Edo Gök

171511001

MAK 413

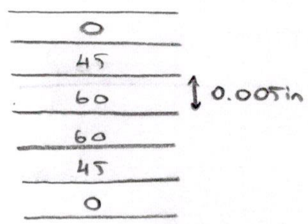
Mechanics of Composite Materials

Homework 5

20.03.2018

1) 4.6 :

$[0/45/60]_s$ laminate (thickness = 0.005 in)



$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1.686\times10^{-8} \\ -6.500\times10^{-8} \\ -2.143\times10^{-7} \end{bmatrix}$$

midplane strains :

$z = -3\cdot(0.005\,in) = -0.015\,in$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} 8.388\times10^{-6} \\ 4.762\times10^{-4} \\ -3.129\times10^{-3} \end{bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\cdot \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

The midplane curvatures;

$$z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}$$

$$-0.015\,in \begin{Bmatrix} k_x \\ k_y \\ k_z \end{Bmatrix} = \begin{Bmatrix} 1.686\times10^{-8} - 8.388\times10^{-6} \\ -6.500\times10^{-8} - 4.762\times10^{-4} \\ -2.129\times10^{-7} + 3.129\times10^{-3} \end{Bmatrix} = \begin{Bmatrix} -8.37114\times10^{-6} \\ -4.76265\times10^{-4} \\ 3.1287871\times10^{-3} \end{Bmatrix}$$

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} 5.58076\times10^{-4} \\ 3.1751\times10^{-2} \\ -0.2085858 \end{Bmatrix} in^{-1}$$

2) 4.7:

For σ_x;

For ply 1: $(\sigma_x)_{mid} = \dfrac{(-3.547\times10^4)+(-2.983\times10^3)}{2} = -19226.5$ psi

For ply 2: $(\sigma_x)_{mid} = \dfrac{(-9.267\times10^3)+(1.658\times10^4)}{2} = 3656.5$ psi

For ply 2: $(\sigma_x)_{mid} = \dfrac{(7.201\times10^3)+(2.435\times10^4)}{2} = 15775.5$ psi

For σ_y;

For ply 1: $(\sigma_y)_{mid} = \dfrac{(-2.425\times10^4)+(-7.087\times10^3)}{2} = -15668.5$ psi

For ply 2: $(\sigma_y)_{mid} = \dfrac{(-1.638\times10^4)+(9.432\times10^3)}{2} = -3474$ psi

For ply 3: $(\sigma_y)_{mid} = \dfrac{(3.155\times10^3+3.553\times10^4)}{2} = 19342.5$ psi

For τ_{xy};

For ply 1: $(\tau_{xy})_{mid} = \dfrac{(-2.946\times10^4)+(-5.564\times10^3)}{2} = -17512$ psi

For ply 2: $(\tau_{xy})_{mid} = \dfrac{(-1.299\times10^4)+(1.317\times10^4)}{2} = 90$ psi

For ply 3: $(\tau_{xy})_{mid} = \dfrac{(5.703\times10^3)+(2.954\times10^4)}{2} = 17621.5$ psi

Portion of N_x taken by 1^{st} ply $= N_x = (-19226.5)(0.005 in) = -96.1325$ lbf/in

Portion of N_x taken by 2^{nd} ply $= N_x = (3656.5)(0.005in) = 18.2825$ lbf/in

Portion of N_x taken by 3^{rd} ply $= N_x = (15775.5)(0.005in) = 78.8775$ lbf/in

Total applied $N_x = -96.1325 + 18.2825 + 78.8775 = 1.0275$ lbf/in

Total force in the x direction $= (1.0275$ lbf/in$)(4in) = 4.11$ lbf

The same calculation will be done for N_y;

$(N_y)_{1st} = (-15668.5)(0.005in) = -78.3425$ lbf/in

$(N_y)_{2nd} = (-3474)(0.005in) = -17.37$ lbf/in

$(N_y)_{3rd} = (19342.5)(0.005in) = 96.7125$ lbf/in

Total applied $N_y = -78.3425 - 17.37 + 96.7125 = 1$ lbf/in

Total force in the y direction $= (1$ lbf/in$)(4in) = 4$ lbf

For the shear force in xy plane;

