TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK 501 ENGINEERING MATHEMATICS

FALL 2016

Due Date: 11/11/2016

HOMEWORK 4

1. Consider a slightly damped vibrating string that satisfies,

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2} - \beta \frac{\partial u}{\partial t}$$

   a) Briefly explain why $\beta > 0$
   b) Determine the solution (by seperation of variables) that satisfies the boundary conditions,

$$u(0,t) = 0$$

$$u(L,t) = 0$$

   and the initial conditions,

$$u(x,0) = f(x)$$

$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

   You can assume that this fractional coefficient β is relatively small. $(\beta^2 < 4\pi^2 \rho_0 T_0 / L^2)$

2. Slove Laplace's equation inside a rectangle $0 \le x \le L, 0 \le y \le H$, with the following boundary conditions,

$$u(0,y) = 0, u(L,y) = 0, u(x,0) - \frac{\partial u}{\partial y}(x,0) = 0, u(x,H) = f(x)$$

3. Slove Laplace's equation inside a $90°$ sector of a circular annulus $(a < r < b, 0 < \theta < \frac{\pi}{2})$ subject to the boundary conditions,

   a) $u(r,0) = 0, u(r,\pi/2) = 0, u(a,\theta) = 0, u(b,\theta) = f(\theta)$
   b) $u(r,0) = 0, u(r,\pi/2) = f(r), u(a,\theta) = 0, u(b,\theta) = 0$

4. Solve the heat equation,

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

$$0<x<L, t>0$$

which subject to,

$$\frac{\partial u}{\partial x}(0,t)=0, t>0$$

$$\frac{\partial u}{\partial x}(L,t)=0, t>0$$

and the initial condition,

$$u(x,0)=6+4\cos\frac{3\pi x}{L}$$

5. Solve the Poisson's equation,

$$\nabla^2 u=e^{2y} sinx$$

subject to the following boundary conditions,

$$u(0,y)=0\, u(x,0)=0\, u(\pi,y)=0\, u(x,L)=f(x)$$