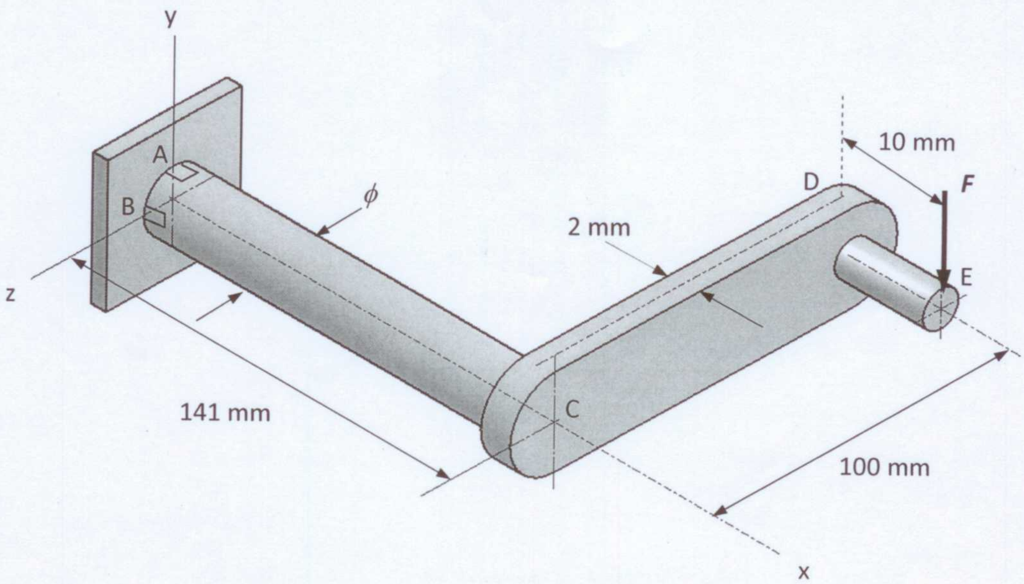
1.1

1. Soru: **(40 Puan)**



Şekil 1

Şekil 1'de verilen krank ankastre olarak bağlanmıştır ve E noktasından F=1500 N'luk yüke maruz bırakılmıştır. Bu kuvveti etkisiyle AC uzvu eğ**ilmeye v**e burulmaya zorlanmaktadır. Malzemenin akma gerilmesi 310 MPa'dır. Uzuvların ağırlıklarını v**e kesmed**en kaynaklanan kayma gerilmesini ihmal ediniz.

a) A ve B noktalarındaki gerilmeleri bulup, her bir sonsuz **küç**üklükteki eleman üzerinde bulduğunuz bu gerilmeleri gösteriniz. (10p)

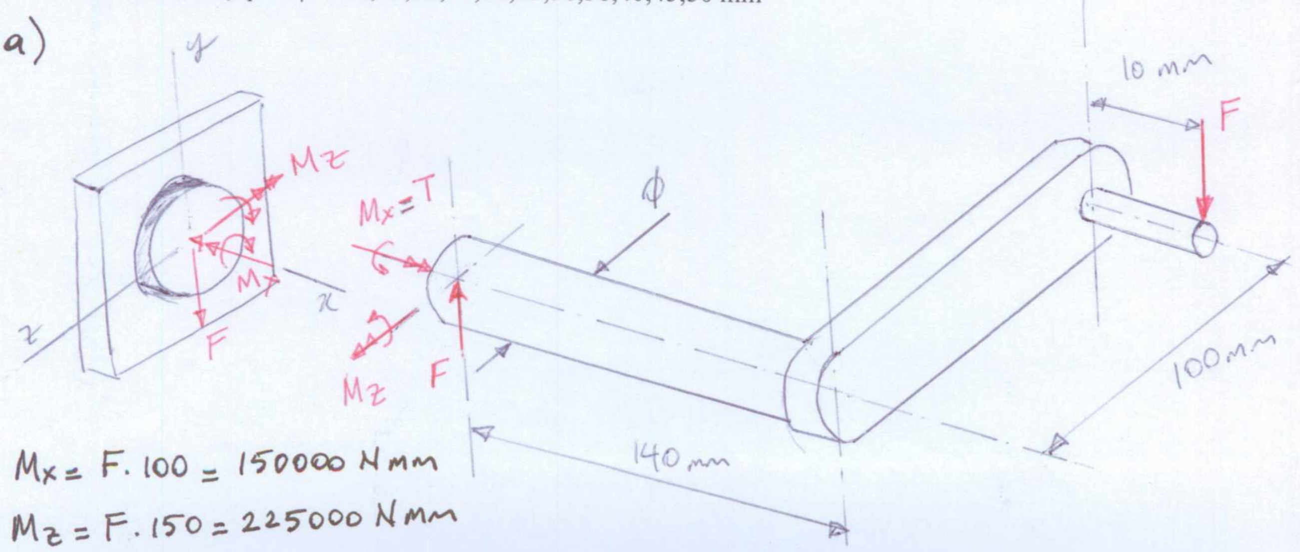
b) Şekil değiştirme enerjisi teoremine göre A noktasını dikkate alarak, AC uzvunun çapını bulunuz ve aşağıda verilen standart çapı seçiniz.(güvenlik katsayısı n=2 alınız) (10p)

