q. 1

$$* \quad a_0 = \frac{1}{3}\int_0^3 f(x)\,dx = \frac{1}{6}$$

$$* \quad a_n = \frac{1}{\left(\frac{3}{2}\right)}\int_0^3 f(x)\cdot \cos\frac{n\pi x}{\left(\frac{3}{2}\right)}\,dx = \frac{2}{3}\int_2^3 (x-2)\cos\left(\frac{2n\pi x}{3}\right)dx$$

$$a_n = \frac{2}{3}\int_2^3 x\cdot\cos\frac{2n\pi}{3}x\,dx - \frac{4}{3}\int_2^3 \cos\underbrace{\frac{2n\pi}{3}}_{a}x\,dx$$

$$a_n = \frac{2}{3}\left[\frac{1}{a^2}\cdot\cos ax + \frac{x}{a}\cdot\sin ax\right]_2^3 - \frac{4}{3}\left[\frac{3}{2n\pi}\cdot\sin ax\right]_2^3$$

$$a_n = \frac{3}{2n^2\pi^2} - \frac{3}{2n^2\pi^2}\cdot\cos\frac{4n\pi}{3} - \cancel{\frac{2}{n\pi}\sin\frac{4n\pi}{3}} + \cancel{\frac{2}{n\pi}\cdot\sin\frac{4n\pi}{3}}$$

$$a_n = \frac{3}{2n^2\pi^2} - \frac{3}{2n^2\pi^2}\cdot\cos\left(\frac{4n\pi}{3}\right)$$

$$* \quad b_n = \frac{2}{3}\int_0^3 f(x)\cdot\sin\left(\frac{2n\pi}{3}\right)x\,dx$$

$$b_n = \frac{2}{3}\int_2^3 (x-2)\cdot\sin\frac{2n\pi}{3}x\,dx$$

$$b_n = \frac{2}{3}\int_2^3 x\cdot\sin\frac{2n\pi x}{3}\,dx - \frac{4}{3}\int_2^3 \sin\frac{2n\pi}{3}x\,dx$$

$$b_n = \frac{2}{3}\left(-x\,\frac{\cos ax}{a} + \frac{\sin ax}{a^2}\right)\Bigg|_2^3 + \frac{4}{3}\cdot\frac{3}{2n\pi}\cdot\cos\frac{2n\pi}{3}x\,\Bigg]_2^3$$

$$b_n = \frac{2}{3}\left[-\frac{9\cos 2n\pi}{2n\pi} + 0 + \frac{2\cos(4n\pi/3)}{(2n\pi/3)} - \sin\left(\frac{4n\pi}{3}\right)\cdot\frac{9}{4n^2\pi^2} \right.$$

$$\left. + \frac{4}{3}\cdot\frac{3}{2n\pi}\cdot\cos\frac{2n\pi x}{3}\right)\Big|_2^3$$

$$b_n = \frac{-3}{n\pi} + \frac{2}{n\pi} - \frac{3}{2n^2\pi^2}\cdot\sin\left(\frac{4n\pi}{3}\right)$$

$$b_n = \frac{-1}{n\pi} - \frac{3}{2n^2\pi^2}\cdot\sin\left(\frac{4n\pi}{3}\right)$$

$$f_{(x)} = a_0 + \sum_{n=1}^{\infty}\left(a_n\cdot\cos\frac{n\pi x}{(\frac{3}{2})} + b_n\sin\frac{n\pi x}{(3/2)}\right)$$

$$f_{(x)} = \frac{1}{6} + \sum_{n=1}^{\infty}\left[\left(\frac{3}{2n^2\pi^2} - \frac{3}{2n^2\pi^2}\cos\left(\frac{4n\pi}{3}\right)\right)\cos\frac{2n\pi x}{3}\right.$$

$$\left. + \left(\frac{-1}{n\pi} - \frac{3}{2n^2\pi^2}\cdot\sin\left(\frac{4n\pi}{3}\right)\right)\cdot\sin\frac{2n\pi x}{3}\right]$$

