1.1

Adı Soyadı : Numarası:

MAK 510
MAKİNA MÜHENDİSLERİ İÇİN SONLU ELEMANLAR YÖNTEMİ
GÜZ 2008 – 2009 DÖNEMİ
ARA SINAVI
1 Kasım 2008

1. **Şekil 1'de gösterilen ve lineer elastik yaylardan oluşan sistemin her bir yay elemanı için serbest cisim diyagramını çizerek eleman denklemlerini oluşturunuz. Daha sonra bu eleman matrislerini birleştirerek sistemin Global Rijitlik(stiffness) matrisini bulunuz.** 20 **Sınır şartlarını her düğüm noktası için** 10 **uygulayarak bilinmeyen yer değiştirmeleri ve kuvvetlerin bulunabileceği global denklem sistemini en sade matris formunda yazınız. Bu denklem sistemi kullanılarak bilinmeyen yer değiştirmeler ve kuvvetlerin nasıl bulunacağını tarif ediniz.** Castigliano (10)

Consider the system of linear elastic springs shown in Figure 1. Assemble the element equations to obtain the force-displacement relations for the entire system. Use boundary conditions to write the condensed equations for the unknown displacements and forces[1]. Please see the additional information in Turkish

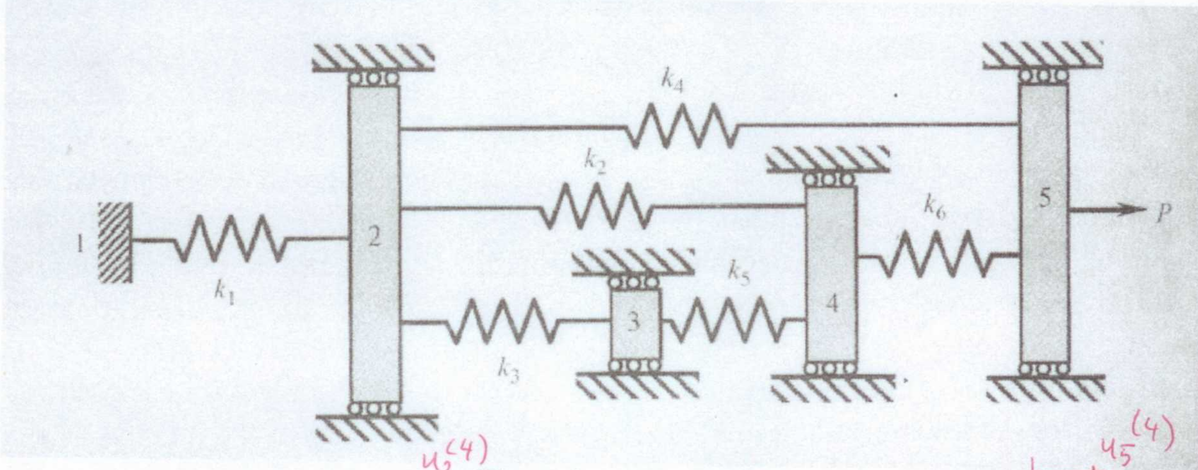
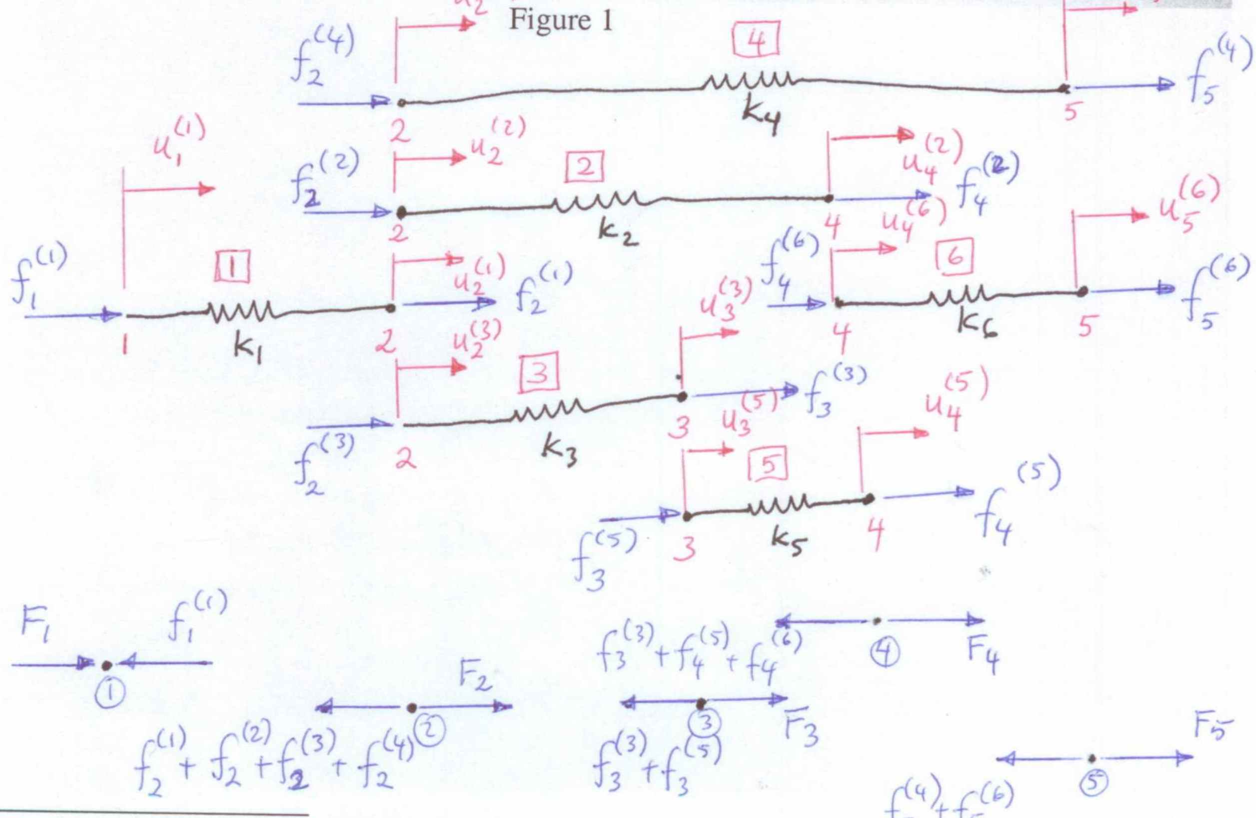


