TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK506 THEORY OF ELASTICITY
FALL 2010
Due date: 25.10.2010
HOMEWORK 3

1. (Pr. 1.18, Elasticity Tensor, Dyadic, and Engineering Approaches, P.C.Chou and N.J. Pagano) The state of stress at a point within a structure relative to an *xyz* coordinate system is given by,

$$[\sigma]=\begin{bmatrix} 500 & -1500 & 0 \\ 0 & 500 & -500\sqrt{2} \\ 500\sqrt{2} & 2 & 2000 \end{bmatrix}\text{MPa}$$

Determine,
a) The principal stresses
b) The direction cosines of one of the principal axes.

2. (Pr. 3.3, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) The stress tensor at P relative to axes $Px_1x_2x_3$ has components in MPa given by the matrix representation

$$[\sigma_{ij}]=\begin{bmatrix} \sigma_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

where σ_{11} is unspecified. Determine a direction $\hat{n}$ at P for which the plane perpendicular to $\hat{n}$ will be stress-free, that is, for which $t^{(\hat{n})}=0$ on that plane. What is the required value of σ_{11} for this condition?

3. (Pr. 3.8, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Relative to cartesian axes $Ox_1x_2x_3$ a stress field is given by the matrix

$$[\sigma_{ij}]=\begin{bmatrix} \left(1-x_1^2\right)x_2+\frac{2}{3}x_2^3 & -\left(4-x_2^2\right)x_1 & 0 \\ -\left(4-x_2^2\right)x & -\frac{1}{3}\left(x_2^3-12x_2\right) & 0 \\ 0 & 0 & \left(3-x_1^2\right)x_2 \end{bmatrix}$$

(a) Show that the equilibrium equations are satisfied everywhere for zero body forces.
(b) Determine the stress vector at the point $P(2,-1,6)$ of the plane whose equation is $3x_1+6x_2+2x_3=12$.

4. (Pr. 3.14, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase)When referred to principal axes at P, the stress matrix in ksi units is

$$\left[\sigma_{ij}^{*}\right]=\begin{bmatrix}2 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 12\end{bmatrix}$$

If the transformation matrix between the principal axes and axes $Px_1x_2x_3$ is

$$\left[a_{ij}\right]=\frac{1}{\sqrt{2}}\begin{bmatrix}-\frac{3}{5} & 1 & -\frac{4}{5}\\ a_{21} & a_{22} & a_{23}\\ -\frac{3}{5} & -1 & -\frac{4}{5}\end{bmatrix}$$

Where a_{21}, a_{22}, and a_{23} are to be determined, calculate $\left[\sigma_{ij}\right]$.