$(N_{xy})_{1st} = (-17512)(0.005in) = -87.56$ lbf/in

$(N_{xy})_{2nd} = (90)(0.005in) = 0.45$ lbf/in

$(N_{xy})_{3rd} = (17621.5)(0.005in) = 88.1075$ lbf/in

Total applied $N_{xy} = -87.56 + 0.45 + 88.1075 = 0.9975$ lbf/in

Total shear force in the xy direction $= (0.9975$ lbf/in$)(4in) = 3.99$ lbf

For moments;

Ply 1:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}_1 = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} z\,dz$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}_1 = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \cdot \frac{1}{2}\left[(-7.5\times10^{-3})^2 - (-2.5\times10^{-3})^2\right]$$

$$= \begin{bmatrix} -1.82265 \times 10^4 \\ -1.56685 \times 10^4 \\ -1.7512 \times 10^4 \end{bmatrix} \cdot \frac{1}{2} \cdot \left[(-7.5 \times 10^{-3})^2 - (-2.5 \times 10^{-3})^2 \right] = \begin{bmatrix} -0.4806625 \\ -0.3917125 \\ -0.4378 \end{bmatrix} \text{lbf.in/in}$$

Ply 2 :

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}_2 = \begin{bmatrix} 3.6565 \times 10^3 \\ -3.474 \times 10^3 \\ 80 \end{bmatrix} \frac{1}{2} \left[(2.5 \times 10^{-3})^2 - (-2.5 \times 10^{-3})^2 \right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{lbf.in/in}$$

Ply 3 :

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}_3 = \begin{bmatrix} 1.57755 \times 10^4 \\ 1.93425 \times 10^4 \\ 1.76215 \times 10^4 \end{bmatrix} \frac{1}{2} \left[(7.5 \times 10^{-3})^2 - (2.5 \times 10^{-3})^2 \right] = \begin{bmatrix} 0.3943875 \\ 0.4835625 \\ 0.4405375 \end{bmatrix} \text{lbf.in/in}$$

For the total moments all the values will be taken positive, becouse negative values indicates the directions.

$$\left| \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}_1 \right| + \left| \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}_2 \right| + \left| \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}_3 \right| = \begin{bmatrix} 0.87505 \\ 0.875275 \\ 0.8783375 \end{bmatrix} \text{lbf.in/in}$$

In order to calculate the moments, we should multiply the results with length which is given as 4in.

→ Bending moment around y-axis : $M_x = 3.5002$ lbf-in

" " " x-axis : $M_y = 3.5011$ lbf.in

Twisting moment : $M_{xy} = 3.51335$ lbf.in

3) 4.8

[0/60/-60] Glass / Epoxy

$E_1 = 5.60$ Msi $= 5.6 \times 10^6$ Psi

$E_2 = 1.20$ Msi $= 1.2 \times 10^6$ Psi

$G_{12} = 0.60$ Msi $= 0.6 \times 10^6$ Psi

$\upsilon_{12} = 0.26$

$$S_{11} = \frac{1}{E_1} = 1.7857 \times 10^{-7} \ Psi^{-1}$$

$$S_{22} = \frac{1}{E_2} = 8.3333 \times 10^{-7} \ Psi^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = 1.6667 \times 10^{-6} \ Psi^{-1}$$

$$S_{12} = -\frac{\upsilon_{12}}{E_1} = -4.6429 \times 10^{-8} \ Psi^{-1}$$

$$[S] = \begin{bmatrix} 1.7857\times10^{-7} & -4.62429\times10^{-8} & 0 \\ -4.62429\times10^{-8} & 8.3333\times10^{-7} & 0 \\ 0 & 0 & 1.6667\times10^{-6} \end{bmatrix} Psi^{-1}$$

$$[Q] = [S]^{-1} = \begin{bmatrix} 5681632.181 & 315286.7697 & 0 \\ 315286.7697 & 1217500.599 & 0 \\ 0 & 0 & 599888.0002 \end{bmatrix} Psi$$