c) Seçtiğiniz standart çapı kullanarak Mohr dairesini oluşturunuz. Asal gerilmeleri, maksimum kayma gerilmesini ve düzlemlerini bulunuz. Sonsuz küçüklükteki eleman üzerinde bulduğunuz bu gerilmeleri gösteriniz. (10p)

d) Seçtiğiniz standart çapı kullanarak, maksimum kayma gerilmesi teoremine göre güvenlik katsayısını bulunuz. Hasar teorisinin şeklini çizerek grafik üzerinde gerekli değerleri gösteriniz. (10p)

Standart çaplar ϕ = 22,25,28,30,32,35,36,38,40,45,50 mm

a)

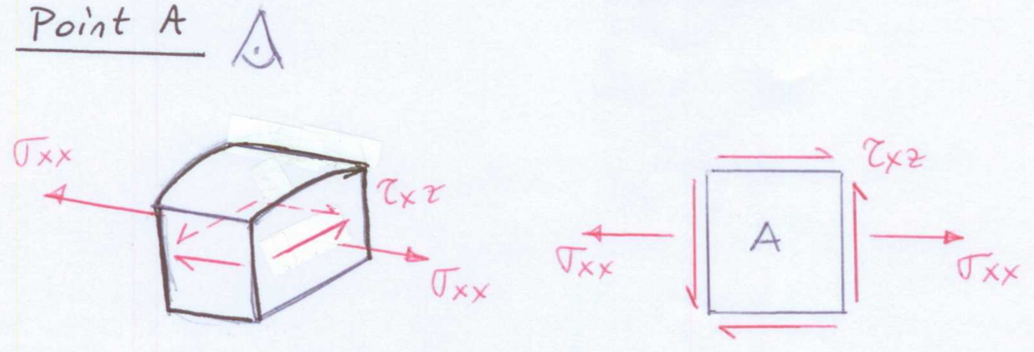


$M_x = F \cdot 100 = 150000$ Nmm

$M_z = F \cdot 150 = 225000$ Nmm

1.Soru b)

Point A



Point B

$$\sigma_{xx} = \frac{Mc}{I} = \frac{225000 \cdot \frac{d}{2}}{\frac{\pi d^4}{64}} = 2291831.18\, d^{-3}\ MPa$$

$$\tau_{xz} = \tau_{xy} = \frac{T.c}{J} = \frac{150000 \cdot \frac{d}{2}}{\frac{\pi d^4}{32}} = 763943.73\, d^{-3}\ MPa$$

b)

For Point A

Von misses stress

$$\sigma' = \sqrt{\sigma_{xx}^2 + 3\tau_{xz}^2} = \sqrt{(2291831.18 d^{-3})^2 + 3(763943.73 d^{-3})^2}$$

$$\sigma' = 2646378.69\, d^{-3}\ MPa$$

From Distortion Energy Theory

$$\sigma' = \frac{S_y}{n} \quad \text{or} \quad n = \frac{S_y}{\sigma'} \Rightarrow 2 = \frac{310\ MPa}{2646378.69\, d^{-3}}$$

