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MAK 413 - Mechanics of Composite Materials

Spring 2018

Homework - 2

97/100

1.) $\theta = 30°$ Find the stresses in the local axes.

$\sigma_x = 4$ MPa

$\sigma_y = 2$ MPa

$\tau_{xy} = -3$ MPa

$c = \cos 30° = 0,866$

$s = \sin 30° = 0,5$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \rightarrow [T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} = \begin{bmatrix} 0,75 & 0,25 & 0,866 \\ 0,25 & 0,75 & -0,866 \\ -0,433 & 0,433 & 0,5 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} 0,75 & 0,25 & 0,866 \\ 0,25 & 0,75 & -0,866 \\ -0,433 & 0,433 & 0,5 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$$

$\sigma_1 = (0,75 . 4) + (0,25 . 2) - (0,866 . 3) = 0,902$

$\sigma_2 = (0,25 . 4) + (0,75 . 2) + (0,866 . 3) = 5,098$

$\tau_{12} = -(0,433 . 4) + (0,433 . 2) - (0,5 . 3) = -2,366$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} 0,902 \\ 5,098 \\ -2,366 \end{bmatrix} \text{MPa}$$

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Why or why not? -3

2.) $E_1 = 204\ GPa$ $\quad c = \cos 60° = 0,5$

$E_2 = 18,50\ GPa$ $\quad s = \sin 60° = 0,866$

$\nu_{12} = 0,23$

$G_{12} = 5,59$

$$T = \begin{bmatrix} 0,25 & 0,75 & 0,866 \\ 0,75 & 0,25 & -0,866 \\ -0,433 & 0,433 & -0,5 \end{bmatrix}$$

$$S_{11} = \frac{1}{204 \cdot 10^9} = 4,9 \cdot 10^{-12}\ 1/Pa \qquad S_{22} = \frac{1}{18,50 \cdot 10^9} = 54,054 \cdot 10^{-12}\ 1/Pa$$

$$S_{12} = \frac{-0,23}{204 \cdot 10^9} = -1,1275 \cdot 10^{-12}\ 1/Pa \qquad S_{66} = \frac{1}{5,59 \cdot 10^9} = 178,8909 \cdot 10^{-12}\ 1/Pa$$

$$\bar{S}_{11} = S_{11} \cdot c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22} \cdot s^4$$

$$= 4,9 \cdot 10^{-12} \cdot (0,5)^4 + (-2 \cdot 1,1275 \cdot 10^{-12} + 178,8909 \cdot 10^{-12})(0,866)^2(0,5)^2$$
$$+ 54,054 \cdot 10^{-12} \cdot (0,866)^2$$

$$\bar{S}_{11} = 0,30625 \cdot 10^{-12} + 33,117 \cdot 10^{-12} + 30,402 \cdot 10^{-12}$$

$$= 63,825 \cdot 10^{-12}\ 1/Pa \longrightarrow \bar{S}_{11} = 6,3825 \cdot 10^{-2}\ 1/GPa$$

$$\bar{S}_{22} = S_{11} \cdot s^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}c^4$$

$$= 4,9 \cdot 10^{-12} \cdot (0,866)^4 + (-2 \cdot 1,1275 \cdot 10^{-12} + 178,8909 \cdot 10^{-12}) \cdot (0,866)^2 \cdot (0,5)^2$$
$$+ 54,054 \cdot 10^{-12} \cdot (0,5)^4$$

$$\bar{S}_{22} = 2,756 \cdot 10^{-12} + 33,117 \cdot 10^{-12} + 3,3784 \cdot 10^{-12}$$

$$= 39,251 \cdot 10^{-12}\ 1/Pa \longrightarrow \bar{S}_{22} = 3,925 \cdot 10^{-2}\ 1/GPa$$

$$\bar{S}_{12} = S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66})s^2c^2$$

$$= -1,1275 \cdot 10^{-12}((0,866)^4 + (0,5)^4) + (4,9 \cdot 10^{-12} + 54,054 \cdot 10^{-12} - 178,8909 \cdot 10^{-12})$$
$$\cdot ((0,866)^2 \cdot (0,5)^2)$$

$$= -0,7046 \cdot 10^{-12} - 22,4868 \cdot 10^{-12} \rightarrow \bar{S}_{12} = -2,3191 \cdot 10^{-2}\ 1/GPa$$