Figure 1



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[1] J.N. Reddy, "An Introduction to the Finite Element Method", 3rd Edition, Mc.Graw Hill, 2006.

Element $\boxed{1}$

$$\begin{bmatrix} k_1 & -k_1 \\ k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix}$$

Element $\boxed{2}$

$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2^{(2)} \\ u_4^{(2)} \end{Bmatrix} = \begin{Bmatrix} f_2^{(2)} \\ f_4^{(2)} \end{Bmatrix}$$

Element $\boxed{3}$

$$\begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_2^{(3)} \\ u_3^{(3)} \end{Bmatrix} = \begin{Bmatrix} f_2^{(3)} \\ f_3^{(3)} \end{Bmatrix}$$

Element $\boxed{4}$

$$\begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_2^{(4)} \\ u_5^{(4)} \end{Bmatrix} = \begin{Bmatrix} f_2^{(4)} \\ f_5^{(4)} \end{Bmatrix}$$

Element $\boxed{5}$

$$\begin{bmatrix} k_5 & -k_5 \\ -k_5 & k_5 \end{bmatrix} \begin{Bmatrix} u_3^{(5)} \\ u_4^{(5)} \end{Bmatrix} = \begin{Bmatrix} f_3^{(5)} \\ f_4^{(5)} \end{Bmatrix}$$

Element $\boxed{6}$

$$\begin{bmatrix} k_6 & -k_6 \\ -k_6 & k_6 \end{bmatrix} \begin{Bmatrix} u_4^{(6)} \\ u_5^{(6)} \end{Bmatrix} = \begin{Bmatrix} f_4^{(6)} \\ f_5^{(6)} \end{Bmatrix}$$