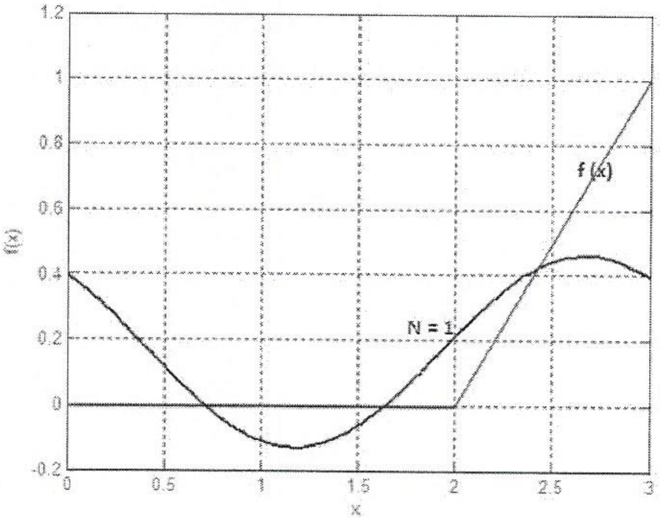
if $x = 2$ & $N = 1$

$\hookrightarrow f_{(2)}\Big|_{N=1} = 0.2144$

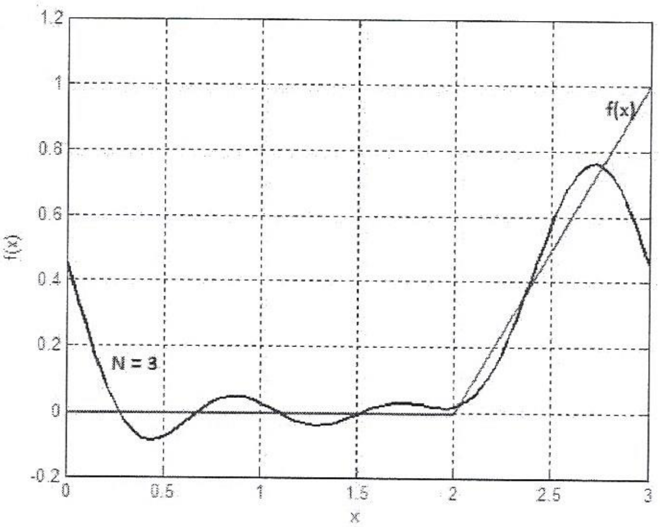
if $x = 2$ & $N = 3$ $\longrightarrow f_{(2)}\Big|_{N=3} = 0.0195$