From Matlab code:

$$[\bar{Q}]_0 = \begin{bmatrix} 5.6823 & 0.3166 & 0 \\ 0.3166 & 1.2176 & 0 \\ 0 & 0 & 0.600 \end{bmatrix} \times 10^6 \ Psi$$

$$[\bar{Q}]_{60} = \begin{bmatrix} 1.6090 & 1.0417 & 0.5481 \\ 1.0417 & 3.8414 & 1.3853 \\ 0.5481 & 1.3853 & 1.3252 \end{bmatrix} \times 10^6 \ Psi$$

$$[\bar{Q}]_{-60^\circ} = \begin{bmatrix} 1.6090 & 1.0417 & -0.5481 \\ 1.0417 & 3.8414 & -1.3853 \\ -0.5481 & -1.3853 & 1.3252 \end{bmatrix} \times 10^6 \text{ Psi}$$

$$h = 0.005 \times 3 = 0.015\text{in} \implies \begin{aligned} h_0 &= -0.0075\text{in} \\ h_1 &= -0.0025\text{in} \\ h_2 &= 0.0025\text{in} \\ h_3 &= 0.0075\text{in} \end{aligned}$$

$$[A] = \begin{bmatrix} 5.6823 & 0.3166 & 0 \\ 0.3166 & 1.2176 & 0 \\ 0 & 0 & 0.600 \end{bmatrix} \times 10^6 \cdot [(-0.0025) - (-0.0075)]$$

$$+ \begin{bmatrix} 1.6090 & 1.0417 & 0.5481 \\ 1.0417 & 3.8414 & 1.3853 \\ 0.5481 & 1.3853 & 1.3252 \end{bmatrix} \times 10^6 [0.0025 - (-0.0025)]$$

$$+ \begin{bmatrix} 1.6090 & 1.0417 & -0.5481 \\ 1.0417 & 3.8414 & -1.3853 \\ -0.5481 & 1.3853 & 1.3252 \end{bmatrix} \times 10^6 [0.0075 - 0.0025]$$

$$[A] = \begin{bmatrix} 4.4501 & 1.200 & 0 \\ 1.200 & 4.4502 & 0 \\ 0 & 0 & 1.6252 \end{bmatrix} \times 10^4 \text{ Psi.in} \checkmark$$

$$[B] = \frac{1}{2} \cdot \begin{bmatrix} 5.6823 & 0.3166 & 0 \\ 0.3166 & 1.2176 & 0 \\ 0 & 0 & 0.600 \end{bmatrix} \times 10^6 \cdot [(-0.0025)^2 - (-0.0075)^2]$$

$$+ \frac{1}{2} \cdot \begin{bmatrix} 1.6090 & 1.0417 & 0.5481 \\ 1.0417 & 3.8414 & 1.3853 \\ 0.5481 & 1.3853 & 1.3252 \end{bmatrix} \times 10^6 \cdot [(0.0025)^2 - (-0.0025)^2]$$

$$+\frac{1}{2}\cdot\begin{bmatrix} 1.6080 & 1.0417 & -0.5481 \\ 1.0417 & 3.8414 & -1.3853 \\ -0.5481 & 1.3853 & 1.3252 \end{bmatrix} \times 10^6 \left[(0.075)^2-(0.025)^2\right]$$

$$[B] = \begin{bmatrix} -101.8339 & 18.1283 & -13.7028 \\ 18.1283 & 65.5938 & -34.6336 \\ -13.7028 & -34.6336 & 18.1280 \end{bmatrix} \text{Psi}\cdot\text{in}^2$$

$$[D] = \frac{1}{3}\begin{bmatrix} 5.6823 & 0.3166 & 0 \\ 0.3166 & 1.2176 & 0 \\ 0 & 0 & 0.600 \end{bmatrix} \times 10^6 \cdot \left[(-0.0025)^3-(-0.0075)^3\right]$$

