Mörvet BEKTAŞ
171511015

MAK 413

(100/100)

HOMEWORK 4

1) Problem 3.2: Hybrid lamina → Fibers ⇒ Glass & Graphite; Matrix ⇒ Epoxy

$V_{f,glass} = 0.4$, $V_{f,graphite} = 0.2$; $V_{m,epoxy} = 1-(0.4+0.2) = 0.4$

$SG_{glass} = 2.6$, $SG_{graphite} = 1.8$; $SG_{epoxy} = 1.2$

1. Mass fractions = ? 2. Density of the composite = ?

$$\text{Specific Gravity} = \frac{\text{Density of material}}{\text{Density of water}} \quad ; \quad \rho_{water} = 1000\ kg/m^3$$

$\Rightarrow \rho_{glass} = 2.6 \times 1000 = 2600\ kg/m^3$, $\rho_{graphite} = 1.8 \times 1000 = 1800\ kg/m^3$

$\Rightarrow \rho_{epoxy} = 1.2 \times 1000 = 1200\ kg/m^3$

$\rightsquigarrow \rho_{composite} = \rho_f V_f + \rho_m V_m = 2600 \times 0.4 + 1800 \times 0.2 + 1200 \times 0.4 = \boxed{1880\ kg/m^3}$ ✓

Mass Fractions $\Rightarrow W_f = \frac{\rho_f}{\rho_c} V_f \rightarrow W_{glass} = \frac{2600}{1880} \times 0.4 = \boxed{0.5532}$; $W_{graphite} = \frac{1800}{1880} \times 0.2 = \boxed{0.1915}$ ✓

$\Rightarrow W_m = \frac{\rho_m}{\rho_c} V_m \rightarrow W_{epoxy} = \frac{1200}{1880} \times 0.4 = \boxed{0.2553}$ ✓

2) Problem 3.4: Resin hybrid lamina → Fiber ⇒ Graphite; Matrices ⇒ Resin A & Resin B

$W_{f,graphite} = 0.4$; $W_{m,A} = 0.3$, $W_{m,B} = 0.3$

$SG_{graphite} = 1.2$; $SG_A = 2.6$, $SG_B = 1.7$

1. Fiber volume fraction = ? 2. Density of composite = ?

$$\text{Fiber Volume Fraction} = \frac{V_f}{v_c} = \frac{v_f}{v_f + v_{mA} + v_{mB}} = \frac{w_f/\rho_f}{w_f/\rho_f + w_{mA}/\rho_{mA} + w_{mB}/\rho_{mB}} \times \frac{1/w_c}{1/w_c} \times \frac{\rho_{water}}{\rho_{water}}$$

$$\Rightarrow V_f = \frac{W_f/SG_f}{W_f/SG_f + W_{mA}/SG_A + W_{mB}/SG_B} = \frac{0.4/1.2}{0.4/1.2 + 0.3/2.6 + 0.3/1.7} = \boxed{0.5332}$$ ✓

$$\text{Density of Composite} ; \frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_{mA}}{\rho_{mA}} + \frac{W_{mB}}{\rho_{mB}} = \frac{0.4}{1200} + \frac{0.3}{2600} + \frac{0.3}{1700} = 6.25 \times 10^{-4}\ m^3/kg$$

$\Rightarrow \boxed{\rho_{composite} = 1599.52\ kg/m^3}$ ✓

3) Problem 3.5 : Glass / Epoxy ; $V_f = 0.4 \Rightarrow$ Elastic moduli = ?

	Table 3.3 / Fibers / Glass	Table 3.4 / Matrices / Epoxy
Axial Modulus	12.33 Msi	0.493 Msi
Transverse M.	12.33 Msi	0.493 Msi

$\Rightarrow E_1 = E_f V_f + E_m V_m = 12.33 \times 0.4 + 0.493 \times (1-0.4) = \boxed{5.2278 \text{ Msi}}$ ✓

$\Rightarrow \frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} = \frac{0.4}{12.33} + \frac{(1-0.4)}{0.493} = 1.2495 \rightsquigarrow \boxed{E_2 = 0.8003 \text{ Msi}}$ ✓

4) Problem 3.9 : For Problem 3.5 ; Fibers → Circularly in Square array $\{\xi = 2\}$

$\Rightarrow$ Halphin - Tsai:

$E_1 = E_f V_f + E_m V_m = 12.33 \times 0.4 + 0.493 \times 0.6 = \boxed{5.2278 \text{ Msi}}$ ✓

$\frac{E_2}{E_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f}$ & $\eta = \frac{(E_f/E_m) - 1}{(E_f/E_m) + \xi} = \frac{12.33/0.493 - 1}{12.33/0.493 + 2} = 0.8889$

$\Rightarrow E_2 = 0.493 \frac{1 + 2 \times 0.8889 \times 0.4}{1 - 0.8889 \times 0.4} = \boxed{1.3091 \text{ Msi}}$ ✓

For the longitudinal Young's modulus, E_1, the equation is same as problem 3.5 and the results are also same.

