



MAK506 THEORY OF ELASTICITY

Fall 2010

Due date: September 27, 2010

HOMEWORK 1

1. (Pr. 1.1, Elasticity, M. H. Sadd) For the given second- and first-order tensors

$$a_{ij} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \quad b_i = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

- compute the following quantities: a_{ii} , a_{ij} , $a_{ij}a_{jk}$, $a_{ij}b_j$, $a_{ij}b_ib_j$, b_ib_j , b_ib_i . For each of the quantity, point out whether the result is a scalar, vector or a higher-order tensor.
2. (Pr. 1.4, Elasticity, M. H. Sadd) Explicitly verify the following properties of the Kronecker delta:

$$\delta_{ij}a_j = a_i$$

$$\delta_{ij}a_{jk} = a_{ik}$$

3. (Pr. 2.5, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) Expand the following expressions involving Kronecker deltas, and simplify where possible.

(a) $\delta_{ij}\delta_{ij}$, (b) $\delta_{ij}\delta_{jk}\delta_{ik}$, (c) $\delta_{ij}\delta_{jk}$, (d) $\delta_{ij}A_{ik}$

4. (Pr. 2.19, Continuum Mechanics for Engineers, G. Thomas Mase and George E. Mase) The angles between the respective axes of the $Ox'_1x'_2x'_3$ and the $Ox_1x_2x_3$ Cartesian systems are given by the table below

	x_1	x_2	x_3
x'_1	45°	90°	45°
x'_2	60°	45°	120°
x'_3	120°	45°	60°

Determine

- (a) The transformation matrix between the two sets of axes, and show that it is a proper orthogonal transform.
- (b) The equation of the plane $x_1 + x_2 + x_3 = 1/\sqrt{2}$ in its primed axes form, that is, in the form $b_1x'_1 + b_2x'_2 + b_3x'_3 = b$