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MAK 411

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HOMEWORK 1

1) Problem 2.2 : Orthotropic Material ;

$E_1 = 4$ Msi , $E_2 = 3$ Msi , $E_3 = 3.1$ Msi
$\nu_{12} = 0.2$, $\nu_{23} = 0.4$, $\nu_{31} = 0.6$
$G_{12} = 6$ Msi , $G_{23} = 7$ Msi , $G_{31} = 2$ Msi

Stiffness Matrix [C] = ?
Compliance Matrix [S] = ?

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix}$$

$S_{11} = \frac{\varepsilon_1}{\sigma_1} = \frac{1}{E_1}$, $S_{12} = \frac{\varepsilon_2}{\sigma_1} = -\frac{\nu_{12}}{E_1}$, $S_{13} = \frac{\varepsilon_3}{\sigma_1} = -\frac{\nu_{13}}{E_1} = -\frac{\nu_{31}}{E_3}$

$S_{22} = \frac{\varepsilon_2}{\sigma_2} = \frac{1}{E_2}$, $S_{23} = \frac{\varepsilon_3}{\sigma_2} = -\frac{\nu_{23}}{E_2}$, $S_{33} = \frac{\varepsilon_3}{\sigma_3} = \frac{1}{E_3}$

$S_{44} = \frac{\gamma_{23}}{\tau_{23}} = \frac{1}{G_{23}}$, $S_{55} = \frac{\gamma_{31}}{\tau_{31}} = \frac{1}{G_{31}}$, $S_{66} = \frac{\gamma_{12}}{\tau_{12}} = \frac{1}{G_{12}}$

$$[S] = \begin{bmatrix} 1/4 & -0.2/4 & -0.6/3.1 & 0 & 0 & 0 \\ -0.2/4 & 1/3 & -0.4/3 & 0 & 0 & 0 \\ -0.6/3.1 & -0.4/3 & 1/3.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/6 \end{bmatrix} = \boxed{\begin{bmatrix} 0.25 & -0.05 & -0.1935 & 0 & 0 & 0 \\ -0.05 & 0.3333 & -0.1333 & 0 & 0 & 0 \\ -0.1935 & -0.1333 & 0.3226 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1429 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1667 \end{bmatrix}}$$

Msi^{-1}

$$[C] = [S]^{-1} = \boxed{\begin{bmatrix} 13.6548 & 6.3781 & 10.8258 & 0 & 0 & 0 \\ 6.3781 & 6.5735 & 6.5418 & 0 & 0 & 0 \\ 10.8258 & 6.5418 & 12.2964 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix}}$$

← From the calculator result

Msi

OR, From the formulas ; $\nu_{21} = \frac{E_2}{E_1}\nu_{12} = 0.15$, $\nu_{32} = \frac{E_3}{E_2}\nu_{23} = 0.4133$, $\nu_{13} = \frac{E_1}{E_3}\nu_{31} = 0.7742$

$\Delta = (1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13}) / E_1E_2E_3 = 0.0066$

$\Rightarrow C_{11} = \frac{1-\nu_{23}\nu_{32}}{E_2E_3\Delta} = \boxed{13.5986}$; $C_{12} = \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2E_3\Delta} = \boxed{6.3539}$; $C_{13} = \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} = \boxed{10.7852}$

$C_{22} = \frac{1-\nu_{13}\nu_{31}}{E_1E_3\Delta} = \boxed{6.5430}$; $C_{23} = \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} = \boxed{6.5164}$; $C_{33} = \frac{1-\nu_{12}\nu_{21}}{E_1E_2\Delta} = \boxed{12.2475}$

Msi

$C_{44} = G_{23} = 7$ Msi ; $C_{55} = G_{31} = 2$ Msi ; $C_{66} = G_{12} = 6$ Msi