Also

$u_1^{(1)} = U_1$

$u_2^{(1)} = U_2$

$u_2^{(2)} = U_2$

$u_2^{(3)} = U_2$

$u_2^{(4)} = U_2$

$u_3^{(5)} = U_3$

$u_3^{(3)} = U_3$

$u_4^{(2)} = U_4$

$u_4^{(6)} = U_4$

$u_4^{(5)} = U_4$

$u_5^{(4)} = U_5$

$u_5^{(6)} = U_5$

We then write the individual element equations as

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (1)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & -k_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & k_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_2^{(4)} \\ 0 \\ f_4^{(2)} \\ 0 \end{Bmatrix} \quad \text{——— (2)}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_2^{(3)} \\ f_3^{(3)} \\ 0 \\ 0 \end{Bmatrix} \quad \text{——— (3)}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_4 & 0 & 0 & -k_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -k_4 & 0 & 0 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_2^{(4)} \\ 0 \\ 0 \\ f_5^{(4)} \end{Bmatrix} \quad \text{——— (4)}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_5 & -k_5 & 0 \\ 0 & 0 & -k_5 & k_5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ f_3^{(5)} \\ f_4^{(5)} \\ 0 \end{Bmatrix} \quad \text{——— (5)}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_6 & -k_6 \\ 0 & 0 & 0 & -k_6 & k_6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ f_4^{(6)} \\ f_5^{(6)} \end{Bmatrix} \quad \text{——— (6)}$$

Adding Equations 1-6, we obtain

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2+k_3+k_4 & -k_3 & -k_2 & -k_4 \\ 0 & -k_3 & k_3+k_5 & -k_5 & 0 \\ 0 & -k_2 & -k_5 & k_2+k_5+k_6 & -k_6 \\ 0 & -k_4 & 0 & -k_6 & k_4+k_6 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \\ 0 \\ 0 \\ P \end{Bmatrix} \quad \text{— (7)}$$

Condensed equations for the unknown displacements (w_2, w_3, w_4, w_5) are

$$\begin{bmatrix} k_1+k_2+k_3+k_4 & -k_3 & -k_2 & -k_4 \\ -k_3 & k_3+k_5 & -k_5 & 0 \\ -k_2 & -k_5 & k_2+k_5+k_6 & -k_6 \\ -k_4 & 0 & -k_6 & k_4+k_6 \end{bmatrix} \begin{Bmatrix} w_2 \\ w_3 \\ w_4 \\ w_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ P \end{Bmatrix} \quad (8)$$

F_1 can be found from the first equation in Eq. (7)

$$k_1 w_1 - k_1 w_2 = F_1$$

since $w_1 = 0$

$$\boxed{F_1 = -k_1 w_2} \quad \text{—— (9)}$$

1.5

Using the strain energy of the whole system

$$U_e = \frac{1}{2} k_1 (u_2 - u_1)^2 + \frac{1}{2} k_2 (u_4 - u_2)^2 + \frac{1}{2} k_3 (u_3 - u_2)^2$$
$$+ \frac{1}{2} k_4 (u_5 - u_2)^2 + \frac{1}{2} k_5 (u_4 - u_3)^2$$
$$+ \frac{1}{2} k_6 (u_5 - u_4)^2 \qquad (10)$$

Using Castigliano's theorem

$$\frac{\partial U_e}{\partial u_1} = F_1 = k_1 (u_2 - u_1)(-1) = k_1 u_1 - k_1 u_2 \qquad (11)$$

$$\frac{\partial U_e}{\partial u_2} = F_2 = k_1 (u_2 - u_1) + k_2 (u_4 - u_2)(-1) + k_3 (u_3 - u_2)(-1)$$
$$+ k_4 (u_5 - u_2)(-1)$$
$$= F_2 = -k_1 u_1 + (k_1 + k_2 + k_3 + k_4) u_2 - k_3 u_3 - k_4 u_5 \qquad (12)$$

$$\frac{\partial U_e}{\partial u_3} = F_3 = k_3 (u_3 - u_2) + k_5 (u_4 - u_3)(-1)$$
$$= F_3 = -k_3 u_2 + (k_3 + k_5) u_3 - k_5 u_4 \qquad (13)$$

$$\frac{\partial U_e}{\partial u_4} = F_4 = k_2 (u_4 - u_2) + k_5 (u_4 - u_3) + k_6 (u_5 - u_4)(-1)$$
$$= F_4 = -k_2 u_2 + (-k_5) u_3 + (k_2 + k_5 + k_6) u_4 - k_6 u_5 \qquad (14)$$

$$\frac{\partial U_e}{\partial u_5} = F_5 = k_4 (u_5 - u_2) + k_6 (u_5 - u_4)$$
$$F_5 = -k_4 u_2 - k_6 u_4 + (k_4 + k_6) u_5 \qquad (15)$$

Equations (11) - (15) are the same as equation 7

Adı Soyadı : Numarası:

2. **Şekil 2'de iki kiriş bir elastic yayla desteklenmektedir. Kirişlerin içi doludur ve kesiti daireseldir (çap** $d = 20$ **mm ve** $E = 80$ **GPa). Yayın elastiklik sabiti 50 N/mm'dir. Buna göre**
   - **a) Global stiffnes matrisini bulunuz ve sınırlandırılmamış (unconstrained) düğüm noktasının global yer değiştirmelerini bulunuz**
   - **b) Reaksiyon kuvvetlerini bulunuz ve statik dengenin sağlanıp sağlanmadığını kontrol ediniz.**
   - **c) Uygulanan yükün yaptığı iş ile elastic gerinim enerjisi birbirine eşit midir gösteriniz**

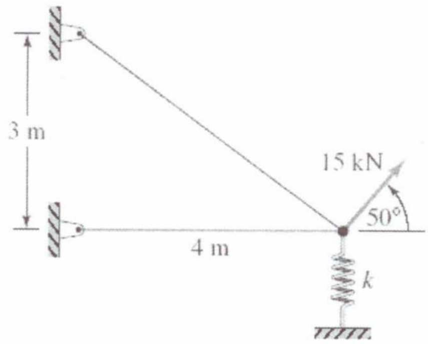
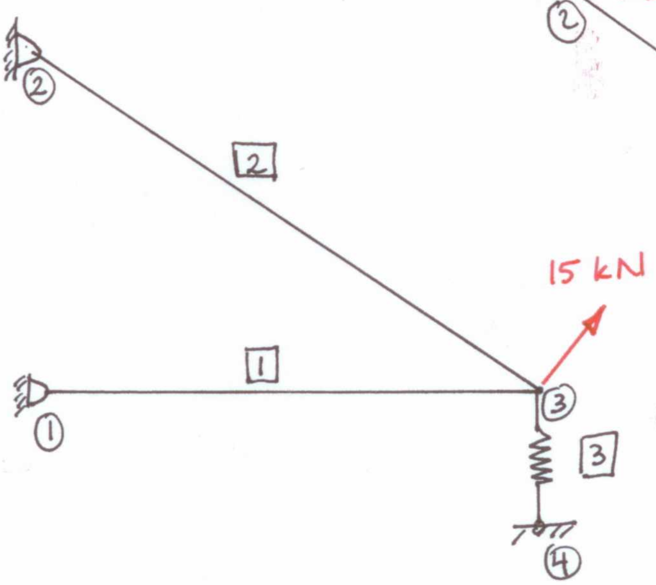


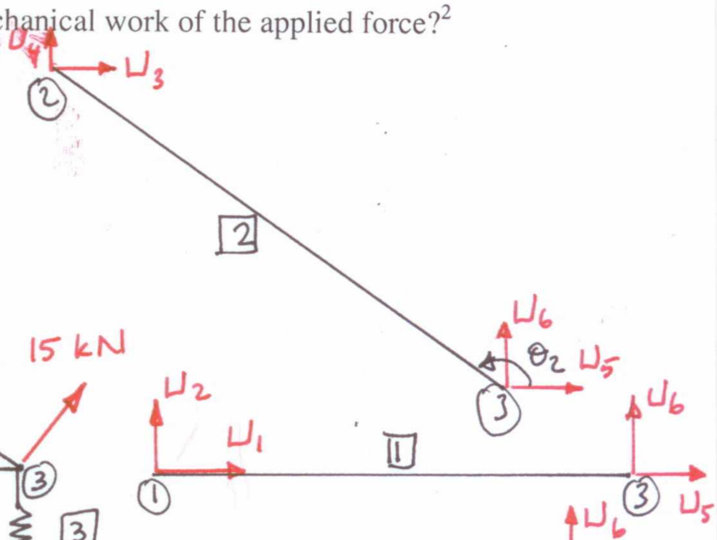
Figure 2 shows a two-member plane truss supported by a linearly elastic spring. The truss members are of a solid circlular cross section having $d = 20$ mm and $E = 80$ GPa. The linear spring has stiffness constant 50 N/mm.

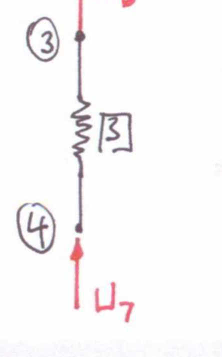
25 a) Assemble the system global stiffness matrix and calculate the global displacements of the unconstrained node.