2.) devamı

$$\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c$$

$$= (2.4,9.10^{-12} + 2.1,1275.10^{-12} - 178,8909.10^{-12}).(0,866).(0,5)^3$$
$$-(2.54,054.10^{-12} + 2.1,1275.10^{-12} - 178,8909.10^{-12}).(0,866)^3.(0,5)$$

$$= -18,06.10^{-12} + 22,2531.10^{-12} \rightarrow \bar{S}_{16} = 4,194.10^{-3} \ 1/GPa$$

$$\bar{S}_{26} = (2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3$$

$$= (2.4,9.10^{-12} + 2.1,1275.10^{-12} - 178,8909.10^{-12}).(0,866)^3.(0,5)$$
$$-(2.54,054.10^{-12} + 2.1,1275.10^{-12} - 178,8909.10^{-12}).(0,866).(0,5)^3$$

$$= -54,1768.10^{-12} + 7,4181.10^{-12} \rightarrow \bar{S}_{26} = -4,676.10^{-2} \ 1/GPa$$

$$\bar{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4)$$

$$= 2(2.4,9.10^{-12} + 2.54,054.10^{-12} + 4.1,1275.10^{-12} - 178,8909.10^{-12})(0,866)^2(0,5)^2$$
$$+ 178,8909.10^{-12}.((0,866)^4 + (0,5)^4)$$

$$= -21,176.10^{-12} + 111,795.10^{-12} \rightarrow \bar{S}_{66} = 9,062.10^{-2} \ 1/GPa$$

$$[\bar{S}] = \begin{bmatrix} 6,3825.10^{-2} & -2,3191.10^{-2} & 4,194.10^{-3} \\ -2,3191.10^{-2} & 3,925.10^{-2} & -4,676.10^{-2} \\ 4,194.10^{-3} & -4,676.10^{-2} & 9,062.10^{-2} \end{bmatrix} 1/GPa$$

Transformed reduced compliance matrix

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$$\left.\begin{aligned} \{\varepsilon\} &= [\bar{S}]\{\sigma\} \\ \{\sigma\} &= [\bar{Q}]\{\varepsilon\} \end{aligned}\right\} \rightarrow [\bar{S}]^{-1} = [Q]$$

$$[\bar{S}]^{-1} = [\bar{Q}] = \begin{bmatrix} 29,08 & 40,43 & 19,52 \\ 40,43 & 122,35 & 61,26 \\ 19,52 & 61,26 & 41,74 \end{bmatrix} GPa$$

✓ 10

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3.) $60°$ angle lamina boron / epoxy

$\sigma_x = 4\,MPa$ $E_1 = 204\,GPa$ $\nu_{12} = 0{,}23$

$\sigma_y = 2\,MPa$ $E_2 = 18{,}50\,GPa$ $G_{12} = 5{,}59\,GPa$

$\tau_{xy} = -3\,MPa$ $c = \cos(60°) = 0{,}5$ $s = \sin(60°) = 0{,}866$

Global strains?
Local stresses and strains?
Principal normal stresses and principal normal strains?
Max shear stress and max shear strain

Önceki soruda hesaplanan değerler için,

a.) $S_{11} = 4{,}9 \cdot 10^{-12}\ 1/Pa$ $S_{22} = 54{,}054 \cdot 10^{-12}\ 1/Pa$

$S_{12} = -1{,}1275 \cdot 10^{-12}\ 1/Pa$ $S_{66} = 178{,}8909 \cdot 10^{-12}\ 1/Pa$

$\bar{S}_{11} = 6{,}3825 \cdot 10^{-2}\ 1/GPa$ $\bar{S}_{22} = 3{,}925 \cdot 10^{-2}\ 1/GPa$ $\bar{S}_{26} = -4{,}676 \cdot 10^{-2}\ 1/GPa$

$\bar{S}_{12} = -2{,}3191 \cdot 10^{-2}\ 1/GPa$ $\bar{S}_{16} = 4{,}194 \cdot 10^{-3}\ 1/GPa$ $\bar{S}_{66} = 9{,}062 \cdot 10^{-2}\ 1/GPa$

$$\{\varepsilon\} = [\bar{S}]\{\sigma\}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \underset{1/GPa}{\begin{bmatrix} 6{,}3825 \cdot 10^{-2} & -2{,}3191 \cdot 10^{-2} & 4{,}194 \cdot 10^{-3} \\ -2{,}3191 \cdot 10^{-2} & 3{,}925 \cdot 10^{-2} & -4{,}676 \cdot 10^{-2} \\ 4{,}194 \cdot 10^{-3} & -4{,}676 \cdot 10^{-2} & 9{,}062 \cdot 10^{-2} \end{bmatrix}} \underset{GPa}{\begin{bmatrix} 4 \cdot 10^{-3} \\ 2 \cdot 10^{-3} \\ -3 \cdot 10^{-3} \end{bmatrix}}$$