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2) Problem 2.3 : Orthotropic Material

$$[C] = \begin{bmatrix} -0.67308 & -1.8269 & -1.0577 & 0 & 0 & 0 \\ -1.8269 & -0.67308 & -1.4423 & 0 & 0 & 0 \\ -1.0577 & -1.4423 & 0.48077 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.5 \end{bmatrix} \text{GPa}$$

a) $\varepsilon_1 = 1\,\mu m/m$, $\varepsilon_2 = 3\,\mu m/m$, $\varepsilon_3 = 2\,\mu m/m$; $\gamma_{23} = 0$, $\gamma_{31} = 5\,\mu m/m$, $\gamma_{12} = 6\,\mu m/m$

$$\Rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = [C] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \underbrace{\begin{bmatrix} -0.67308 & -1.8269 & -1.0577 & 0 & 0 & 0 \\ -1.8269 & -0.67308 & -1.4423 & 0 & 0 & 0 \\ -1.0577 & -1.4423 & 0.48077 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.5 \end{bmatrix}}_{\times 10^9} \underbrace{\begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \\ 5 \\ 6 \end{bmatrix}}_{\times 10^{-6} = 10^3} \times 10^3 = \boxed{\begin{bmatrix} -8.26918 \\ -6.73074 \\ -4.42306 \\ 0 \\ 10 \\ 9 \end{bmatrix} \text{kPa}}$$

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b)

$$[S] = [C]^{-1} = \boxed{\begin{bmatrix} 0.5 & -0.5 & -0.4 & 0 & 0 & 0 \\ -0.5 & 0.3 & -0.2 & 0 & 0 & 0 \\ -0.4 & -0.2 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6667 \end{bmatrix} \text{GPa}^{-1}}$$

← by a calculator

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c) $E_1 = \frac{1}{S_{11}} = 2$, $E_2 = \frac{1}{S_{22}} = 3.3333$, $E_3 = \frac{1}{S_{33}} = 1.6667$) GPa

$\nu_{12} = -\frac{S_{12}}{S_{11}} = 1$, $\nu_{23} = -\frac{S_{23}}{S_{22}} = 0.6667$, $\nu_{31} = -\frac{S_{31}}{S_{33}} = 0.6667$

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$G_{12} = \frac{1}{S_{66}} = 1.5$, $G_{23} = \frac{1}{S_{44}} = 4$, $G_{31} = \frac{1}{S_{55}} = 2$) GPa

d) $W = \frac{1}{2}\left(\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3 + \tau_{12}\gamma_{12} + \tau_{23}\gamma_{23} + \tau_{31}\gamma_{31}\right)$

$= \frac{1}{2}\left(-8.26918 \times 1 + (-6.73074 \times 3) + (-4.42306 \times 2) + 9 \times 6 + 0 + 10 \times 5\right) \times 10^{-3}$

$\boxed{= 33.34624 \times 10^{-3}\ N/m^3}$

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3) **Problem 2.10**: For a unidirectional lamina of boron/epoxy {Table 2.1}, [Q] & [S] = ?

From Table 2.1: $E_1 = 204$ GPa, $E_2 = 18.50$ GPa; $\nu_{12} = 0.23$; $G_{12} = 5.59$ GPa

$$\Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad \left\} \begin{array}{l} S_{11} = \dfrac{\varepsilon_1}{\sigma_1} = \dfrac{1}{E_1}, \quad S_{12} = \dfrac{\varepsilon_2}{\sigma_1} = -\dfrac{\nu_{12}}{E_1} \\ S_{22} = \dfrac{\varepsilon_2}{\sigma_2} = \dfrac{1}{E_2}, \quad S_{66} = \dfrac{\gamma_{12}}{\tau_{12}} = \dfrac{1}{G_{12}} \end{array} \right.$$

$$[S] = \begin{bmatrix} 1/204 & -0.23/204 & 0 \\ -0.23/204 & 1/18.5 & 0 \\ 0 & 0 & 1/5.59 \end{bmatrix} = \boxed{\begin{bmatrix} 0.0049 & -0.0011 & 0 \\ -0.0011 & 0.0541 & 0 \\ 0 & 0 & 0.1789 \end{bmatrix} \text{GPa}^{-1}}$$

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$$\Rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad \left\} \begin{array}{l} Q_{11} = \dfrac{E_1}{1-\nu_{21}\nu_{12}}, \quad Q_{12} = \dfrac{\nu_{12}E_2}{1-\nu_{21}\nu_{12}} \\ Q_{22} = \dfrac{E_2}{1-\nu_{21}\nu_{12}}, \quad Q_{66} = G_{12} \end{array} \right. ; \quad \nu_{21} = \frac{\nu_{12}}{E_1} E_2 = 0.0209$$

$$[Q] = \begin{bmatrix} 204/0.9952 & 0.23\times 18.5/0.9952 & 0 \\ 0.23\times 18.5/0.9952 & 18.5/0.9952 & 0 \\ 0 & 0 & 5.59 \end{bmatrix} = \boxed{\begin{bmatrix} 204.98 & 4.2755 & 0 \\ 4.2755 & 18.5892 & 0 \\ 0 & 0 & 5.59 \end{bmatrix} \text{GPa}}$$