10 b) Compute the reaction forces and check the equilibrium conditions. Check the energy balance.

5 c) Is the strain energy in balance with the mechanical work of the applied force?[2]







[2] David V. Hutton, "Fundementals of Finite Element Analysis", 1st Edition, Mc.Graw Hill, 2004

Element Displacement Correspondence Table

	Element I	Element II	Element III
1	1	0	0
2	2	0	0
3	0	~~1~~ 3	0
4	0	~~2~~ 4	0
5	3	~~3~~ 1	0
6	4	~~4~~ 2	2
7	0	0	1

$\theta_1 = 0$

$$\theta_2 = 180 - \tan^{-1}\frac{3}{4} = 143.13^\circ$$

$L_1 = 4m$

$L_2 = 5m$

For element I : $\cos\theta_1 = 1$, $\sin\theta_1 = 0$

For element II : $\cos\theta_2 = -\frac{4}{5}$, $\sin\theta_2 = \frac{3}{5}$

$$\cos^2\theta_2 = c^2\theta_2 = 0.64 \quad , \quad s^2\theta_2 = \sin^2\theta_2 = 0.36$$

$$[K^{(1)}] = k_1 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array} \quad ; \quad k_1 = 6283 \text{ N/mm}$$

$$[K^{(2)}] = k_2 \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \\ 3 & 4 & \cancel{5} & \cancel{6} \end{array} \\ \begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix} \end{array} \quad ; \quad k_2 = 5026 \text{ N/mm}$$

$$k_1 = \frac{A_1 E_1}{L_1} = \frac{\frac{\pi (0.020)^2}{4} \cdot 80\,10^9}{4} = 6.283\,10^6 \text{ N/m} = 6283 \text{ N/mm}$$

$$k_2 = \frac{A_2 E_2}{L_2} = \frac{\frac{\pi (0.020)^2}{4} \cdot 80\,10^9}{5} = 5.026\,10^6 \text{ N/m} = 5026 \text{ N/mm}$$

$$[K^{(3)}] = 50 \begin{array}{c} \begin{array}{cc} 7 & 6 \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 7 \\ 6 \end{array} \quad \text{N/mm}$$

Global stiffness matrix

$$[K] = \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} & & & k_{13}^{(1)} & k_{14}^{(1)} & \\ k_{21}^{(1)} & k_{22}^{(1)} & & & k_{23}^{(1)} & k_{24}^{(1)} & \\ & & k_{11}^{(2)} & k_{12}^{(2)} & k_{13}^{(2)} & k_{14}^{(2)} & \\ & & k_{21}^{(2)} & k_{22}^{(2)} & k_{23}^{(2)} & k_{24}^{(2)} & \\ k_{13}^{(1)} & & k_{31}^{(2)} & k_{32}^{(2)} & k_{33}^{(1)} + k_{33}^{(2)} & k_{34}^{(1)} + k_{34}^{(2)} & \\ k_{14}^{(1)} & & k_{41}^{(2)} & k_{42}^{(2)} & k_{43}^{(2)} & k_{44}^{(1)} + k_{44}^{(2)} + k_{22}^{(3)} & k_{12}^{(3)} \\ & & & & & & k_{11}^{(3)} \end{bmatrix} \end{array} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{Bmatrix}$$

$$
[K] = \begin{array}{c}
\begin{array}{ccccccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad & \quad 5 \quad & \quad 6 \quad & \quad 7 \quad \end{array} \\
\begin{bmatrix}
6283 & 0 & 0 & 0 & -6283 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3217 & -2412.5 & -3217 & 2412.5 & 0 \\
0 & 0 & -2412.5 & 1809.4 & 2412.5 & -1809.4 & 0 \\
-6283 & 0 & -3217 & 2412.5 & 9500 & -2412.5 & 0 \\
0 & 0 & 2412.5 & -1809.4 & -2412.5 & 1859.56 & -50 \\
0 & 0 & 0 & 0 & 0 & -50 & 50
\end{bmatrix}
\end{array} \quad [N/mm]
$$

The only nonzero global displacement components are U_5 and U_6 which can be found from the reduced (condensed) equations. Also the nodal forces are

$$
\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ 15 . \cos 50 \\ 15 \sin 50 \\ F_7 \end{Bmatrix}
$$

$$\begin{bmatrix} 9500 & -2412.5 \\ -2412.5 & 1859.56 \end{bmatrix} \begin{Bmatrix} u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 9642 \\ 11490.7 \end{Bmatrix} \quad [N]$$