$$+\frac{1}{3}\cdot\begin{bmatrix} 1.6080 & 1.0417 & 0.5481 \\ 1.0417 & 3.8414 & 1.3853 \\ 0.5481 & 1.3853 & 1.3252 \end{bmatrix} \times 10^6 \cdot \left[(0.0025)^3-(-0.0025)^3\right]$$

$$+\frac{1}{3}\cdot\begin{bmatrix} 1.6080 & 1.0417 & -0.5481 \\ 1.0417 & 3.8414 & -1.3853 \\ -0.5481 & 1.3853 & 1.3252 \end{bmatrix} \times 10^6 \cdot \left[(0.0075)^3-(0.0025)^3\right]$$

$$[D] = \begin{bmatrix} 1.0041 & 0.1848 & -0.0685 \\ 0.1848 & 0.7251 & -0.1732 \\ -0.0685 & -0.1732 & 0.2745 \end{bmatrix} \text{Psi}\cdot\text{in}^3$$

$$\rho_c = V_m \cdot \rho_m + V_f \cdot \rho_f$$

$$\rho_f = 2500\,\text{kg/m}^3 = 2500 \times 0.000036127\ \text{lb/in}^3 = 0.0903175\ \text{lb/in}^3$$

$$\rho_m = 1200\,\text{kg/m}^3 = 1200 \times 0.000036127\ \text{lb/in}^3 = 0.0433524\ \text{lb/in}^3$$

$V_f = 0.45 \qquad V_m = 0.55$

$$\rho_c = (0.55).(0.0433524\ lb/in^3) + (0.45).(0.0903175\ lb/in^3)$$

$$\rho_c = 0.064486695\ lb/in^3$$

$$V = 5 \times 7 \times 0.015 = 0.525\ in^3$$

$$m_c = \rho_c \cdot V = (0.064486695\ lb/in^3).(0.525\ in^3)$$

$$m_c = 0.03385\ lbM$$ ✓

4) 4.13

Graphite / Epoxy [0/60/-60]

$$E_1 = 181\ GPa = 1.81 \times 10^{11}\ Pa$$

$$E_2 = 10.30\ GPa = 1.03 \times 10^{10}\ Pa$$

$$G_{12} = 7.17\ GPa = 7.17 \times 10^{9}\ Pa$$

$$\upsilon_{12} = 0.28$$

$$S_{11} = \frac{1}{E_1} = 5.5249 \times 10^{-12}\ Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = 9.7087 \times 10^{-11}\ Pa^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = 1.3947 \times 10^{-10}\ Pa^{-1}$$

$$S_{12} = -\frac{\upsilon_{12}}{G_{12}} = -1.5470 \times 10^{-12}\ Pa^{-1}$$

$$[S] = \begin{bmatrix} 5.5249\times10^{-12} & -1.5470\times10^{-12} & 0 \\ -1.5470\times10^{-12} & 9.7087\times10^{-11} & 0 \\ 0 & 0 & 1.3947\times10^{-10} \end{bmatrix} Pa^{-1}$$

$$[Q] = [S]^{-1} = \begin{bmatrix} 1.818\times10^{11} & 2.8\times10^{9} & 0 \\ 2.8\times10^{9} & 1.03\times10^{10} & 0 \\ 0 & 0 & 7.1\times10^{9} \end{bmatrix} Pa$$

From the Matlab code

$$[\bar{Q}_0] = \begin{bmatrix} 1.8181 & 0.0280 & 0 \\ 0.0280 & 0.1035 & 0 \\ 0 & 0 & 0.0717 \end{bmatrix} \times 10^{11} \text{ Pa}$$

$$[\bar{Q}_{60}] = \begin{bmatrix} 0.2365 & 0.3247 & 0.2006 \\ 0.3247 & 1.0838 & 0.5420 \\ 0.2006 & 0.5420 & 0.3674 \end{bmatrix} \times 10^{11} \text{ Pa}$$

$$[\bar{Q}_{-60}] = \begin{bmatrix} 0.2365 & 0.3247 & -0.2006 \\ 0.3247 & 1.0838 & -0.5420 \\ -0.2006 & -0.5420 & 0.3674 \end{bmatrix} \times 10^{11} \text{ Pa}$$