For the transverse Young's modulus, E_2, the reinforcing factor (ξ) is used by Halphin-Tsai due to their experimental results. So the result gives higher value in comparison with problem 3.5. However, the lower result in problem 3.5 with the strength solution is more conservative. ✓

5) Problem 3.10 : Glass / Epoxy ; $V_f = 0.7$ $\xrightarrow{\text{Replaced}}$ Graphite / Epoxy $\Rightarrow V_f = ?$

	Table 3.1 / Glass ~ Graphite		Table 3.2 / Epoxy
Axial Mod.	85 GPa	230 GPa	3.4 GPa

→ Glass / Epoxy $\Rightarrow E_1 = E_f V_f + E_m V_m = 85 \times 0.7 + 3.4 \times (1-0.7) = 60.52$ GPa

→ Graphite / Epoxy $\Rightarrow 60.52 = 230 \times V_f + 3.4 \times (1 - V_f) \Rightarrow \boxed{V_f = 0.2521 \rightarrow \%25.21}$ ✓

6) Problem 3.16 : Glass/Epoxy ; $V_f = 0.4$ ⇒ 5 Strength parameters = ?

	Table 3.3/ Glass	Table 3.4/ Epoxy
E_1	12.33 Msi	0.493 Msi
E_2	12.33 Msi	0.493 Msi
ν_{12}	0.20	0.30
G	5.136 Msi	0.1897 Msi
σ^T	224.8 ksi	10.44 ksi
τ_{ult}	5.08 ksi	4.93 ksi
σ^C	224.8 ksi	14.79 ksi

I ⇒ $(\sigma_1^T)_{ult} = (\sigma_f^T)_{ult} V_f + (\varepsilon_f)_{ult} E_m (1-V_f)$ where $(\varepsilon_f)_{ult} = \frac{(\sigma_f^T)_{ult}}{E_f} = \frac{224.8 \times 10^3}{12.33 \times 10^6} = 0.01823$

$= 224.8 \times 0.4 + 0.01823 \times 0.493 \times 10^3 \times (1-0.4) =$ **95.3124 ksi** ✓

II ⇒ $(\sigma_1^C)_{ult} = \frac{E_1 (\varepsilon_2^T)_{ult}}{\nu_{12}}$ where * $E_1 = E_f V_f + E_m V_m = 12.33 \times 0.4 + 0.493 \times (1-0.4) = 5.2278$ Msi

* $(\varepsilon_2^T)_{ult} = (\varepsilon_m^T)_{ult} (1 - V_f^{1/3}) = \frac{(\sigma_m^T)_{ult}}{E_m} (1 - V_f^{1/3}) = \frac{10.44}{0.493 \times 10^3} (1 - 0.4^{1/3}) = 5.5735 \times 10^{-3}$

* $\nu_{12} = \nu_f V_f + \nu_m V_m = 0.2 \times 0.4 + 0.3 \times (1-0.4) = 0.26$

$(\sigma_1^C)_{ult} = \frac{5.2278 \times 10^3 \times 5.5735 \times 10^{-3}}{0.26} =$ **112.0659 ksi**

II ⇒ $(\sigma_1^C)_{ult} = \min[S_1^c, S_2^c]$ where * $S_1^c = 2\left[V_f + V_m \frac{E_m}{E_f}\right] \sqrt{\frac{V_f E_m E_f}{3(1-V_f)}} = 0.9856$ Msi

* $S_2^c = \frac{G_m}{1-V_f} = \frac{0.1897}{0.6} = 0.3162$ Msi

$(\sigma_1^C)_{ult} = \min[S_1^c, S_2^c] = 0.3162$ Msi $=$ **316.2 ksi**

II ⇒ $(\sigma_1^C)_{ult} = 2[(\tau_f)_{ult} V_f + (\tau_m)_{ult} V_m] = 2[5.08 \times 0.4 + 4.93 \times 0.6] =$ **9.98 ksi**

CHOOSE MINIMUM

⇒ **$(\sigma_1^C)_{ult} = 9.98$ ksi** ✓

III ⇒ $(\sigma_2^T)_{ult} = E_2 (\varepsilon_2^T)_{ult}$ where $\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \rightarrow E_2 = 0.8003 \times 10^3$ ksi & $(\varepsilon_2^T)_{ult} = 5.5735 \times 10^{-3}$