Therefore

$$\begin{Bmatrix} u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 3.85 \\ 11.18 \end{Bmatrix} \quad [mm]$$

1. denklemden (Global stiffness matrix)

$$6283\,\cancel{u_1}^{\,0} - 6283\,u_5 = F_1 \quad \Rightarrow \quad F_1 = -24.19\text{ kN}$$

3 ve 4. denklemden (Global stiffness matrix)

$$3217\,\cancel{u_3}^{\,0} - 2412.5\,\cancel{u_4}^{\,0} - 3217\,u_5 + 2412.5\,u_6 = F_3$$

$$-2412.5\,\cancel{u_3}^{\,0} + 1809.4\,\cancel{u_4}^{\,0} + 2412.5\,u_5 - 1809.4\,u_6 = F_4$$

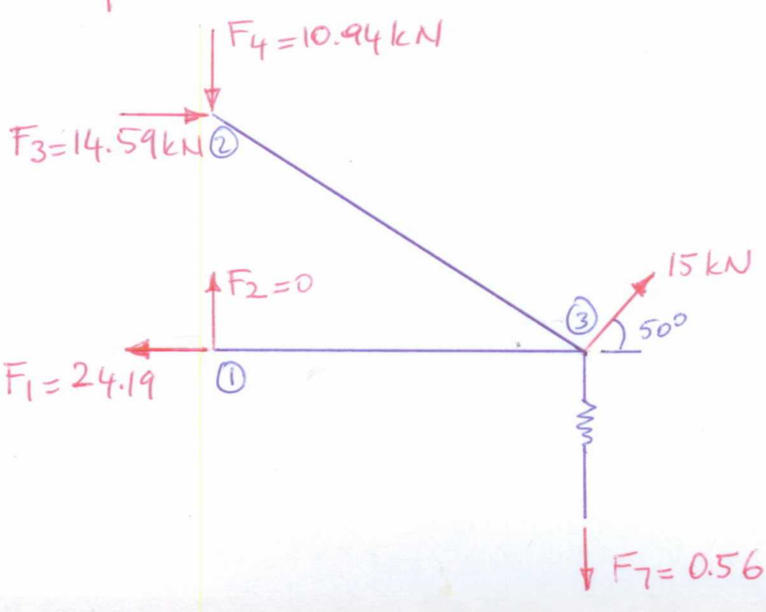
$$F_3 = 14.59\text{ kN}$$

$$F_4 = -10.94\text{ kN}$$

7. denklemden (Global stiffness matrix)

$$-50\,u_6 + 50\,\cancel{u_7}^{\,0} = F_7 \quad \Rightarrow \quad F_7 = -0.56\text{ kN}$$

Equilibrium.



Moment about node 3

$$\circlearrowleft^{+}\ \Sigma M = 0$$

$$14.59\,(3) - 10.94\,(4) = 0.01 \sim 0$$

$$\xrightarrow{+}\ \Sigma F_x = 0$$

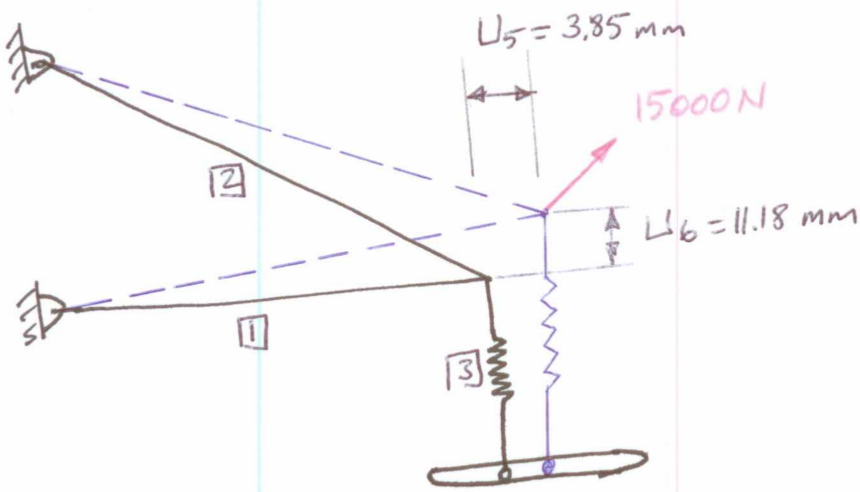
$$14.59 - 24.19 + 15\cos(50) = 0.04 \sim 0$$

