TOBB ETÜ
University of Economics & Technology

MAK 413 Mechanics of Composite Materials

Spring 2018 Midterm examination

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Question No	Max. Point	Point
1	20	12
2	20	20
3	25	25
4	25	25
5	30	30
Total	120	112

Instructions

1. Yükseköğretim Kurumları 2015 Öğrenci Disiplin Yönetmeliği Madde 5-d ve 7-e'ye göre "sınavlarda kopyaya teşebbüs veya kopya çekmek yapmak veya yaptırmak veya bunlara teşebbüs etmek" fiilinin suçu YÜKSEKÖĞRETİM KURUMUNDAN BİR VEYA İKİ YARIYIL İÇİN UZAKLAŞTIRMA cezasıdır.

UYARI VE KURALLARI OKUDUM.

Signature: ____________

Good luck!

Question 1 (20 points)

1. Composite materials are classified based on
(a) Type of matrix (b) Size and shape of reinforcement (c) Both (d) None

2. Major load carrier in dispersion-strengthened composites
(a) Matrix (b) Fiber (c) Both (d) Can't define

3. Usually softer constituent of a composite is
(a) Matrix (b) Reinforcement (c) Both are of equal strength (d) Can't define

4. Usually harder constituent of a composite is
(a) Matrix (b) Reinforcement (c) Both are of equal strength (d) Can't define

5. Last constituent to fail in fiber reinforced compsites
(a) Matrix (b) Fiber (c) Both fails at the same time (d) Can't define

6. Al-alloys for engine/automobile parts are reinforced to increase their
(a) Strength (b) Wear resistance (c) Elastic modulus (d) Density

7. Mechanical properties of fiber-reinforced composites depend on
(a) Properties of constituents
(b) Interface strength
(c) Fiber length, orientation, and volume fraction
(d) All the above

8. Longitudinal strength of fiber reinforced composite is mainly influenced by
(a) Fiber strength (b) Fiber orientation (c) Fiber volume fraction (d) Fiber length

9. The following material can be used for filling in sandwich structures
(a) Polymers (b) Cement (c) Wood (d) All

10. Not an example for laminar composite
(a) Wood (b) Bimetallic (c) Coatings/Paints (d) Claddings

Question 2 (20 points)

The reduced stiffness matrix $[Q]$ is given for a unidirectional lamina is given as follows:

$$[Q] = \begin{bmatrix} 5.6810 & 0.3164 & 0 \\ 0.3164 & 1.2170 & 0 \\ 0 & 0 & 0.6006 \end{bmatrix} Msi.$$

What are the four engineering constants, E_1, E_2, ν_{12}, and G_{12}, of the lamina ?

$$[S] = [Q]^{-1} = \begin{bmatrix} 0.1786 & -0.0464 & 0 \\ -0.0464 & 0.8338 & 0 \\ 0 & 0 & 1.6650 \end{bmatrix} Msi^{-1}$$

$$E_1 = \frac{1}{S_{11}} = \frac{1}{0.1786} = 5.5991 \text{ Msi}$$

$$E_2 = \frac{1}{S_{22}} = \frac{1}{0.8338} = 1.1993 \text{ Msi}$$

$$\nu_{12} = -\frac{S_{12}}{S_{11}} = -\frac{-0.0464}{0.1786} = 0.2598$$

$$G_{12} = \frac{1}{S_{66}} = \frac{1}{1.6650} = 0.6006 \text{ Msi}$$

Question 3 (25 points)

What is the relationship between the elements of the transformed compliance matrix $[\bar{S}]$ for a 0 and 90° lamina?

$0° \rightarrow \sin 0° = 0$; $\cos 0° = 1$		$90° \rightarrow \cos 90° = 0$; $\sin 90° = 1$
$\bar{S}_{11} = S_{11} c^4 = S_{11}$		$\bar{S}_{11} = S_{22} s^4 = S_{22}$
$\bar{S}_{12} = S_{12} (c^4) = S_{12}$	=	$\bar{S}_{12} = S_{12} (s^4) = S_{12}$
$\bar{S}_{22} = S_{22} c^4 = S_{22}$		$\bar{S}_{22} = S_{11} s^4 = S_{11}$
$\bar{S}_{16} = 0$	=	$\bar{S}_{16} = 0$
$\bar{S}_{26} = 0$	=	$\bar{S}_{26} = 0$
$\bar{S}_{66} = S_{66} (c^4) = S_{66}$	=	$\bar{S}_{66} = S_{66} (s^4) = S_{66}$

For a 0° and 90° lamina, $\bar{S}_{12}$, $\bar{S}_{16}$, $\bar{S}_{26}$ and $\bar{S}_{66}$ have same values for both of them.

$\bar{S}_{11}/0° = \bar{S}_{22}/90°$ ✓

$\bar{S}_{22}/0° = \bar{S}_{11}/90°$ ✓

Question 4 (25 points)

The tensile modulus of a 0°, 90°, and 45° graphite/epoxy ply is measured as follows to give $E_1 = 26.25$ Msi, $E_2 = 1.494$ Msi, $E_x = 2.427$ Msi for the 45° ply, respectively.

(a) What is the value Ex for a 60° ply?

(b) Can you calculate the values of ν_{12} and G_{12} from the previous three measured values of elastic moduli?