$(\sigma_2^T)_{ult} = 0.8003 \times 10^3 \times 5.5735 \times 10^{-3} =$ **4.4605 ksi** ✓

IV

$\Rightarrow (\sigma_2^c)_{ult} = E_2 (\varepsilon_2^c)_{ult}$ where $(\varepsilon_2^c)_{ult} = \left[\frac{d}{s}\frac{E_m}{E_f} + 1 - \frac{d}{s}\right](\varepsilon_m^c)_{ult}$

$* (\varepsilon_m^c)_{ult} = \frac{(\sigma_m^c)_{ult}}{E_m} = \frac{14.79}{0.493\times10^3} = 0.03$ $* d/s = (4V_f/\pi)^{1/2} = 0.7136$

$\rightarrow (\varepsilon_2^c)_{ult} = \left[0.7136\,\frac{0.493}{12.33} + 1 - 0.7136\right] \times 0.03 = 9.448\times10^{-3}$

$(\sigma_2^c)_{ult} = 0.8003\times10^3 \times 9.448\times10^{-3} = \boxed{7.5612\ ksi}$ ✓

V

$\Rightarrow (\tau_{12})_{ult} = G_{12}(\gamma_{12})_{ult}$ where $* \frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} = \frac{0.4}{5.136} + \frac{0.6}{0.1897} \Rightarrow G_{12} = 0.3086\ Msi$

$* (\gamma_{12})_{ult} = \left[\frac{d}{s}\frac{G_m}{G_f} + 1 - \frac{d}{s}\right](\gamma_{12})_{mult} \rightarrow (\gamma_{12})_{mult} = \frac{(\tau_{12})_{mult}}{G_m} = \frac{4.93}{0.1897\times10^3} = 0.026$

$\rightarrow (\gamma_{12})_{ult} = \left[0.7136\,\frac{0.1897}{5.136} + 1 - 0.7136\right] \times 0.026 = 8.1317\times10^{-3}$

$(\tau_{12})_{ult} = 0.3086\times10^3 \times 8.1317\times10^{-3} = \boxed{2.5094\ ksi}$ ✓

7) Problem 3.17 : $P = 1400\,N$, $N = 2$; Steel or Graphite/Epoxy ($V_f = 0.66$)

	Steel	Table 3.1 / Graphite	Table 3.2 / Epoxy
E_1	210 GPa	230 GPa	3.4 GPa
ν_{12}	0.3	0.3	0.3
σ^T	450 MPa	2067 MPa	72 MPa
SG	7.8	1.8	1.2

The cost of Graphite / Epoxy = 5× Steel

$* (\sigma_1^T)_{ult} = (\sigma_f^T)_{ult} V_f + (\varepsilon_f)_{ult} E_m V_m$; $(\varepsilon_f)_{ult} = \frac{(\sigma_f)_{ult}}{E_f} = \frac{2067}{230\times10^3} = 8.9869\times10^{-3}$

$= 2067\times0.66 + 8.9869\times10^{-3}\times3.4\times10^3\times0.34 = 1374.6089\ MPa$

Max. shear stress theory; $\frac{\sigma_1 - \sigma_2}{2} \leq \frac{\sigma_T}{N}$ where $\sigma_1 = \frac{P}{A} = \frac{1400}{\pi d^2/4}$, $\sigma_2 = 0$

$* \text{Steel} \Rightarrow \frac{1400}{\pi d^2/2} \leq \frac{450}{2} \rightarrow 1.9903 \leq d_s$

$* \text{Graphite/Epoxy} \Rightarrow \frac{1400}{\pi d^2/2} \leq \frac{1374.6089}{2} \rightarrow 1.1387 \leq d_{ge}$

$* \rho_{comp} = \rho_f V_f + \rho_m V_m = 1800\times0.66 + 1200\times0.34 = 1596\ kg/m^3$

MASS

$\frac{m_s}{m_{ge}} = \frac{\pi/4\, d_s^2\, l\, \rho_s}{\pi/4\, d_{ge}^2\, l\, \rho_{ge}} = \frac{1.9903^2\times7800}{1.1387^2\times1596} = 14.9307$

$* m_{steel} = 14.9306 \times m_{composite}$ ⇒ Mass of steel is approximately 15 times heavier;

* Cost of steel is 5 times cheaper than composite. However, 15 steels are more expensive like 3 times, so I choose the composite! ✓