$h = 0.125 \times 3 = 0.375$ mm $\Longrightarrow$ $h_0 = -0.1875$ mm $= -1.875 \times 10^{-4}$ m

$h_1 = -0.0625$ mm $= -6.25 \times 10^{-5}$ m

$h_2 = 0.0625$ mm $= 6.25 \times 10^{-5}$ m

$h_3 = 0.1875$ mm $= 1.875 \times 10^{-4}$ m

$$[A] = \begin{bmatrix} 1.8181 & 0.0280 & 0 \\ 0.0280 & 0.1035 & 0 \\ 0 & 0 & 0.0717 \end{bmatrix} \times 10^{11} [(-6.25 \times 10^{-5} - (-1.875 \times 10^{-4})]$$

$$+ \begin{bmatrix} 0.2365 & 0.3247 & 0.2006 \\ 0.3247 & 1.0838 & 0.5420 \\ 0.2006 & 0.5420 & 0.3674 \end{bmatrix} \times 10^{11} [6.25 \times 10^{-5} - (-6.25 \times 10^{-5})]$$

$$+ \begin{bmatrix} 0.2365 & 0.3247 & -0.2006 \\ 0.3247 & 1.0838 & -0.5420 \\ -0.2006 & -0.5420 & 0.3674 \end{bmatrix} \times 10^{11} [1.875 \times 10^{-4} - 6.25 \times 10^{-5})]$$

$$[A] = \begin{bmatrix} 2.8638 & 0.8478 & 0 \\ 0.8478 & 2.8640 & 0 \\ 0 & 0 & 1.0081 \end{bmatrix} \times 10^{7} \text{ Pa}\cdot\text{m}$$

$$[B] = \frac{1}{2} \cdot \begin{bmatrix} 1.8181 & 0.0280 & 0 \\ 0.0280 & 0.1035 & 0 \\ 0 & 0 & 0.0717 \end{bmatrix} \times 10^{11} \cdot \left[(-6.25\times10^{-5})^2 - (-1.875\times10^{-4})^2 \right]$$

$$+ \frac{1}{2} \begin{bmatrix} 0.2365 & 0.3247 & 0.2006 \\ 0.3247 & 1.0838 & 0.5420 \\ 0.2006 & 0.5420 & 0.3674 \end{bmatrix} \times 10^{11} \left[(6.25\times10^{-5})^2 - (-6.25\times10^{-5})^2 \right]$$

$$+ \frac{1}{2} \cdot \begin{bmatrix} 0.2365 & 0.3247 & -0.2006 \\ 0.3247 & 1.0838 & -0.5420 \\ -0.2006 & -0.5420 & 0.3674 \end{bmatrix} \times 10^{11} \left[(1.875\times10^{-4})^2 - (6.25\times10^{-5})^2 \right]$$

$$[B] = \begin{bmatrix} -2.4713 & 0.4620 & -0.3134 \\ 0.4620 & 1.5475 & -0.8468 \\ -0.3134 & -0.8468 & 0.4620 \end{bmatrix} \times 10^{3} \quad Pa.m^2$$

$$[D] = \frac{1}{3} \cdot \begin{bmatrix} 1.8181 & 0.0280 & 0 \\ 0.0280 & 0.1035 & 0 \\ 0 & 0 & 0.0717 \end{bmatrix} \times 10^{11} \cdot \left[(-6.25\times10^{-5})^3 - (-1.875\times10^{-4})^3 \right]$$

$$+ \frac{1}{3} \cdot \begin{bmatrix} 0.2365 & 0.3247 & 0.2006 \\ 0.3247 & 1.0838 & 0.5400 \\ 0.2006 & 0.5420 & 0.3674 \end{bmatrix} \times 10^{11} \left[(6.25\times10^{-5})^3 - (-6.25\times10^{-5})^3 \right]$$