$d = 25.75$ mm

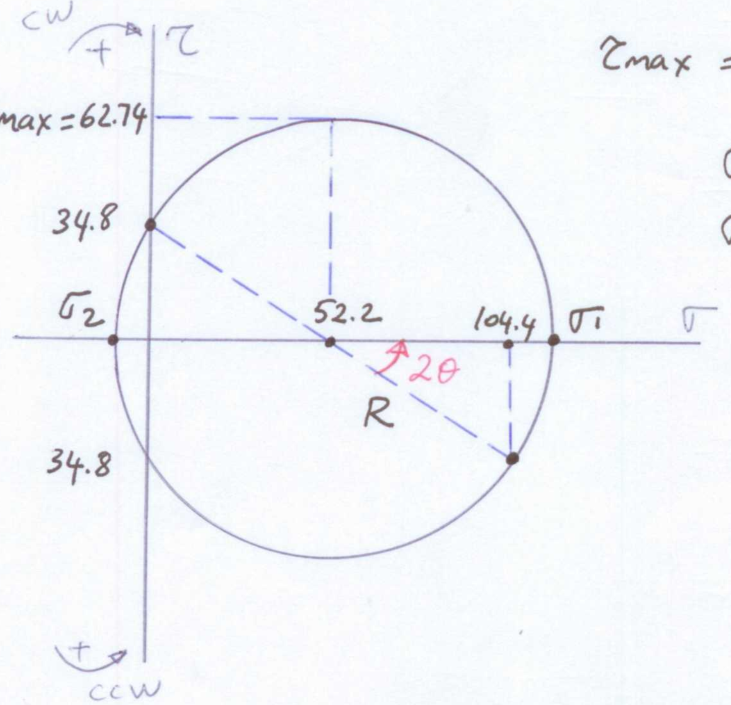
From the standard diameters given

$d = 28$ mm

For $d = 28$ mm

$$\sigma_{xx} = 2291831.18\,(28)^{-3} = 104.4 \text{ MPa}$$

$$\tau_{xz} = 763943.73\,(28)^{-3} = 34.8 \text{ MPa}$$



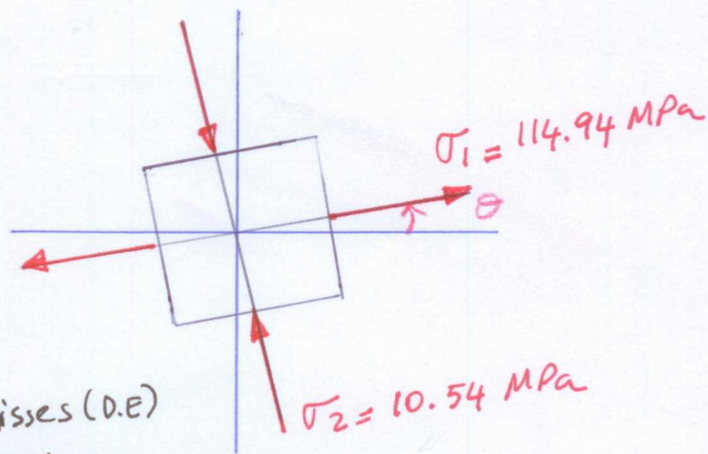
$$\tau_{max} = R = \sqrt{52.2^2 + 34.8^2} = 62.74 \text{ MPa}$$

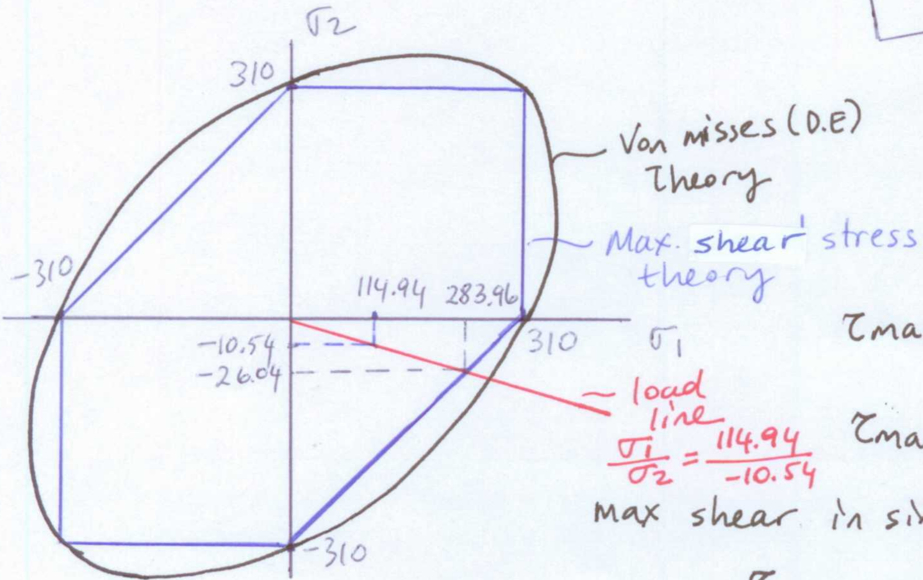
$$\sigma_1 = 52.2 + R = 114.94 \text{ MPa}$$

