TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK 501 ENGINEERING MATHEMATICS

FALL 2016

Due Date:21.10.2016- Friday* (08:30)

HOMEWORK 3

1. Solve below problems using separation of variables. Take the separation constant to be $-\lambda^2$. Express your result by superposing X an T. If the PDE cannot be separated, state that. (If the last digit of your ID number is odd, solve a and c; if even, solve b and d)

a. $u_{xx} = u_t + 3u$

b. $u_{xx} + 2u_x = u_t$ HINT: In this case you should find that the value of K that needs to be distinguished [as we distinguished the case K=0 in (9) and (10)] is K=1, not K=0.

c. $u_{xx} + 2u_{xt} = u_t$

d. $u_{xx} + 2u_{xt} = u_{tt}$

2. Can we use superposition to conclude from

$$X = Acos\lambda X + Bsin\lambda X + D + Ex$$

and

$$T = Fexp(-\lambda^2\alpha^2 t) + G$$

that

$$u(x,t) = [Acos\lambda X + Bsin(\lambda x) + D + Ex][F\,exp(-\lambda^2\alpha^2 t) + G]$$

Explain.

3. The temperature distribution u(x,t) in a 2-m long brass rod is governed by the problem

$$\alpha^2 u_{xx} = u_t, (0 < x < 2, 0 < t < \infty)$$

$$u(0,t) = u(2,t) = 0, (t > 0)$$

$$u(x,0) = \begin{cases} 50x, & (0 < x < 1) \\ 100 - 5x, & (1 < x < 2) \end{cases}$$

where $\alpha^2 = 2.9 * 10^{-5}\ m^2/sec$

a. Determine the solution for $u(x,t)$

b. Compute the temperature at the midpoint of the rod at the end of 1 hour.

c. Compute the time it will take for the temperature at that point to diminish to 5°C.

d. Compute the time it will take for the temperature at that point to diminish to 1°C.

4. Constant forcing function, the governing PDE is

$$c^2 y_{xx} = y_{tt} + g \tag{1}$$

Solve PDE subject to conditions

$$y(0,t) = 0, y(L,t) = 0,$$

$$y(x,0) = f(x), y_t(x,0) = 0,$$

leaving expansion coefficients in integral form. HINT: The form y(x,t)=X(x)T(t) gives

$$\frac{X''}{X} = \frac{1}{c^2}\frac{T''}{T} + \frac{g}{c^2 XT} \tag{2}$$

Because of the last term in above equation, which contains both x and t dependence, we are unable to successfully complete the separation process (i.e., we are unable to get all of the x dependence on one side of the equation and all of the t dependence on the other). Thus, we suggest seeking y in the form

$$y(x,t) = y_p(x) + X(x)T(t) \tag{3}$$

instead. Putting Eq. 3 into Eq. 1, obtain

$$c^2 y_p'' + c^2 X''T = XT'' + g \tag{4}$$

Thus, we can remove the unwelcome g term by setting $c^2 y_p'' = g$. Then we can complete the separation of variable in Eq. 4 as usual. Mathematically, $y_p(x)$ is a particular solution of the nonhomogeneus equation Eq. 1 since it satisfies the full equation (7.1), and XT is a solution of the associated homogeneous equation $c^2 y_{xx} = y_{tt}$. But in physical terms you will find that it is simply the "static sag" of the string due to gravity, satisfying the problem

$$c^2 y_p''(x) = g, y_p(0) = 0, y_p(L) = 0$$

Due date is **Friday 21th of October**. For each day delay **15 points** will be reduced.

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