$$+ \frac{1}{3} \cdot \begin{bmatrix} 0.2365 & 0.3247 & -0.2006 \\ 0.3247 & 1.0838 & -0.5420 \\ -0.2006 & -0.5420 & 0.3674 \end{bmatrix} \times 10^{11} \cdot \left[(1.875\times10^{-4})^3 - (6.25\times10^{-5})^3 \right]$$

$$[D] = \begin{bmatrix} 0.4386 & 0.0801 & -0.0382 \\ 0.0801 & 0.2711 & -0.1058 \\ -0.0382 & -0.1058 & 0.0888 \end{bmatrix} \quad Pa.m^3$$

$$
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
2.8639\times10^{7} & 0.8479\times10^{7} & 0 & -2.4713\times10^{3} & 0.4620\times10^{3} & -0.3134\times10^{3} \\
0.8478\times10^{7} & 2.8640\times10^{7} & 0 & 0.4620\times10^{3} & 1.5475\times10^{3} & -0.8468\times10^{3} \\
0 & 0 & 1.0081\times10^{7} & -0.3134\times10^{3} & -0.8468\times10^{3} & 0.4620\times10^{3} \\
-2.4713\times10^{3} & 0.4620\times10^{3} & -0.3134\times10^{3} & 0.4386 & 0.0801 & -0.0392 \\
0.4620\times10^{3} & 1.5475\times10^{3} & -0.8468\times10^{3} & 0.0801 & 0.2711 & -0.1058 \\
-0.3134\times10^{3} & -0.8468\times10^{3} & 0.4620\times10^{3} & -0.0392 & -0.1058 & 0.0988
\end{bmatrix}
\begin{bmatrix} \varepsilon_x^{\circ} \\ \varepsilon_y^{\circ} \\ \gamma_{xy}^{\circ} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}
$$

$$0 = 2.8639\times10^{7}\cdot\varepsilon_x^{\circ} + 0.8479\times10^{7}\cdot\varepsilon_y^{\circ} + 0 - 2.4713\times10^{3}\,k_x + 0.4620\times10^{3}\,k_y$$
$$-0.3134\times10^{3}\,k_{xy}$$

$$0 = 0.8479\times10^{7}\,\varepsilon_x^{\circ} + 2.8640\times10^{7}\,\varepsilon_y^{\circ} + 0 + 0.4620\times10^{3}\,k_x + 1.5475\times10^{3}\,k_y$$
$$-0.8468\times10^{3}\,k_{xy}$$

$$0 = 1.0081\times10^{7}\,\gamma_{xy} - 0.3134\times10^{3}\,k_x - 0.8468\times10^{3}\,k_y + 0.4620\times10^{3}\,k_{xy}$$

$$50 = -2.4713\times10^{3}\,\varepsilon_x^{\circ} + 0.4620\times10^{3}\,\varepsilon_y^{\circ} - 0.3134\times10^{3}\,\gamma_{xy}^{\circ} + 0.4386\,k_x$$
$$+0.0801\,k_y - 0.0392\,k_{xy}$$

$$0 = 0.4620\times10^{3}\,\varepsilon_x^{\circ} + 1.5475\times10^{3}\,\varepsilon_y^{\circ} - 0.8468\times10^{3}\,\gamma_{xy}^{\circ} + 0.0801\,k_x +$$
$$0.2711\,k_y - 0.1058\,k_{xy}$$

$$0 = -0.3134\times10^{3}\,\varepsilon_x^{\circ} - 0.8468\times10^{3}\,\varepsilon_y^{\circ} + 0.4620\times10^{3}\,\gamma_{xy}^{\circ} - 0.0392\,k_x$$
$$-0.1059\,k_y + 0.0988\,k_{xy}$$

