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MAK 411

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HOMEWORK 3

1) **Problem 2.23:** Bidirectional woven composite ply ;

$E_1 = E_2 = 69$ GPa , $\nu_{12} = 0.3$, $G_{12} = 20$ GPa

⇒ For $G_{xy,max}$; Angle of the ply : $\boxed{\theta = 45^\circ}$ from Figure 2.27

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$\rightarrow \quad G_{xy/45^\circ} = \dfrac{E_1}{1 + 2\nu_{12} + \dfrac{E_1}{E_2}} = \dfrac{69}{2.6} = \boxed{26.5385 \text{ GPa}}$ ✓

⇒ For $G_{xy,min}$; Angle of the ply : $\boxed{\theta = 0^\circ}$ from Figure 2.27

$$\rightarrow \quad G_{xy/0^\circ} = \frac{1}{\bar{S}_{66/0^\circ}} = \frac{1}{2(2S_{11}+2S_{22}-4S_{12}-S_{66})\underbrace{(\sin^2 0^\circ \cos^2 0^\circ)}_{0} + S_{66}(\underbrace{\sin^4 0^\circ}_{0} + \underbrace{\cos^4 0^\circ}_{1})}$$

$$= \frac{1}{S_{66}} = G_{12} = \boxed{20 \text{ GPa}}$$ ✓

2) **Problem 2.25 :** Uniaxial load , 10° ply , $\sigma_x = 123\,\varepsilon_x$

$E_1 = 180$ GPa , $E_2 = 10$ GPa , $\nu_{12} = 0.25$

⇒ $G_{12} = ?$ $\quad S_{11} = 1/E_1 = 0.0055 \text{ GPa}^{-1}$, $S_{12} = -\nu_{12}/E_1 = -0.001389 \text{ GPa}^{-1}$

$\quad S_{22} = 1/E_2 = 0.1 \text{ GPa}^{-1}$, $S_{66} = ?$

$$\bar{S}_{11} = \frac{1}{E_x} = \frac{\varepsilon_x}{\sigma_x} = 0.00813 \text{ GPa}^{-1} = S_{11}\cos^4 10 + (2S_{12}+S_{66})\sin^2 10 \cos^2 10 + S_{22}\sin^4 10$$

$$= 0.0052353 + 0.0292444\, S_{66}$$

$\rightsquigarrow S_{66} = 0.0989831 \text{ GPa}^{-1} \Rightarrow G_{12} = 1/S_{66} = \boxed{10.1027 \text{ GPa}}$ ✓

⇒ $E_{x/60^\circ} = ?$ $\quad \bar{S}_{11} = 0.0055\cos^4 60^\circ + (2\times(-0.001389) + 0.0989831)\sin^2 60^\circ \cos^2 60^\circ$

$\quad + 0.1 \sin^4 60^\circ$

$\bar{S}_{11} = 0.0746322 \text{ GPa}^{-1}$

$\rightsquigarrow E_{x/60^\circ} = \dfrac{1}{\bar{S}_{11}} = \boxed{13.3990 \text{ GPa}}$ ✓

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3) Problem 2.26: $0° \rightarrow E_1 = 26.25$ Msi, $90° \rightarrow E_2 = 1.494$, $45° \rightarrow E_x = 2.427$ Msi

$\Rightarrow 30° \rightarrow E_x = ?$

$* \; S_{11} = 1/E_1 = 0.038095238 \; Msi^{-1}$

$* \; S_{22} = 1/E_2 = 0.669344042 \; Msi^{-1}$

$\rightsquigarrow E_x/_{45°} = \dfrac{1}{\bar{S}_{11}}$; $\bar{S}_{11} = \dfrac{1}{2.427} = 0.412031314 \; Msi^{-1}$

$* \; \bar{S}_{11} = S_{11}\cos^4 45 + (2S_{12} + S_{66})\sin^2 45 \cos^2 45 + S_{22}\sin^4 45$