$$+\uparrow\ \Sigma F_y = 0$$

$$-10.94 - 0.56 + 15\sin(50) = 0.01 \sim 0$$

Equilibrium is satisfied

c.) Elastic Gerinim enerjisi



1. Eleman : Final length $= \sqrt{(4000+3.85)^2 + 11.18^2} = 4003.8656$ mm

Elongation $= 4003.8656 - 4000 = 3.8656$ mm

2. Eleman : Final length $= \sqrt{(3000-11.18)^2 + (4000+3.85)^2}$

$= 4996.384674$

Elongation $= 4996.384674 - 5000 = -3.6153$

3. Eleman : (sadece yay yönünde uzadığı varsayılırsa

Elongation $= 11.18$ mm

Total strain Energy $= \frac{1}{2} k_1 \delta_1^2 + \frac{1}{2} k_2 \delta_2^2 + \frac{1}{2} k_3 \delta_3^2$

$= \frac{1}{2} 6283 (3.8656)^2 + \frac{1}{2} 5026 (3.6153)^2$

$+ \frac{1}{2} 50 (11.18)^2 = 82.9$ m

Work done on a system by external forces

$$W = \int F . du = \int k u \, du = \frac{1}{2} k u^2 = \frac{1}{2} (ku) . u = \frac{1}{2} F . u$$

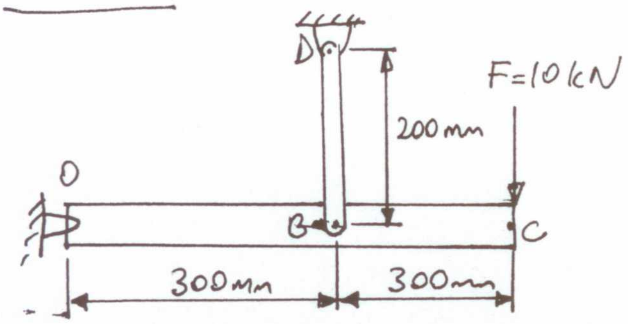
$W = \frac{1}{2} F . u$

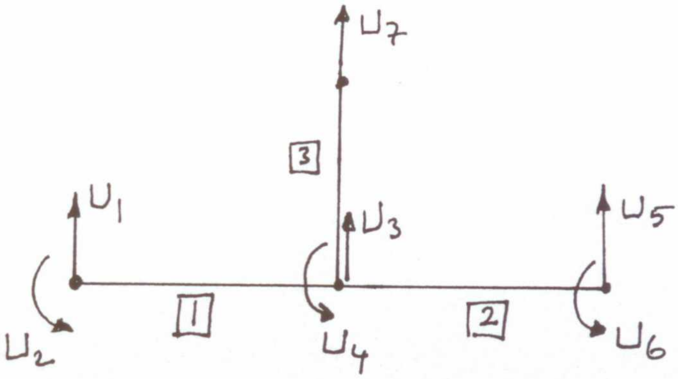
Note : u must be in the direction of F

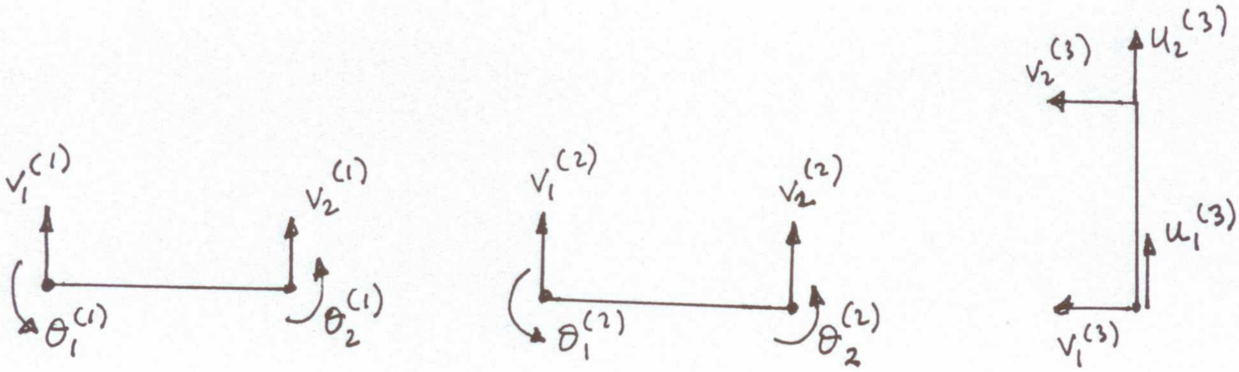
$$W = \frac{1}{2} (15) \cos 50 \, (U_5) + \frac{1}{2} (15) \sin 50 \, (U_6) = 82.8 \text{ N.m}$$