$$\varepsilon_x = 25{,}53 \cdot 10^{-5} - 4{,}6382 \cdot 10^{-5} - 1{,}2582 \cdot 10^{-5} = 19{,}6336 \cdot 10^{-5}$$

$$\varepsilon_y = -9{,}2764 \cdot 10^{-5} + 7{,}85 \cdot 10^{-5} + 14{,}028 \cdot 10^{-5} = 12{,}6016 \cdot 10^{-5}$$

$$\gamma_{xy} = 1{,}6776 \cdot 10^{-5} - 9{,}352 \cdot 10^{-5} - 27{,}186 \cdot 10^{-5} = -34{,}8604 \cdot 10^{-5}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 196{,}336 \\ 126{,}016 \\ -348{,}604 \end{bmatrix} \frac{\mu m}{m} \ (\cdot 10^{-6})$$ #

Global strains in the x-y plane

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b.) $$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{12} \end{bmatrix}$$

For 60° Transformation Matrix

$$[T] = \begin{bmatrix} 0{,}25 & 0{,}75 & 0{,}866 \\ 0{,}75 & 0{,}25 & -0{,}866 \\ -0{,}433 & 0{,}433 & -0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

3.) devamı

b.)
$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}0{,}25 & 0{,}75 & 0{,}866\\ 0{,}75 & 0{,}25 & -0{,}866\\ -0{,}433 & 0{,}433 & -0{,}5\end{bmatrix}\begin{bmatrix}4\\ 2\\ -3\end{bmatrix}$$

Local stresses

$\sigma_1 = 1 + 1{,}5 - 2{,}598 = -0{,}098$ MPa

$\sigma_2 = 3 + 0{,}5 + 2{,}598 = 6{,}098$ MPa

$\tau_{12} = -1{,}732 + 0{,}866 + 1{,}5 = 0{,}634$ MPa

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$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}-9{,}8 \cdot 10^{-2}\\ 6{,}098\\ 6{,}34 \cdot 10^{-1}\end{bmatrix} \text{MPa}$$ #

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}/2\end{bmatrix} = \begin{bmatrix}0{,}25 & 0{,}75 & 0{,}866\\ 0{,}75 & 0{,}25 & -0{,}866\\ -0{,}433 & 0{,}433 & -0{,}5\end{bmatrix}\begin{bmatrix}196{,}336 \cdot 10^{-6}\\ 126{,}016 \cdot 10^{-6}\\ -348{,}604 \cdot 10^{-6}/2\end{bmatrix}$$

$\varepsilon_1 = 49{,}084 \cdot 10^{-6} + 94{,}512 \cdot 10^{-6} - 150{,}945 \cdot 10^{-6} = -7{,}35 \cdot 10^{-6}$

$\varepsilon_2 = 147{,}252 \cdot 10^{-6} + 31{,}504 \cdot 10^{-6} + 150{,}945 \cdot 10^{-6} = 329{,}7 \cdot 10^{-6}$

$\gamma_{12} = (-85{,}013 \cdot 10^{-6} + 54{,}565 \cdot 10^{-6} + 87{,}151 \cdot 10^{-6}) \cdot 4 = 113{,}406 \cdot 10^{-6}$

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}-7{,}35\\ 329{,}7\\ 113{,}406\end{bmatrix} \frac{\mu m}{m} (\cdot 10^{-6})$$ #

Local strains

c.)
$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4+2}{2} \pm \sqrt{\left(\frac{4-2}{2}\right)^2 + (-3)^2}$$

principal normal stresses

$\sigma_{max} = 6{,}1623$ MPa

$\sigma_{min} = -0{,}1623$ MPa #

The angle at which the max normal stresses occur,

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \rightarrow \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\cdot(-3)}{4-2}\right)$$

$\theta_p = -35{,}78°$

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$$\varepsilon_{max,min} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{196{,}336 + 126{,}016}{2} \pm \sqrt{\left(\frac{196{,}336 - 126{,}016}{2}\right)^2 + \left(\frac{-348{,}604}{2}\right)^2}$$

principal normal strains

$\varepsilon_{max} = 338{,}989$ $(\cdot 10^{-6}) \frac{\mu m}{m}$

$\varepsilon_{min} = -16{,}637$ $(\cdot 10^{-6}) \frac{\mu m}{m}$ #

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3.) devamı

d.) $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\tau_{max} = 3,162 \ MPa$ # maximum shear stress