if $x = 2$ & $N = 5$ $\longrightarrow f_{(2)}\Big|_{N=5} = 0.0099$

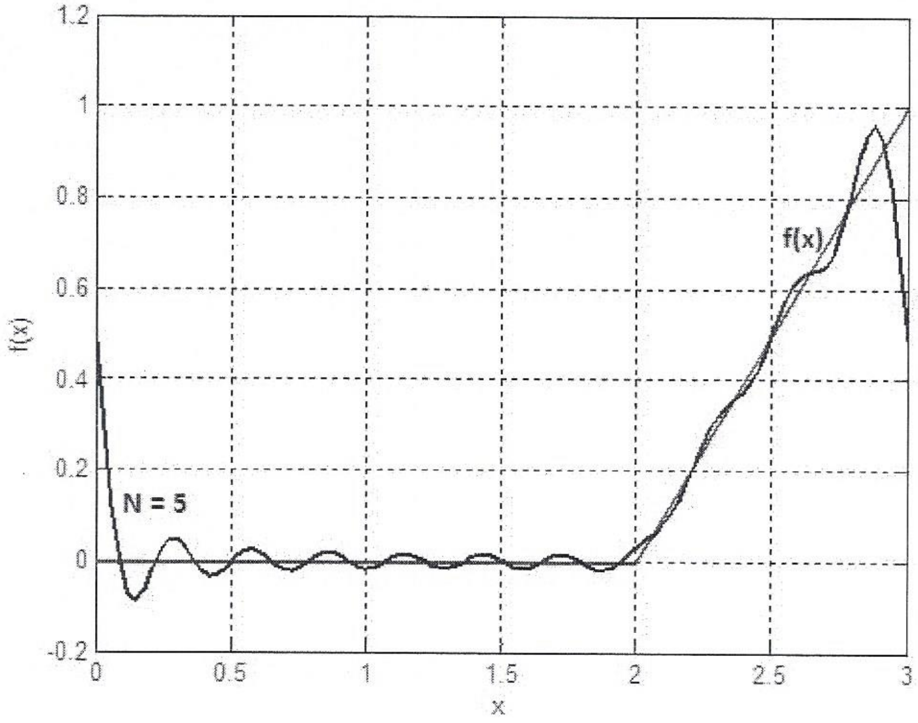
Q1.
N = 1 & x = 2



N = 3 & x = 2



Q1.
N = 5 & x = 2



q4.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad u = X.Y$$

$$X''Y + X.Y'' = 0 \quad \rightarrow \quad \frac{X''}{X} + \frac{Y''}{Y} = 0$$

$$\frac{X''}{X} = \frac{-Y''}{Y} = \text{constant} = K$$

$$\rightarrow K = +\mu^2 \quad \rightarrow X'' - \mu^2 X = 0 \quad \rightarrow X_{(x)} = c_1 e^{\mu x} + c_2 e^{-\mu x}$$

$$\rightarrow Y'' + \mu^2 Y = 0 \quad \rightarrow Y_{(y)} = c_3 \cos\mu y + c_4 \sin\mu y$$

$$\rightarrow k = 0 \quad \rightarrow X'' = 0 \quad \rightarrow X = c_5 x + c_6$$

$$\rightarrow Y'' = 0 \quad \rightarrow Y = c_7 y + c_8$$

$$(c_6 + c_5 x)(c_8 + c_7 y) + (c_1 e^{\mu x} + c_2 e^{-\mu x})(c_3 \cos\mu y + c_4 . \sin\mu y)$$

$$(A + By + Cx + D) + (E e^{\mu x} \cos\mu y + F e^{\mu x} \sin\mu y) + (I e^{-\mu x} \cos\mu y + G e^{-\mu x} \sin\mu y)$$

B.C :

$x \rightarrow \infty$ $\quad u$ must be finite $\Rightarrow$ $C = 0$, $E = 0$, $F = 0$

$$S + By + e^{-\mu x}\left(I \cos\mu y + G \sin\mu y\right)$$

$$u(x,0) = 10 \longrightarrow S + e^{-\mu x}\left(I \cos(0) + G \sin(0)\right) = 0$$

$$S = 10$$
$$I = 0$$

$$u(x,y) = 10 + By + G e^{-\mu x} . \sin(\mu y)$$

$$u_y(x,1) = 0 \longrightarrow B + G . e^{-\mu x} . \mu . \cos\mu = 0 \longrightarrow B = 0$$

$$\cos\mu = 0 \rightarrow \mu = \frac{n\pi}{2} \longrightarrow n = 1, 3, 5, \dots$$

$$u(x,y) = 10 + G_n . e^{\left(-\frac{n\pi x}{2}\right)} . \sin \frac{n\pi y}{2} \qquad n = 1, 3, 5, \dots$$

$$u(0,y) = 0 \Rightarrow 10 + G_n . \sin\left(\frac{n\pi y}{2}\right) = 0$$

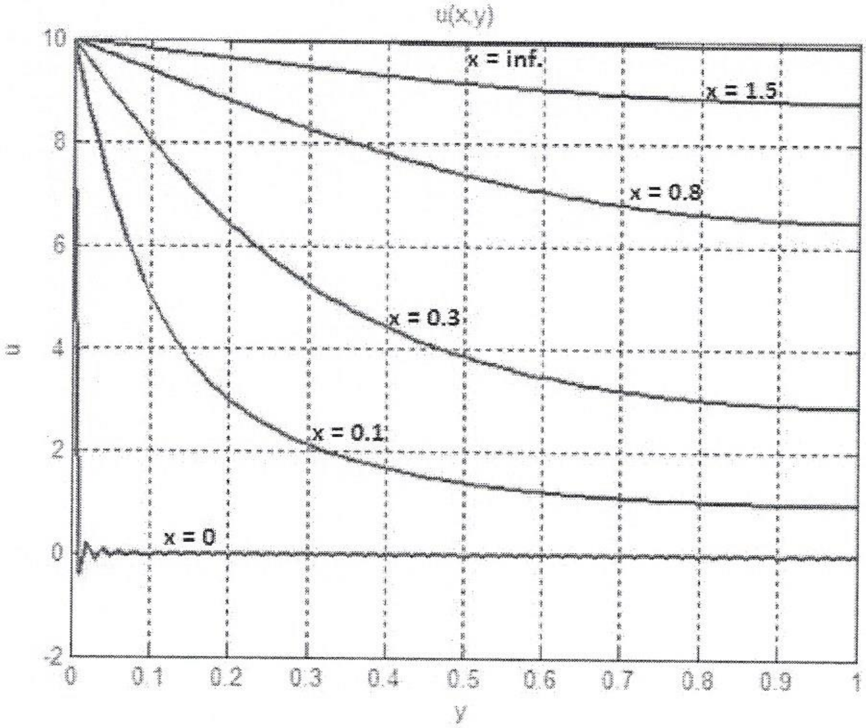
$$\sum_{n=1,3,5,\dots}^{\infty} G_n . \sin\left(\frac{n\pi y}{2}\right) = -10$$