Work done = strain energy stored

3. Soru







Displacement scheme

Global		Element 1	Element 2	Element 3
1	U_1	$v_1^{(1)}$	0	0
2	U_2	$\theta_1^{(1)}$	0	0
3	U_3	$v_2^{(1)}$	$v_1^{(2)}$	$u_1^{(3)}$
4	U_4	$\theta_2^{(1)}$	$\theta_1^{(2)}$	0
5	U_5	0	$v_2^{(2)}$	0
6	U_6	0	$\theta_2^{(2)}$	0
7	U_7	0	0	$u_2^{(3)}$

Element Displacement Correspondence

Global	Element 1	Element 2	Element 3
1	1	0	0
2	2	0	0
3	3	1	1
4	4	2	0
5	0	3	0
6	0	4	0
7	0	0	3

(3.2)

Beam elements

$$I_z = \frac{bh^3}{12} = \frac{40(40^3)}{12} = 213333 \text{ mm}^4$$

For Elements 1 & 2

$$\frac{EI_z}{L^3} = \frac{207(10^3)(213333)}{300^3} = 1635.6 \text{ N/mm}$$

Element stiffness matrix for beam elements 1 & 2

$$[k_e] = \frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$[k^{(1)}] = [k^{(2)}] = 1635.6\begin{bmatrix} 12 & 1800 & -12 & 1800 \\ 1800 & 360000 & -1800 & 180000 \\ -12 & -1800 & 12 & -1800 \\ 1800 & 180000 & -1800 & 360000 \end{bmatrix}$$

for element (3)

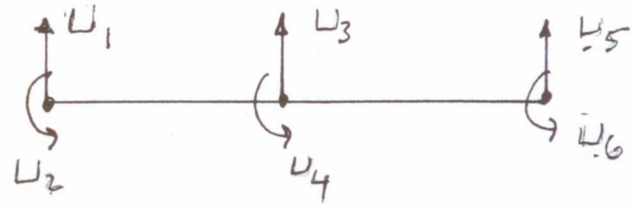
$$[k^{(3)}] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 27096\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

	1	2	3	4	5	6	7
1	19627.2	2.944×10^6	-19.627.2	2.944×10^6	0	0	
2		5.888×10^8	-2.944×10^6	2.944×10^8	0	0	
3			66350.4	0	-19,627.2	2.944×10^6	-27096
4				11.78×10^8	-2.944×10^6	2.944×10^8	0
5		Symmetric			19627.2	-2.944×10^6	0
6						5.889×10^8	0
7						0	27096

Constraint conditions $U_1 = U_7 = 0$

Right hand side vector

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \\ F_4 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 0 \\ 0 \\ 0 \\ -10000\,N \\ 0 \\ R_4 \end{Bmatrix}$$



Axial stress in member BD

$$\sigma_{BD} = E\,\varepsilon^{(e)} = E\,\frac{du^{(e)}(x)}{dx} = E\,\frac{d^{(e)}}{dx}\begin{bmatrix} N_1(x) & N_2(x) \end{bmatrix}\begin{bmatrix} R \end{bmatrix}\begin{Bmatrix} U_1^{(e)} \\ U_2^{(e)} \\ U_3^{(e)} \\ U_4^{(e)} \end{Bmatrix}$$

$$\sigma_{BD} = 69(10^3)\begin{bmatrix} -\frac{1}{200} & \frac{1}{200} \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ U_3 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} K \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 0 \\ 0 \\ 0 \\ -10000 \\ 0 \\ R_4 \end{Bmatrix} \Rightarrow \begin{aligned} U_2 &= \checkmark \\ U_3 &= \checkmark \\ U_4 &= \checkmark \\ U_5 &= \checkmark \\ U_6 &= \checkmark \end{aligned}$$