Show that, from boundary condition 6,

$$D_3 = \frac{3q}{4c}\left(\frac{1}{15} - \frac{L^2}{6c^2}\right),$$

so that finally

$$\sigma_{11} = \frac{q}{2I}\left(x_1^2 - \frac{2}{3}x_2^2 + \frac{1}{15}c^2 - \frac{1}{6}L^3\right)x_2.$$