Solving these six equations,

$\varepsilon_x^{\circ} = 0.0333050$

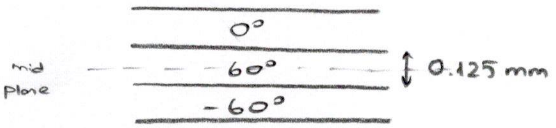
$\varepsilon_y^{\circ} = -0.0090900$

$\gamma_{xy}^{\circ} = 0.0008100$

$k_x = 331.6149800$ 1/m

$$\kappa_y = -68.1617000 \ 1/m$$

$$\kappa_{xy} = 82.2634000 \ 1/m$$



$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_{60°\_Top} = \begin{bmatrix} \varepsilon_x^{\circ} \\ \varepsilon_y^{\circ} \\ \gamma_{xy}^{\circ} \end{bmatrix} + z \cdot \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 0.033305 \\ -0.009090 \\ 0.000810 \end{bmatrix} + (-0.0625 \times 10^{-3}) \cdot \begin{bmatrix} 331.61498 \\ -68.1617000 \\ 82.2634000 \end{bmatrix}$$

$$= \begin{bmatrix} 0.012579 \\ -0.004830 \\ -0.004331 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{60°\_Top} = \begin{bmatrix} \bar{Q}_{60°} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 0.2365 & 0.3247 & 0.2006 \\ 0.3247 & 1.0939 & 0.5420 \\ 0.2006 & 0.5420 & 0.3674 \end{bmatrix} \times 10^{11} \cdot \begin{bmatrix} 0.012579 \\ -0.004830 \\ -0.004331 \end{bmatrix}$$

$$= \begin{bmatrix} 53875000 \\ -354784550 \\ -168649690 \end{bmatrix} Pa$$

$$\begin{bmatrix} T_{60} \end{bmatrix} = \begin{bmatrix} 0.25 & 0.75 & 0.86 \\ 0.75 & 0.25 & -0.86 \\ -0.433 & 0.433 & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_{60°\_Top} = \begin{bmatrix} 0.25 & 0.75 & 0.86 \\ 0.75 & 0.25 & -0.86 \\ -0.433 & 0.433 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 53875000 \\ -354784550 \\ -168649690 \end{bmatrix}$$

$$= \begin{bmatrix} -398670294 \\ 97760744.04 \\ -92624740.15 \end{bmatrix} \checkmark \ Pa.$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$M_x = B_{11} \cdot \varepsilon_x^o + B_{12} \cdot \varepsilon_y^o + B_{16} \cdot \gamma_{xy}^o + D_{11} \cdot \kappa_x + D_{12} \cdot \kappa_y + D_{16} \cdot \kappa_{xy}$$

1. $M_x/0^\circ = (-2840.625) \times (0.033305) + (-45.265625) \times (-0.008080) + (0) \times (0.000810) + (0.3846678688) \times (331.61498) + (0.0061287200053) \times (-68.16170) + (0) \times (82.26340)$
$= 32.94828755$ N.m/m $\cong 32.95$ N.m/m

2. $M_x/60^\circ = 0 + 0 + 0 + (0.0038492838) \times (331.61498) + (0.0052832031) \times (-68.1617) + (0.0032633463) \times (82.2634) = 1.184822028$ N.m/m $\cong 1.185$ Nm/m

3. $M_x/-60^\circ = (368.53125) \times (0.033305) + (507.1875) \times (-0.008080) + (-313.28125) \times (0.000810) + (0.05004068) \times (331.61498) + (0.06868164) \times (-68.16170) + (-0.042423502) \times (82.26340)$
$= 15.86602865$ N.m/m $\cong 15.866$ N.m/m

$M_x = M_x/0^\circ + M_x/60^\circ + M_x/-60^\circ \cong 50$ N.m/m

$0^\circ \rightarrow \dfrac{32.95}{50} = 65.90\%$

$60^\circ \rightarrow \dfrac{1.185}{50} = 2.37\%$

$-60^\circ \rightarrow \dfrac{15.866}{50} = 31.73\%$

5) 4.14

[0/60/-60] Graphite/Epoxy

$K_x = 0.1\ in^{-1}$ $K_y = 0.1\ in^{-1}$ thickness = 0.005 in

$E_1 = 26.25\ Msi = 26.25 \times 10^6\ Psi$

$E_2 = 1.49\ Msi = 1.49 \times 10^6\ Psi$

$G_{12} = 1.040\ Msi = 1.040 \times 10^6\ Psi$

$\nu_{12} = 0.28$

$$S_{11} = \frac{1}{E_1} = 3.8095 \times 10^{-8}\ Psi^{-1}$$

$$S_{22} = \frac{1}{E_2} = 6.7114 \times 10^{-7}\ Psi^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = 9.6154 \times 10^{-7}\ Psi^{-1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -1.0667 \times 10^{-8}\ Psi^{-1}$$