$E_1/0° = 26.25$ Msi

$E_2/90° = 1.494$ Msi

$E_x/45° = 2.427$ Msi

$\Big\} \; E_x/60° = ?$

(a) $S_{11} = \frac{1}{E_1} = \frac{1}{26.25} = 0.0381 \text{ Msi}^{-1}$; $S_{22} = \frac{1}{E_2} = \frac{1}{1.494} = 0.6693 \text{ Msi}^{-1}$

$$\bar{S}_{11} = S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4$$

$$\Rightarrow E_x/45° = 2.427 \text{ Msi} = \frac{1}{\bar{S}_{11}/45°} = \frac{1}{0.0381\cos^4 45 + (2S_{12}+S_{66})\sin^2 45\cos^2 45 + 0.6693\sin^4 45}$$

$$2.427 \text{ Msi} = \frac{1}{0.009525 + 0.25(2S_{12}+S_{66}) + 0.167325}$$

$$\rightsquigarrow (2S_{12} + S_{66}) = 0.57078505/0.60675 \cong 0.9407 \text{ Msi}^{-1}$$

$$\Rightarrow E_x/60° = \frac{1}{\bar{S}_{11}/60°} = \frac{1}{0.0381\cos^4 60 + 0.9407\sin^2 60\cos^2 60 + 0.6693\sin^4 60} \quad 0.5552$$

$$\boxed{E_x/60° = 1.8010 \text{ Msi}}$$

✓

(b) $\nu_{12} = -\frac{S_{12}}{S_{11}}$; $G_{12} = \frac{1}{S_{66}}$

The values of S_{12} and S_{66} are not known. So, the values of ν_{12} and G_{12} cannot be calculated.

(b) Maximum Strain Failure $\Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [S] \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$; $-(\varepsilon_1^c)_{ult} < \varepsilon_1 < (\varepsilon_1^T)_{ult}$, $-(\varepsilon_2^c)_{ult} < \varepsilon_2 < (\varepsilon_2^T)_{ult}$, $-(\gamma_{12})_{ult} < \gamma_{12} < (\gamma_{12})_{ult}$

$$(\varepsilon_1^c)_{ult} = \frac{(\sigma_1^c)_{ult}}{E_1} = \frac{1500}{181000} = 0.0083$$

$$(\varepsilon_1^T)_{ult} = \frac{(\sigma_1^T)_{ult}}{E_1} = \frac{1500}{181000} = 0.0083$$

$$(\varepsilon_2^c)_{ult} = \frac{(\sigma_2^c)_{ult}}{E_2} = \frac{246}{10300} = 0.0239$$

$$(\varepsilon_2^T)_{ult} = \frac{(\sigma_2^T)_{ult}}{E_2} = \frac{40}{10300} = 0.0039$$

$$(\gamma_{12})_{ult} = \frac{(\tau_{12})_{ult}}{G_{12}} = \frac{68}{7170} = 0.0095$$

$$\varepsilon_1 = 0.0055 \times 10^{-3}(\sigma_1) - 0.0015 \times 10^{-3}(\sigma_2) + 0 = -0.004\sigma \times 10^{-3}$$

$$\varepsilon_2 = -0.0015(\sigma_1) + 0.0971(\sigma_2) + 0 = -0.0956\sigma \times 10^{-3}$$

$$\gamma_{12} = 0 + 0 + 0.1395(\tau_{12}) = 0$$

$$\Rightarrow \begin{array}{l} -0.0083 < -0.000004\sigma < 0.0083 \quad (2075,\ -2075) \\ -0.0239 < -0.0000956\sigma < 0.0039 \quad (250,\ -40.795) \\ -0.0095 < 0\gamma_{12} < 0.0095 \end{array} \longrightarrow \boxed{-40.795 < \sigma < 250 \text{ MPa}}$$

(c) Tsai-Hill Failure $\Rightarrow \left[\frac{\sigma_1}{(\sigma_1^T)_{ult}}\right]^2 - \left[\frac{\sigma_1\sigma_2}{(\sigma_1^T)_{ult}^2}\right] + \left[\frac{\sigma_2}{(\sigma_2^T)_{ult}}\right]^2 + \left[\frac{\tau_{12}}{(\tau_{12})_{ult}}\right]^2 < 1$

$$\Rightarrow \left(\frac{-\sigma}{1500}\right)^2 - \frac{\sigma^2}{1500^2} + \left(\frac{-\sigma}{40}\right)^2 + 0 < 1 \longrightarrow \boxed{-40 < \sigma < 40 \text{ MPa}}$$

(d) Tsai-Wu Failure $\Rightarrow H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 <$

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^c)_{ult}} = \frac{1}{1500} - \frac{1}{1500} = 0 \qquad H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^c)_{ult}} = \frac{1}{40} - \frac{1}{246} = 0.0209 \text{ MPa}$$

$$H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^c)_{ult}} = \frac{1}{1500 \cdot 1500} = 4.44 \times 10^{-7} \text{ MPa}^{-2} \qquad H_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^c)_{ult}} = \frac{1}{40 \cdot 246} = 1.02 \times 10^{-4} \text{ MPa}$$

$$H_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^c)_{ult}(\sigma_2^T)_{ult}(\sigma_2^c)_{ult}}} = -\frac{1}{2}\sqrt{\frac{1}{1500^2 \times 40 \times 246}} = -3.36 \times 10^{-6} \text{ MPa}^{-2}$$

$$\Rightarrow 0.0209(-\sigma) + 4.44 \times 10^{-7}(-\sigma)^2 + 1.02 \times 10^{-4}(-\sigma)^2 + 2(-3.36 \times 10^{-6})(-\sigma)(-\sigma) < 1$$

$$9.5724 \times 10^{-5}\sigma^2 - 0.0209\sigma - 1 < 0 \qquad \sigma_{1,2} = -40.379\ ;\ 258.715$$

$$\longrightarrow \boxed{-40.379 < \sigma < 258.715 \text{ MPa}}$$