$$G_n = \frac{2}{1}\int_0^1 -10 . \sin \frac{n\pi y}{2}\, dy = +20 . \frac{2}{n\pi}\left(\cos \frac{n\pi y}{2}\right)\Big|_0^1$$

$$G_n = \frac{+20}{n\pi} \times 2 \times \left[\underbrace{\cos \frac{n\pi}{2}}_{0} - \cos(0)\right] = \frac{40}{n\pi}\left[0 - 1\right] = \frac{-40}{n\pi}$$

$$u(x,y) = 10 + \sum_{n=1,3,5}^{\infty} \frac{-40}{n\pi} \cdot \sin\left(\frac{n\pi y}{2}\right) \cdot e^{-\left(\frac{n\pi}{2} x\right)}$$

Q4.


u(x,y)
x = inf.
x = 1.5
x = 0.8
x = 0.3
x = 0.1
x = 0
u
y
10
8
6
4
2
0
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

9.5

$\nabla^2 u = 0$

$u(b,\theta) = f(\theta)$ $\qquad f(\theta) = \begin{cases} 50 & ; \ 0<\theta<\pi \\ 0 & ; \ \pi<\theta<2\pi \end{cases}$

$$\nabla^2 u = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$$

$$u(r,\theta) = R(r) \cdot \Theta(\theta)$$

$$R''\Theta + \frac{1}{r} \cdot R'\Theta + \frac{1}{r^2} R\Theta'' = 0$$

$$r^2 R''\Theta + rR'\Theta + R\Theta'' = 0$$

$$\frac{r^2 R'' + rR'}{R} = -\frac{\Theta''}{\Theta} = \text{constant} = \mu$$

case 1) $\mu = +k^2$ $\qquad r^2 R'' + rR' - k^2 R = 0$

$$R = r^\lambda \rightarrow r^2 \cdot \lambda(\lambda-1) r^{\lambda-2} + r\lambda r^{\lambda-1} - k^2 r^\lambda = 0$$

$$r^\lambda\left(\lambda(\lambda-1) + \lambda - k^2\right) = 0$$

$$R(r) = Ar^k + Br^{-k}$$

$$\Theta'' + k^2\Theta = 0 \longrightarrow \Theta(\theta) = C \sin k\theta + D \cos k\theta$$

Case 2) $\mu = 0$

$$r^2 R'' + r R' = 0$$

$$R(r) = E \ln r + F$$

$$\Theta'' = 0 \longrightarrow \Theta(\theta) = I + H\theta$$

$$u(r,\theta) = (I + H\theta)(F + E \ln r) + (A r^k + B r^{-k})(C \sin k\theta + D \cos k\theta)$$

$$r \to 0 \Rightarrow u(r,\theta) \longrightarrow \text{finite} \Rightarrow \begin{cases} E = 0 \\ B = 0 \end{cases}$$

$$u(r,\theta) = F(I + H\theta) + A r^k (C \sin k\theta + D \cos k\theta)$$

$$u(r,\theta) = G + P\theta + r^k (T \sin k\theta + S \cos k\theta)$$

2π - periodic in θ:

$$u(r, \theta + 2\pi) = u(r,\theta)$$

$$P = 0$$

$$\cos k\theta = \cos k(\theta + 2\pi) \Rightarrow$$

$$\cos k\theta = \cos k\theta . \cos 2\pi k - \sin k\theta \sin 2\pi k$$

$\cos 2\pi k = 1 \quad \rightarrow \quad k = 1, 2, 3, \ldots$

$\sin 2\pi k = 0 \quad \rightarrow \quad k = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