$$\sigma_2 = 52.2 - R = -10.54 \text{ MPa}$$

$$\tan 2\theta = \frac{34.8}{52.2} \Rightarrow 2\theta = 33.69^\circ$$

$$\theta = 16.85^\circ$$





$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{114.94 - (-10.54)}{2}$$

$$\tau_{max} = 62.74 \text{ MPa}$$

max shear in simple tension test

$$\tau_{test} = \frac{S_y}{2} = \frac{310}{2} = 155 \text{ MPa}$$

Factor of safety according to max shear stress theory

$$n = \frac{\tau_{test}}{\tau_{max}} = \frac{155}{62.74} = 2.47$$

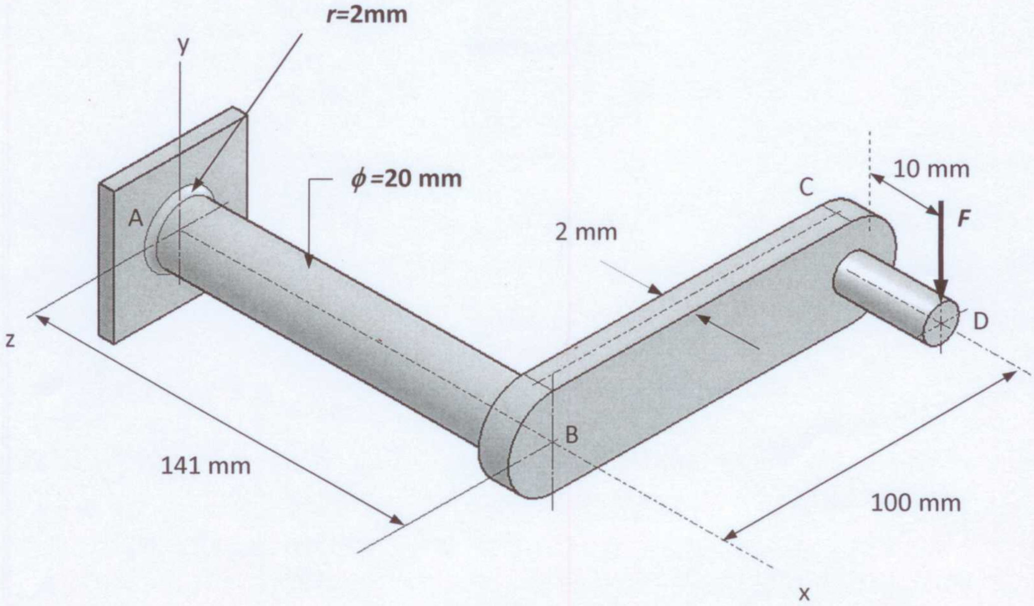
For $n = 1 = \dfrac{\tau_{test}}{\tau_{max}} = \dfrac{155}{\tau_{max}} \Rightarrow \tau_{max} = 155 = \dfrac{\sigma_1 - \sigma_2}{2} \Rightarrow \sigma_1 - \sigma_2 = 310$

