TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK 501 ENGINEERING MATHEMATICS

FALL 2016

HOMEWORK 5

Due 24 Nov 2016

1.

Solve Laplace Equation inside a circular cylinder subject to the boundary conditions,

$$u(r,\theta,0) = \alpha(r) sin7\theta, \;\; u(r,\theta,H) = 0, \;\; u(a,\theta,z) = 0$$

2.

Solve the heat equation,

$$\frac{\partial u}{\partial t} = k\nabla^2 u$$

inside a quarter circular cylinder ($0 < \theta < \frac{\pi}{2}$ with radius a and height H) subject to the initial condition

$$u(r,\theta,z,0) = f(r,\theta,z)$$

Briefly explain what temperature distribution you expect to be approached as t goes to infinity. Consider the following boundary conditions,

$$\frac{\partial u}{\partial z}(r,\theta,0) = 0, \;\; \frac{\partial u}{\partial z}(r,\theta,H) = 0, \;\; \frac{\partial u}{\partial \theta}(r,0,z) = 0, \;\; \frac{\partial u}{\partial \theta}(r,\pi/2,z) = 0, \;\; \frac{\partial u}{\partial r}(a,\theta,z) = 0$$

3.

Use the method of eigenfunction expansions to solve the given problem in the unit disk,

$$\nabla^2 u = 2 + r^3 cos3\theta, \;\; u(1,\theta) = 0$$

4.

a)

you are asked to use the eigenfunction expansions method to solve the nonhomogeneous heat boundary value problem, with time-dependent heat source,

$$\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right) + q(r,\theta,t),$$
$$u(a,\theta,t) = 0,$$
$$u(r,\theta,0) = f(r,\theta),$$

where $0 < r < a$, $0 < \theta < 2\pi$, and $t > 0$. Justify the following steps.

(a) Let

$$u(r,\theta,t) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} J_m(\lambda_{mn} r)(A_{mn}(t)\cos m\theta + B_{mn}(t)\sin m\theta),$$
$$f(r,\theta) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} J_m(\lambda_{mn} r)(a_{mn}\cos m\theta + b_{mn}\sin m\theta),$$
$$q(r,\theta,t) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} J_m(\lambda_{mn} r)(c_{mn}(t)\cos m\theta + d_{mn}(t)\sin m\theta).$$

(Why should this be your starting point?) What are a_{mn}, b_{mn}, $c_{mn}(t)$, and $d_{mn}(t)$, in terms of f and q?

(b) Show that A_{mn} and B_{mn} are solutions of the following initial value problems:

$$A'_{mn}(t) + \lambda_{mn}^2 A_{mn}(t) = c_{mn}(t), \quad A_{mn}(0) = a_{mn};$$
$$B'_{mn}(t) + \lambda_{mn}^2 B_{mn}(t) = d_{mn}(t), \quad B_{mn}(0) = b_{mn}.$$

(c) Complete the solution by showing that

$$A_{mn}(t) = e^{-\lambda_{mn}^2 t}\left(a_{mn} + \int_0^t e^{\lambda_{mn}^2 s} c_{mn}(s)\,ds\right)$$

and

$$B_{mn}(t) = e^{-\lambda_{mn}^2 t}\left(b_{mn} + \int_0^t e^{\lambda_{mn}^2 s} d_{mn}(s)\,ds\right).$$

when $c = 1$, $f = 1$, and $q(r,\theta,t) = e^{-t}$

b)

Plot the temperature of the center and describe what happens as $t \to \infty$.