$\Rightarrow 0.412031314 = 0.17685982 + 0.25(2S_{12} + S_{66})$

$\Rightarrow 2S_{12} + S_{66} = 0.940685976 \; Msi^{-1}$

$$\rightsquigarrow E_x/_{30°} = \frac{1}{S_{11}\cos^4 30 + (2S_{12} + S_{66})\sin^2 30 \cos^2 30 + S_{22}\sin^4 30}$$

$$\boxed{E_x/_{30°} = 4.1729 \; Msi}$$

$\Rightarrow \nu_{12} = -\dfrac{S_{12}}{S_{22}}$; $G_{12} = \dfrac{1}{S_{66}}$

We know the sum of $2S_{12}$ and S_{66}, but we do not know their values. So, ν_{12} and G_{12} cannot be calculated !

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4) **Problem 2.35:** Off-axis shear strength & mode of failure of 60° boron/epoxy

From Table 2.1; $(\sigma_1^T)_{ult} = 1260$ MPa, $(\sigma_1^c)_{ult} = 2500$ MPa, $E_1 = 204$ GPa, $E_2 = 18.5$ GPa

$(\sigma_2^T)_{ult} = 61$ MPa, $(\sigma_2^c) = 202$ MPa, $(\tau_{12})_{ult} = 67$ MPa, $G_{12} = 5.59$ GPa, $\nu_{12} = 0.23$

$$\Rightarrow \begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix}\begin{bmatrix}0\\0\\ \tau\end{bmatrix} \Rightarrow \left.\begin{aligned}\sigma_1 &= 0.866\tau\\ \sigma_2 &= -0.866\tau\\ \tau_{12} &= -0.5\tau\end{aligned}\right\}$$

For off-axis shear strength, $\sigma_x = \sigma_y = 0$; $\tau_{xy} = \tau$; and $\theta = 60°$

⇒ Max. Stress Failure

$$-(\sigma_1^c)_{ult} < \sigma_1 < (\sigma_1^T)_{ult} \rightarrow -2500 < 0.866\tau < 1260$$
$$-(\sigma_2^c)_{ult} < \sigma_2 < (\sigma_2^T)_{ult} \rightarrow -202 < -0.866\tau < 61$$
$$-(\tau_{12})_{ult} < \tau_{12} < (\tau_{12})_{ult} \rightarrow -67 < -0.5\tau < 67$$

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$$\rightsquigarrow \left.\begin{aligned}-2886.836028 &< \tau < 1454.965358\\ -70.43879908 &< \tau < 233.256351\\ -134 &< \tau < 134\end{aligned}\right\}$$

Largest Negative $\boxed{\tau = -70.4388 \text{ MPa}}$
Largest Pozitive $\boxed{\tau = 134.0000 \text{ MPa}}$

$$\Rightarrow \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0\\ -\nu_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} \Rightarrow \begin{aligned}\varepsilon_1 &= 5.2215\times10^{-6}\,\tau\\ \varepsilon_2 &= -47.7872\times10^{-6}\,\tau\\ \gamma_{12} &= -89.4454\times10^{-6}\,\tau\end{aligned}$$

⇒ Max. Strain

$$-(\sigma_1^c)_{ult}/E_1 < \varepsilon_1 < (\sigma_1^T)_{ult}/E_1 \rightarrow -0.012254901 < \varepsilon_1 < 0.006176471$$
$$-(\sigma_2^c)_{ult}/E_2 < \varepsilon_2 < (\sigma_2^T)_{ult}/E_2 \rightarrow -0.010918918 < \varepsilon_2 < 0.003297297$$
$$-(\tau_{12})_{ult}/G_{12} < \gamma_{12} < (\tau_{12})_{ult}/G_{12} \rightarrow -0.011985688 < \gamma_{12} < 0.011985688$$

$$\rightsquigarrow \left.\begin{aligned}-2347.007756 &< \tau < 1182.892081\\ -68.999586 &< \tau < 228.490433\\ -134.000049 &< \tau < 134.000049\end{aligned}\right\}$$