From load line $\dfrac{\sigma_1}{\sigma_2} = \dfrac{114.94}{-10.54}$

$\sigma_1 = 283.96$

$\sigma_2 = -26.04$

2. Soru: **(30 Puan)**



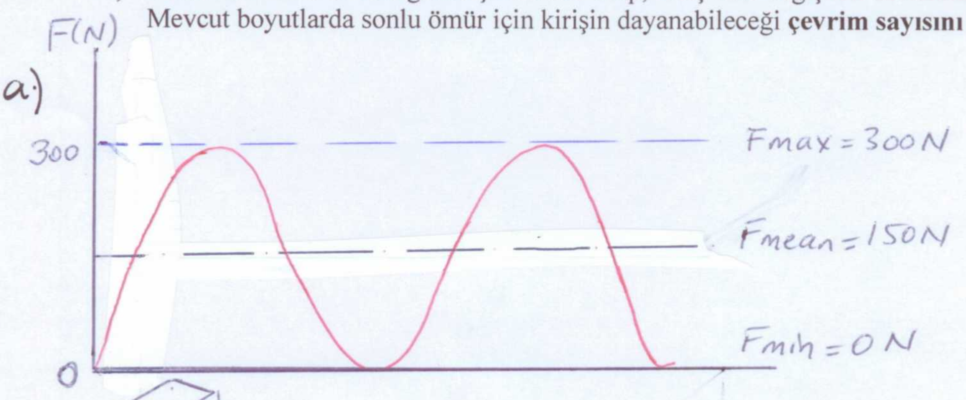
Şekil 2

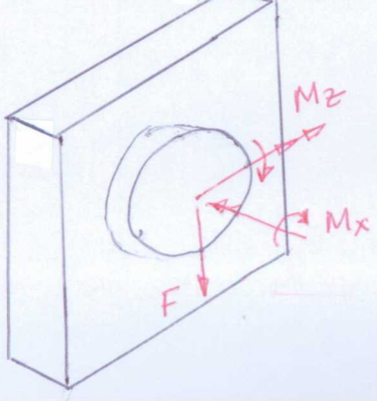
Şekil 2'de verilen ankastre bağlanmış 20 mm çaplı AB mili için malzeme AISI 1018 sıcak haddelenmiş (hot rolled) çelik seçilecek olup; ankastre bağlantı noktasındaki 2mm yarıçaplı yuvarlatma için teorik gerilme yığılma faktörleri $K_t = K_{ts} = 1.4$ alınacaktır. Çalışma sıcaklığı 20 ^{0}C olup; tasarım %90 güvenirlik için yapılacaktır. (S_{ut} = 400 MPa, S_y = 220 MPa, Bhn<200, f = 0.9)

a) F kuvveti 0-300 N aralığında çevrimsel olup, kiriş değişken zorlanmaya maruz kalmaktadır. **Güvenlik katsayılarını** Soderberg ve mod-Goodman yorulma kriterlerini kullanarak sonsuz ömür için hesaplayınız. (20p)

b) F kuvveti ±800 N aralığında çevrimsel olup, kiriş tam-değişken zorlanmaya maruz kalmaktadır. Mevcut boyutlarda sonlu ömür için kirişin dayanabileceği **çevrim sayısını** belirleyiniz. (10p)

a)





$T = M_x = F \cdot 100 \Rightarrow$ torque

$M = M_z = F \cdot 150 \Rightarrow$ bending moment

$T_{max} = F_{max} \cdot 100 = (300)(100) = 30000$ N mm

$T_{mean} = F_{mean} \cdot 100 = (150)(100) = 15000$ N mm

$M_{max} = F_{max} \cdot 150 = (300)(150) = 45000$ N mm

$M_{mean} = F_{mean} \cdot 150 = (150)(150) = 22500$ N mm

$S_{ut} = 400 MPa$

$S_y = 220 MPa$

$Bhn < 200$

$K_t = K_{ts} = 1.4 \quad ; \quad r = 2mm$

$q = 0.7 \quad ; \quad q_s = 0.9$

$K_f = 1 + q(K_t - 1) = 1 + 0.7(1.4 - 1) \Rightarrow \quad \underline{K_f = 1.28}$ 1p

$$\sigma_{max} = K_f \cdot \frac{M_{max} \cdot c}{I} = 1.28 \frac{45000 \cdot (10)}{\frac{\pi (20)^4}{64}} = 73.34 \text{ MPa}$$ 1p

$$\sigma_{mean} = K_f \cdot \frac{M_{mean} \cdot c}{I} = 1.28 \frac{22500 (10)}{\frac{\pi (20)^4}{64}} = 36.67 \text{ MPa}$$ 1p

$K_{fs} = 1 + q_s (K_{ts} - 1) = 1 + 0.9 (1.4 - 1) \underline{= 1.36}$ 1p

$$\tau_{max} = K_{fs} \cdot \frac{T_{max} \cdot c}{J} = 1.36 \frac{30000 (10)}{\frac{\pi (20)^4}{32}} = 25.97 \text{ MPa}$$ 1p

$$\tau_{mean} = K_{fs} \cdot \frac{T_{mean} \cdot c}{J} = 1.36 \frac{15000 (10)}{\frac{\pi (20)^4}{32}} = 12.99 \text{ MPa}$$ 1p

$\sigma_{min} = 0$

$\tau_{min} = 0$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{73.34 - 0}{2} = 36.67 \text{ MPa}$$ 1p

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{73.34 + 0}{2} = 36.67 \text{ MPa}$$ 1p

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{25.97 - 0}{2} = 12.99 \text{ MPa}$$ 1p

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{25.97 + 0}{2} = 12.99 \text{ MPa}$$ 1p