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4) **Problem 2.11**: $\sigma_1 = 4$ MPa, $\sigma_2 = 2$ MPa, $\tau_{12} = -3$ MPa; Unidir. boron/epoxy lamina

$$\Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0049 & -0.0011 & 0 \\ -0.0011 & 0.0541 & 0 \\ 0 & 0 & 0.1789 \end{bmatrix} \times 10^{-9} \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix} \times 10^{6} = \begin{bmatrix} 17.4 \\ 103.8 \\ -536.7 \end{bmatrix} \times 10^{-6} \quad \left\} \boxed{\begin{array}{l} \varepsilon_1 = 17.4 \ \mu m/m \\ \varepsilon_2 = 103.8 \ \mu m/m \\ \gamma_{12} = -536.7 \ \mu m/m \end{array}} \right.$$

-2

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$\varepsilon_2 = 103.59 \ \mu m/m$

5) **Problem 2.14** : Unidirectional continuous fiber composite; $[\sigma]=[Q][\varepsilon]$

$$\Rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad \begin{aligned} &\to \sigma_1 = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 \\ &\to \sigma_2 = Q_{12}\varepsilon_1 + Q_{22}\varepsilon_2 \\ &\to \tau_{12} = Q_{66}\gamma_{12} \end{aligned}$$

* Apply a load in direction 1: $\sigma_1 \neq 0$, $\sigma_2 = 0$, $\tau_{12} = 0$

$$\to \begin{bmatrix} \sigma_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad \begin{aligned} &\to \sigma_1 = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 = \varepsilon_1 \cdot E_1 \Rightarrow Q_{11} + Q_{12}\frac{\varepsilon_2}{\varepsilon_1} = E_1 \\ &\to 0 = Q_{12}\varepsilon_1 + Q_{22}\varepsilon_2 \Rightarrow Q_{12}\varepsilon_1 = -Q_{22}\varepsilon_2 \to \varepsilon_2/\varepsilon_1 = -Q_{12}/Q_{22} \end{aligned}$$

$$\rightsquigarrow E_1 = Q_{11} + Q_{12}\left(-\frac{Q_{12}}{Q_{22}}\right) = \boxed{Q_{11} - \frac{Q_{12}^2}{Q_{22}}}$$ 5 ✓

$$* \; \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} = -\left(-\frac{Q_{12}}{Q_{22}}\right) = \boxed{\frac{Q_{12}}{Q_{22}}}$$ 5 ✓

* Apply a load in direction 2: $\sigma_1 = 0$, $\sigma_2 \neq 0$, $\tau_{12} = 0$

$$\to \begin{bmatrix} 0 \\ \sigma_2 \\ 0 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad \begin{aligned} &\to 0 = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 \Rightarrow Q_{11}\varepsilon_1 = -Q_{12}\varepsilon_2 \to \varepsilon_1/\varepsilon_2 = -Q_{12}/Q_{11} \\ &\to \sigma_2 = Q_{12}\varepsilon_1 + Q_{22}\varepsilon_2 = \varepsilon_2 E_2 \Rightarrow Q_{12}\frac{\varepsilon_1}{\varepsilon_2} + Q_{22} = E_2 \end{aligned}$$

$$\rightsquigarrow E_2 = Q_{12}\left(-\frac{Q_{12}}{Q_{11}}\right) + Q_{22} = \boxed{Q_{22} - \frac{Q_{12}^2}{Q_{11}}}$$ 5 ✓

$$* \; \nu_{21} = -\frac{\varepsilon_1}{\varepsilon_2} = -\left(-\frac{Q_{12}}{Q_{11}}\right) = \boxed{\frac{Q_{12}}{Q_{11}}}$$ 5 ✓

* Apply a shear loading: $\sigma_1 = 0$, $\sigma_2 = 0$, $\tau_{12} \neq 0$

$$\to \begin{bmatrix} 0 \\ 0 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \rightsquigarrow \tau_{12} = Q_{66}\gamma_{12} = G_{12}\gamma_{12} \Rightarrow \boxed{G_{12} = Q_{66}}$$ 5 ✓

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