$= \sqrt{\left(\frac{4-2}{2}\right)^2 + (-3)^2}$

$\gamma_{max} = 2 \cdot \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

maximum shear strain

$\gamma_{max} = 355,626 \ \frac{\mu m}{m}$ $(\cdot 10^{-6})$ #

$= 2 \cdot \sqrt{\left(\frac{196,336 - 126,016}{2}\right)^2 + \left(\frac{-348,604}{2}\right)^2}$

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4.) Glass/epoxy

$E_1 = 5{,}60$ Msi $\qquad \tau_{xy} = 0{,}4$ ksi

$E_2 = 1{,}20$ Msi $\qquad \gamma_{xy} = 468{,}3 \ \frac{\mu in}{in}$

$G_{12} = 0{,}60$ Msi

$\nu_{12} = 0{,}26$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \cancel{\sigma_x} \to 0 \\ \cancel{\sigma_y} \to 0 \\ \tau_{xy} \end{bmatrix}$$

$$S_{11} = \frac{1}{5{,}60} = 0{,}1786 \cdot 10^{-6} \ 1/psi$$

$$S_{12} = \frac{-0{,}26}{5{,}60} = 0{,}046 \cdot 10^{-6} \ 1/psi$$

$$S_{22} = \frac{1}{1{,}20} = 0{,}833 \cdot 10^{-6} \ 1/psi$$

$$S_{66} = \frac{1}{0{,}60} = 1{,}667 \cdot 10^{-6} \ 1/psi$$

$$\bar{S}_{66} = \left[2(2 \cdot 0{,}179 + 2 \cdot 0{,}833 + 4 \cdot 0{,}046 - 1{,}667) s^2c^2 + 1{,}667 \cdot (s^4+c^4)\right] \cdot 10^{-6}$$

$$\bar{S}_{66} = \left[1{,}082 \cdot s^2c^2 + 1{,}667 (s^4+c^4)\right] 10^{-6}$$

$s = \sin\theta = x$

$c = \cos\theta \Rightarrow c^2 = \cos^2\theta = 1 - x^2 \qquad \gamma_{xy} = \bar{S}_{66} \cdot \tau_{xy}$

$c^4 = (1-x^2)^2$

$$468{,}3 = \left[1{,}082 \cdot x^2 \cdot (1-x^2) + 1{,}667(x^4 + (1-x^2)^2)\right] \cdot 400$$

$$1{,}17 = 1{,}082x^2 - 1082x^4 + 1{,}667x^4 + 1{,}667 - 3{,}334x^2 + 1{,}667x^4$$

$$-0{,}4969 = 2{,}252x^4 - 2{,}252x^2$$

$$-0{,}4969 = 2{,}252(x^4 - x^2)$$

$$-0{,}22067 = x^4 - x^2$$

$$\Downarrow$$

$$x^4 - x^2 + 0{,}22067 = 0 \quad (y = x^2)$$

$$y^2 - y + 0{,}22067 = 0$$

$$\Delta = b^2 - 4ac = 0{,}1173$$

$$\frac{-b \mp \sqrt{\Delta}}{2a} = \frac{1 \pm \sqrt{0{,}1173}}{2} \quad (\sqrt{0{,}1173} \to 0{,}3425)$$

$y_1 = 0{,}67125$

$y_2 = 0{,}32875$

$x^2 = y \qquad \sqrt{y_1} = \mp 0{,}8193$

$\sqrt{y_2} = \mp 0{,}5733$

$x = \sin\theta = 0{,}5733$

$= 0{,}8193$

$$\boxed{\theta = 34{,}98^\circ}$$

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5.) Find six constants for 60° boron / epoxy

$E_1 = 29,59$ Msi $\quad \nu_{12} = 0,23 \quad c = \cos(60°) = 0,5$

$E_2 = 2,683$ Msi $\quad s = \sin(60°) = 0,866$

$G_{12} = 0,811$ Msi

$$S_{11} = \frac{1}{29,59.10^6} = 33,8.10^{-9}\ 1/psi \qquad S_{22} = \frac{1}{2,683.10^6} = 372,72.10^{-9}\ 1/psi$$

$$S_{12} = \frac{-0,23}{29,59.10^6} = -7,77.10^{-9}\ 1/psi \qquad S_{66} = \frac{1}{0,811.10^6} = 1233,05.10^{-9}\ 1/psi$$

