TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK 501 ENGINEERING MATHEMATICS

FALL 2016

Due Date: 14.12.2016- Wednesday* (18:30)

HOMEWORK 6

1. The reciprocity relations some entries from transform tables are:

$$F\{\hat{f}(x)\} = 2\pi f(-\omega) \tag{1}$$

$$F^{-1}\{f(-\omega)\} = \frac{\hat{f}(x)}{2\pi} \tag{2}$$

$$e^{-a|x|}(a > 0) \Rightarrow \frac{2a}{\omega^2 + a^2} \tag{3}$$

$$H(x + a) - H(x - a) \Rightarrow \frac{2\sin \omega a}{\omega} \tag{4}$$

$$H(-x)e^{ax}(Rea > 0) \Rightarrow \frac{1}{a - i\omega} \tag{5}$$

a Derive the relations 1 and 2.

b To illustrate, use 1 and 3 to show that

$$F\left\{\frac{2a}{x^2 + a^2}\right\} = 2\pi e^{-a|\omega|}, (a > 0)$$

or equivalently,

$$F\left\{\frac{1}{x^2 + a^2}\right\} = \frac{\pi}{a}e^{-a|\omega|}, (a > 0)$$

c Use 1 and 4 to show that

$$F\left\{\frac{\sin ax}{x}\right\} = \pi\left[H(\omega + a) - H(\omega - a)\right], (a > 0)$$

d Use 1 and 5 to show that

$$F\left\{\frac{1}{a - ix}\right\} = 2\pi H(\omega)e^{-a\omega}, (Re(a) > 0)$$

2. PDE of a rad which is translating rightward with constant speed v is $\alpha^2 u_{xx} = u_t + V u_x$, where $V = v/c$ and c is the specific heat of the material. Use the Fourier transform to solve the problem.

$$\alpha^2 u_{xx} = u_t + V u_x, \quad (-\infty < x < \infty, 0 < t < \infty)$$

$$u(x,0) = f(x), \quad (-\infty < x < \infty)$$

where $u \to 0$ and $u_x \to 0$ as $x \to \pm\infty$.

3. Consider the boundary-value problem

$$u'' - 9u = 50e^{-2x}, \quad (0 < x < \infty) \tag{6}$$

$$u(0) = u_0, \quad u(\infty) - bounded \tag{7}$$

Solve using Laplace transform. (HINT: When you take the transform of u'' you will be faced with a $u'(0)$ term, which is not prescribed in 7. Thus, call that quantity C, say, and evaluate it by imposing on your solution the condition that $u(\infty)$ be bounded, at the end.)!YOU DO NOT HAVE TO FOLLOW THE HINT!

4. Solve the following ordinary differential equation using Laplace transform.

$$y'' + 5y' - 6y = \begin{cases} 0 & 0 < t < 3 \\ e^t & t > 3 \end{cases}$$

$$y(0) = 3, \quad y'(0) = 7$$

Due date is **Monday 12th of December**. For each day delay **15 points** will be reduced.

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