Von misses stresses

$$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{36.67^2 + 3(12.99)^2} = 43.02 \text{ MPa}$$ 1p

$$\sigma_m' = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{36.67^2 + 3(12.99)^2} = 43.02 \text{ MPa}$$ 1p

2.3

$S_{ut} = 400 \text{ MPa}$, $S_y = 220 \text{ MPa}$

$k_a = a\, S_{ut}^{\,b}$ Hot rolled ($a = 57.7$, $b = -0.718$)

$k_a = 57.7\,(400)^{-0.718} = 0.781$

$k_b = 1.24\,[0.370\, d\,]^{-0.107} = 1.24\,[0.370\,(20)]^{-0.107} = 1$

$k_c = 1$

$k_d = 1$

$k_e = 0.897$ for %90 reliability

$S_e' = 0.5\, S_{ut} = 0.5\,(400) = 200 \text{ MPa}$

$S_e = k_a\, k_b\, k_c\, k_d\, k_e\, S_e'$

$S_e = (0.781)(1)(1)(1)(0.897)\,200$ 4p

$S_e = 140.1 \text{ MPa}$

Factor of safety for Soderberg criteria

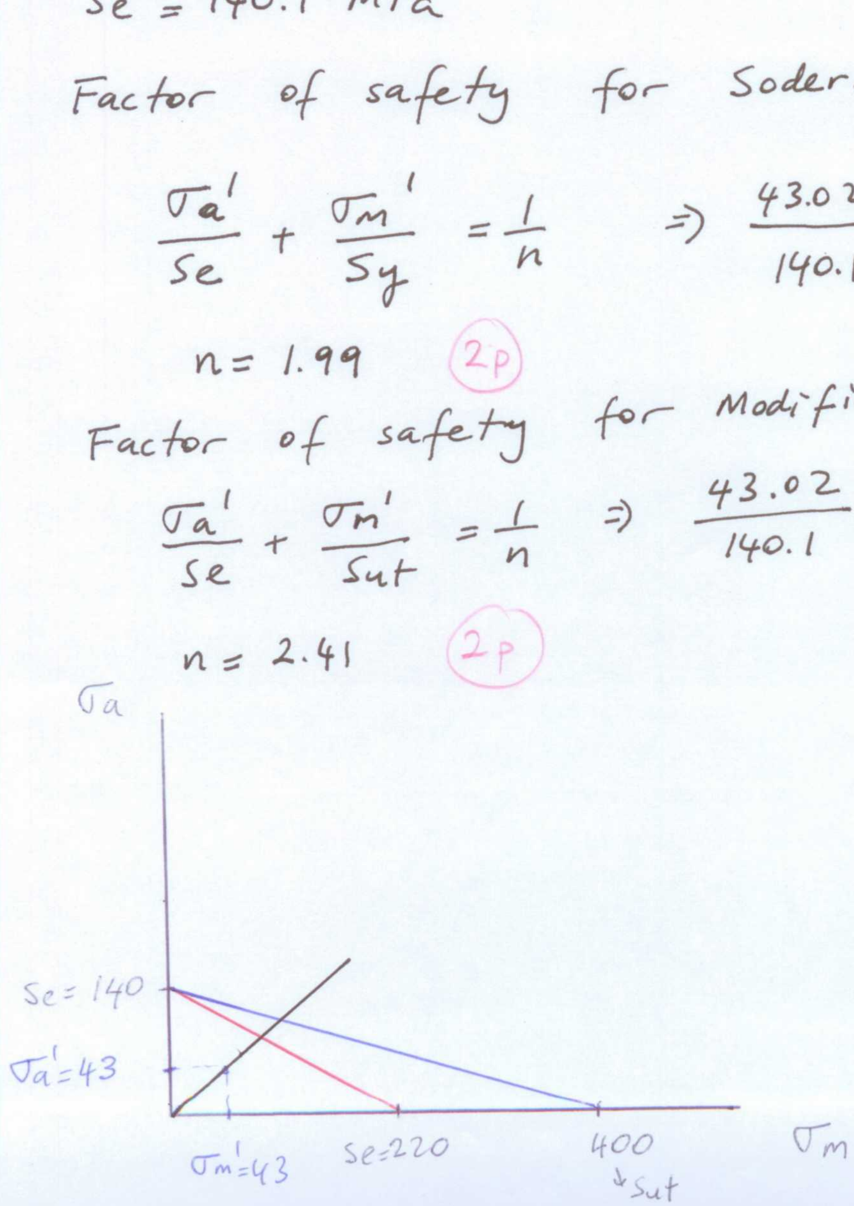
$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_y} = \frac{1}{n} \Rightarrow \frac{43.02}{140.1} + \frac{43.02}{220} = \frac{1}{n}$$