$\Rightarrow \quad k = n = 1, 2, 3, \ldots$

$$u_n(r,\theta) = G + r^n (T \sin n\theta + S \cos n\theta)$$

$$u(r,\theta) = G + \sum_{n=1}^{\infty} r^n (T_n \cdot \sin n\theta + S_n \cdot \cos n\theta)$$

$$u(b,\theta) = u(1,\theta) = f(\theta) = G + \sum_{n=1}^{\infty} (1)^n (T_n \cdot \sin n\theta + S_n \cdot \cos n\theta)$$

$$G = \frac{1}{2\pi} \int_0^{2\pi} f(\theta)\, d\theta = \frac{1}{2\pi} \int_0^{\pi} 50\, d\theta = \frac{50}{2\pi} \cdot \frac{\theta}{1} \Big]_0^{\pi} = \frac{50}{2}$$

$$S_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cdot \cos n\theta\, d\theta = \frac{1}{\pi} \int_0^{\pi} 50 \cos n\theta\, d\theta$$

$$S_n = \frac{50}{\pi} \cdot \frac{\sin n\theta}{n} \Big]_0^{\pi} = \frac{50 \sin n\pi}{n} = 0$$

$$T_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cdot \sin n\theta\, d\theta = \frac{1}{\pi} \int_0^{\pi} 50 \cdot \sin n\theta\, d\theta = \frac{-50}{\pi} \cdot \frac{\cos n\theta}{n} \Big|_0^{\pi}$$

$$T_n = \frac{50}{n\pi} \left(1 - (-1)^n\right)$$

$$u(r,\theta) = \frac{50}{2} + \frac{50}{\pi} \sum_{n=1}^{\infty} \frac{r^n}{1} \cdot \frac{1 - (-1)^n}{n} \cdot \sin(n\theta)$$

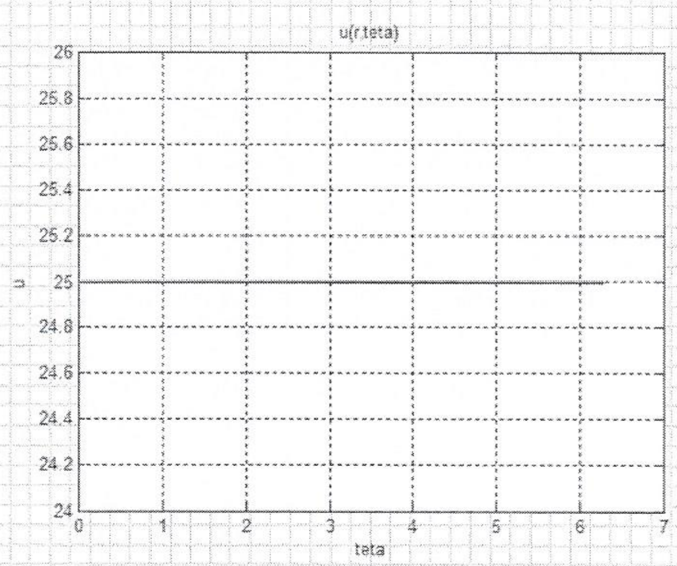
$$u(r,\theta) = \frac{50}{2} + \frac{50}{\pi} \sum_{n=1}^{\infty} r^n \cdot \frac{1 - (-1)^n}{n} \cdot \sin n\theta$$

TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

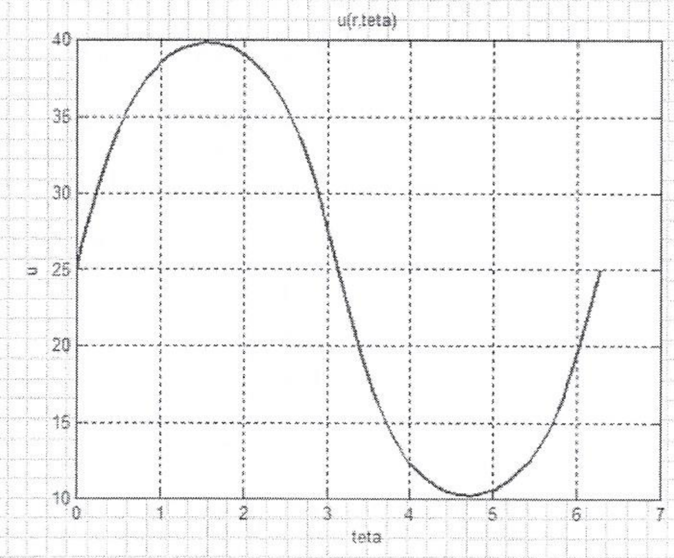
MAK 501 Engineering Mathematics – 2014 Fall

Q.5

at r = 0



at r = 0.5



Q.5

at r = 1