Largest Neg. $\boxed{\tau = -68.9996 \text{ MPa}}$
Largest Poz. $\boxed{\tau = 134.0000 \text{ MPa}}$

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⇒ Tsai-Hill → $\left(\sigma_1/(\sigma_1^T)_{ult}\right)^2 - \left(\sigma_1\sigma_2/(\sigma_1^T)^2_{ult}\right) + \left(\sigma_2/(\sigma_2^T)_{ult}\right)^2 + \left(\tau_{12}/(\tau_{12})_{ult}\right)^2 < 1 \rightarrow \boxed{-62.2352 < \tau < 62.2352}$ MPa

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$$\Rightarrow H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^c)_{ult}} = 3.9365\times10^{-4}\ \text{MPa}^{-1},\quad H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^c)_{ult}} = 0.0114\ \text{MPa}^{-1},\quad H_6 = 0$$

$$H_{11} = 1/(\sigma_1^T)(\sigma_1^c)_{ult} = 3.1746\times10^{-7}\ \text{MPa}^{-2},\quad H_{22} = 1/(\sigma_2^T)(\sigma_2^c)_{ult} = 8.1156\times10^{-5}\ \text{MPa}^{-2},\quad H_{66} = 1/(\tau_{12})^2_{ult} = 2.2277\times10^{-4}\ \text{MPa}^{-2}$$

$$H_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^c)_{ult}(\sigma_2^T)_{ult}(\sigma_2^c)_{ult}}} = -2.5379\times10^{-6}\ \text{MPa}^{-2}$$

⇒ Tsai-Wu → $H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$

$\rightsquigarrow -9.5315\times10^{-3}\tau + 1.206\times10^{-4}\tau^2 < 1 \rightarrow \boxed{-59.7476 < \tau < 138.7816}$ MPa

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5) Problem 2.41 : Off-axis test, Tsai-Wu failure theory, Boron/epoxy system

$(\sigma_1^T)_{ult} = 188$ ksi , $(\sigma_1^C)_{ult} = 361$ ksi , $(\sigma_2^T)_{ult} = 9$ ksi , $(\sigma_2^C)_{ult} = 45$ ksi, $(\tau_{12})_{ult} = 10$ ksi

A 15° specimen fails at a uniaxial load of 33.546 ksi $\Rightarrow H_{12} = ?$

$$\Rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} \sigma_1 = 0.933\,\sigma_x \\ \sigma_2 = 0.06699\,\sigma_x \\ \tau_{12} = -0.25\,\sigma_x \end{array}$$

$* \; H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}} = 2.5491 \times 10^{-3}\ \text{ksi}^{-1}$ $\qquad * \; H_{11} = \frac{1}{(\sigma_1^T)(\sigma_1^C)_{ult}} = 1.4734 \times 10^{-5}\ \text{ksi}^{-2}$

$* \; H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}} = 0.0889\ \text{ksi}^{-1}$ $\qquad * \; H_{22} = \frac{1}{(\sigma_2^T)(\sigma_2^C)_{ult}} = 2.4691 \times 10^{-3}\ \text{ksi}^{-2}$

$* \; H_6 = 0$ $\qquad * \; H_{66} = \frac{1}{(\tau_{12})^2_{ult}} = 0.01\ \text{ksi}^{-2}$

$\Rightarrow$ Tsai-Wu $\rightarrow H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$

$$8.3337 \times 10^{-3}\,\sigma_x + 0.6489 \times 10^{-3}\,\sigma_x^2 + 0.125\,\sigma_x^2 H_{12} < 1$$

$\sigma_x \rightarrow 33.546$ ksi $\Rightarrow 1.0098 + 140.6668\,H_{12} < 1$

$\rightsquigarrow H_{12} < -6.9668 \times 10^{-5}\ \text{ksi}^{-2}$

$\Rightarrow$ To check satisfying $H_{12}^2 < H_{11}H_{22}$

$$\underbrace{4.8537 \times 10^{-9} < 36.3797 \times 10^{-9}}_{\text{It satisfies}} \quad \checkmark$$

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