$n = 1.99$ 2p

Factor of safety for Modified Goodman

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{1}{n} \Rightarrow \frac{43.02}{140.1} + \frac{43.02}{400} = \frac{1}{n}$$

$n = 2.41$ 2p



b.) $F_{max} = 800\ N$, $F_{min} = -800\ N$ $\Big\}$ $M_{max} = 800(150) = 120000\ N\,mm$; $T_{max} = 800(100) = 80000\ N\,mm$

$$\sigma_{max} = K_f \cdot \frac{M_{max} \cdot c}{I} = 1.28\ \frac{120000 \cdot (10)}{\frac{\pi (20)^4}{64}} = 195.6\ MPa$$

$$\sigma_{min} = -\sigma_{max} = -195.6\ MPa \Rightarrow \sigma_a = 195.6\ MPa$$ 2p

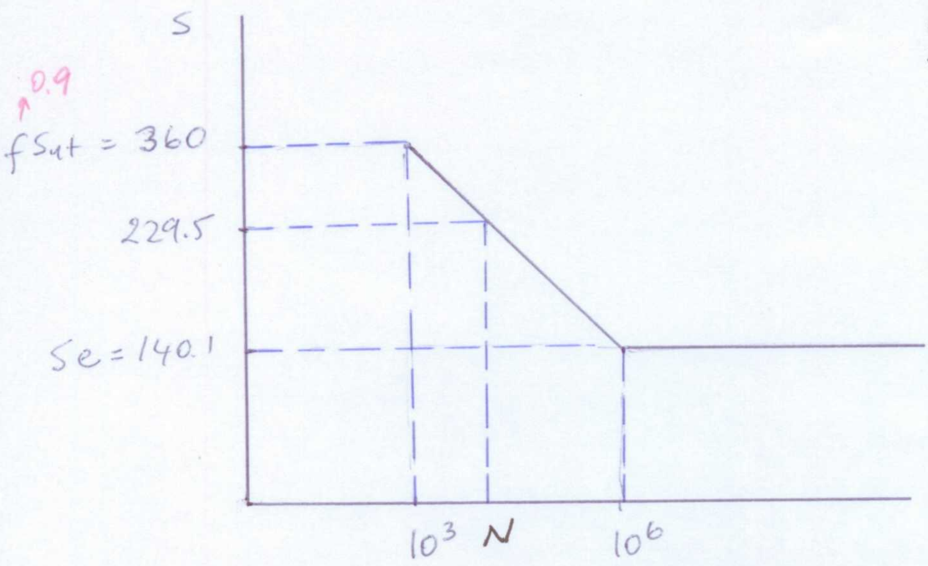
$$\tau_{max} = K_{fs} \cdot \frac{T_{max} \cdot c}{J} = 1.36\ \frac{80000 \cdot (10)}{\frac{\pi (20)^4}{32}} = 69.3\ MPa$$ 2p

$$\tau_{min} = -\tau_{max} = -69.3 \Rightarrow \tau_a = 69.3\ MPa$$

$$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{195.6^2 + 3(69.3)^2} = 229.5\ MPa$$ 2p

at 10^3 cycles $S = f \cdot S_{ut} = 0.9(400) = 360\ MPa$

at 10^6 cycles $S = S_e = 140.1\ MPa$



$$\frac{\log 360 - \log 229.5}{\log N - \log 10^3} = \frac{\log 360 - \log 140.1}{\log 10^6 - \log 10^3}$$