$$\bar{S}_{11} = S_{11}.c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4$$

$$= 33,8.10^{-9}.(0,5)^4 + (-2.7,77.10^{-9} + 1233,05.10^{-9})(0,866)^2(0,5)^2 + 372,72.10^{-9}.(0,866)^4$$

$$= (2,1125 + 228,27 + 209,63).10^{-9} \rightarrow \bar{S}_{11} = 440.10^{-9}\ 1/psi$$

$$\bar{S}_{12} = S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66})s^2c^2$$

$$= -7,77.10^{-9}((0,866)^4 + (0,5)^4) + (33,8 + 372,72 - 1233,05).10^{-9}.(0,866)^2.(0,5)^2$$

$$= -4,86.10^{-9} - 154,97.10^{-9} \rightarrow \bar{S}_{12} = -159,82$$

$$\bar{S}_{12} \cong -160.10^{-9}\ 1/psi$$

$$\bar{S}_{22} = S_{11}.s^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}.c^4$$

$$= 33,8.10^{-9}.(0,866)^4 + (-2.7,77.10^{-9} + 1233,05.10^{-9})(0,866)^2(0,5)^2 + 372,72.10^{-9}.(0,5)^4$$

$$= (19,01 + 228,27 + 23,295).10^{-9} \rightarrow \bar{S}_{22} = 270,575$$

$$\bar{S}_{22} \cong 270,6.10^{-9}\ 1/psi$$

$$\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c$$

$$= [(2.33,8 + 2.7,77 - 1233,05).(0,866)(0,5)^3 - (2.372,72 + 2.7,77 - 1233,05).(0,866)^3(0,5)].10^{-9}$$

$$= [\underbrace{(-1150).(0,866).(0,5)^3}_{-124,48} - \underbrace{(-472,07).(0,866)^3.(0,5)}_{153,3}].10^{-9}$$

$$\bar{S}_{16} = 28,8.10^{-9}\ 1/psi$$

devamı

$$\bar{S}_{26} = (2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3$$

$$= [(2 \cdot 33{,}8 + 2 \cdot 7{,}77 - 1233{,}05)(0{,}866)^3(0{,}5) - (2 \cdot 372{,}72 + 2 \cdot 7{,}77 - 1233{,}05) \cdot (0{,}866) \cdot (0{,}5)^3] \cdot 10^{-9}$$

$$= [\underbrace{(-1150) \cdot (0{,}866)^3(0{,}5)}_{-373{,}44} - \underbrace{(-472{,}07) \cdot (0{,}866)(0{,}5)^3}_{51{,}1}] \cdot 10^{-9}$$

$$\bar{S}_{26} = -322{,}34 \cdot 10^{-9} \ 1/psi$$

$$\bar{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4)$$

$$= [2(2 \cdot 33{,}8 + 2 \cdot 372{,}72 + 4 \cdot 7{,}77 - 1233{,}05)(0{,}866)^2(0{,}5)^2 + 1233{,}05 \cdot ((0{,}866)^4 + (0{,}5)^4)] \cdot 10^{-9}$$

$$= [-145{,}84 + 770{,}57] \cdot 10^{-9} \rightarrow \bar{S}_{66} = 624{,}73 \cdot 10^{-9} \ 1/psi$$

$$E_x = \frac{1}{440 \cdot 10^{-9}} = 2{,}2727 \cdot 10^6 psi = 2{,}2727 \ Msi \ \#$$

$$\nu_{xy} = -\frac{-160 \cdot 10^{-9}}{440 \cdot 10^{-9}} = 0{,}363 \ \#$$

$$\frac{1}{m_x} = -\frac{1}{\bar{S}_{16} \cdot E_1} = -\frac{1}{28{,}8 \cdot 10^{-9} \cdot 29{,}59 \cdot 10^6} \Rightarrow m_x = -0{,}8522 \ \#$$

$$E_y = \frac{1}{270{,}6 \cdot 10^{-9}} = 3{,}696 \cdot 10^6 psi = 3{,}696 \ Msi \ \#$$

$$G_{xy} = \frac{1}{624{,}73 \cdot 10^{-9}} = 1{,}6 \cdot 10^6 psi = 1{,}6 \ Msi \ \#$$

$$\frac{1}{m_y} = -\frac{1}{\bar{S}_{26} \cdot E_1} = -\frac{1}{-322{,}34 \cdot 10^{-9} \cdot 29{,}59 \cdot 106} \Rightarrow m_y = 9{,}53804 \ \#$$

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