$$[S] = \begin{bmatrix} 3.8095\times10^{-8} & -1.0667\times10^{-8} & 0 \\ -1.0667\times10^{-8} & 6.7114\times10^{-7} & 0 \\ 0 & 0 & 9.6154\times10^{-7} \end{bmatrix} Psi^{-1}$$

$$[Q] = [S]^{-1} = \begin{bmatrix} 2.6\times10^{7} & 413081.3278 & 0 \\ 413081.3278 & 1.5\times10^{6} & 0 \\ 0 & 0 & 1\times10^{6} \end{bmatrix} Psi$$

From Matlab code;

$$[\bar{Q}]_{0^\circ} = \begin{bmatrix} 2.6367 & 0.0419 & 0 \\ 0.0419 & 0.1479 & 0 \\ 0 & 0 & 0.1040 \end{bmatrix} \times 10^7\ Psi$$

$$[\bar{Q}]_{60^\circ} = \begin{bmatrix} 0.3427 & 0.4707 & 0.2910 \\ 0.4707 & 1.5863 & 0.7861 \\ 0.2910 & 0.7861 & 0.5328 \end{bmatrix} \times 10^{7} \text{ Psi}$$

$$[\bar{Q}]_{-60^\circ} = \begin{bmatrix} 0.3427 & 0.4707 & -0.2910 \\ 0.4707 & 1.5863 & -0.7861 \\ -0.2910 & -0.7861 & 0.5328 \end{bmatrix} \times 10^{7} \text{ Psi}$$

$$h = 0.005 \times 3 = 0.015\text{in} \implies \begin{aligned} h_0 &= -0.0075\text{in} \\ h_1 &= -0.0025\text{in} \\ h_2 &= 0.0025\text{in} \\ h_3 &= 0.075\text{in} \end{aligned}$$

With the same method as the previous questions [A], [B] and [D] matrix results are founded as follows

$$[A] = \begin{bmatrix} 1.6611 & 0.4916 & 0 \\ 0.4916 & 1.6612 & 0 \\ 0 & 0 & 0.5848 \end{bmatrix} \times 10^{5} \text{ Psi-in}$$

$$[B] = \begin{bmatrix} -573.4977 & 107.1973 & -72.7441 \\ 107.1973 & 359.1664 & -196.5165 \\ -72.7441 & -196.5165 & 107.1986 \end{bmatrix} \text{ Psi-in}^2$$

$$[D] = \begin{bmatrix} 4.0704 & 0.7432 & -0.3637 \\ 0.7432 & 2.5161 & -0.9826 \\ -0.3637 & -0.9826 & 0.9178 \end{bmatrix} \text{ Psi-in}^2$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \to 0 \\ \varepsilon_y^0 \to 0 \\ \gamma_{xy}^0 \to 0 \end{bmatrix}}_{0} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \to 0.1 \\ \kappa_y \to 0.1 \\ \kappa_{xy} \to 0 \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} -573.4377 & 107.1973 & -72.7441 \\ 107.1973 & 259.1664 & -196.5165 \\ -72.7441 & -196.5165 & 107.1986 \end{bmatrix} \cdot \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} -46.6300 \\ 46.6364 \\ -26.9261 \end{bmatrix} \frac{lbf}{in}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \underbrace{\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}}_{0} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \cdot \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} 0.4814 \\ 0.3259 \\ -0.1346 \end{bmatrix} \frac{lbf.in}{in}$$