$N = 26984$

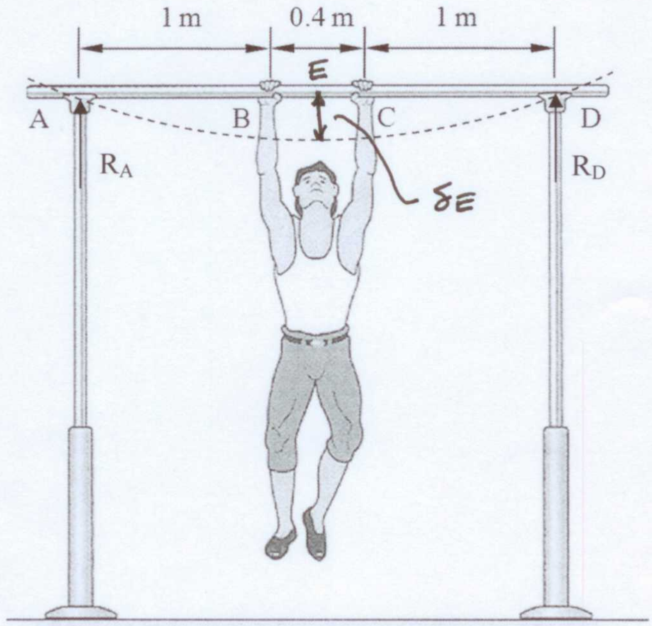
Or $S_f = a N^b$

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{(360)^2}{140.1} = 925.05$$ 1p

$$b = -\frac{1}{3} \log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3} \log\left(\frac{360}{140.1}\right) = -0.1366$$ 1p

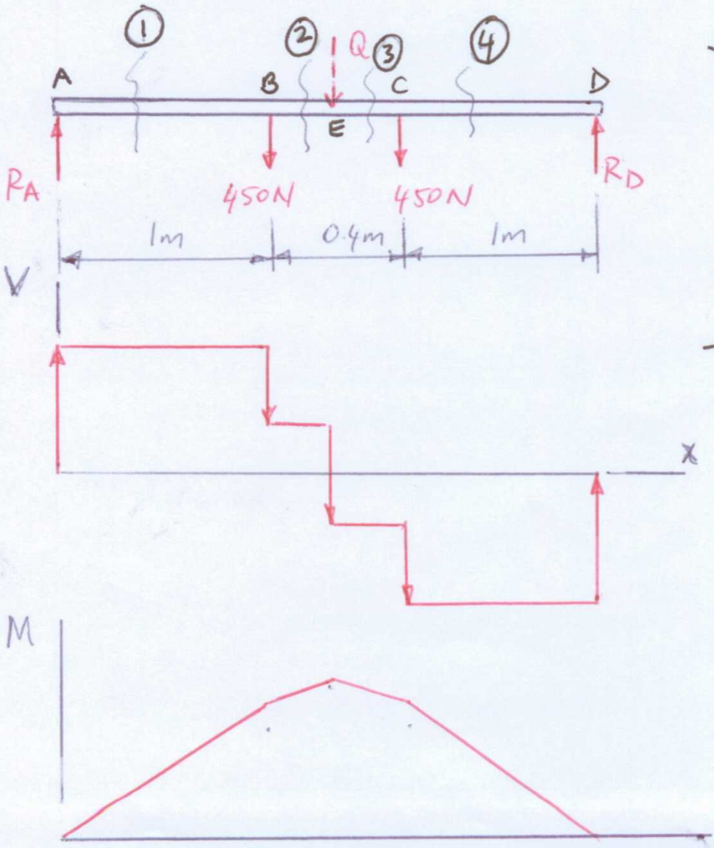
$$N = \left(\frac{S_f}{a}\right)^{1/b} = \left(\frac{229.5}{925.05}\right)^{-\frac{1}{0.1366}} = 27026$$ 2p

3. Soru: (25 puan)



Şekil 3

900 N ağırlığındaki sporcu barfiks çubuğunda Şekil 3'te gösterildiği gibi asılı olarak durmaktadır. Çelik malzemeden imal edilmiş barfiks çubuğunun dış çapı 30 mm olup et kalınlığı 4 mm'dir. Barfiks çubuğunda sehimin en fazla olduğu noktadaki sehim değerini Castigliano Teoremini kullanarak hesaplayınız. (E=200 GPa), (Sporcunun her iki kolunun da eşit yük uyguladığını ve mesnetlerin moment taşımadığını varsayınız. Kesmeden kaynaklanan kayma gerilmesini ihmal ediniz.)



since the loading is symmetric

$R_A = 450 + \frac{Q}{2}$

$R_D = 450 + \frac{Q}{2}$

or $\sum M_A = 0$

$-R_D \cdot (2.4) + Q(1.2) + 450(1.4) + 450(1) = 0$

$R_D = 450 + \frac{Q}{2}$ ——— (3.1)

1p

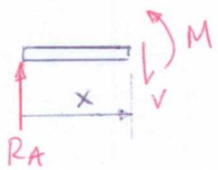
$+\uparrow \sum F_y = 0$

$R_A - 450 - Q - 450 + R_D = 0$

$R_D = 450 + \frac{Q}{2}$ ——— (3.2)

1p

Section ① $0 \leq x \leq 1$

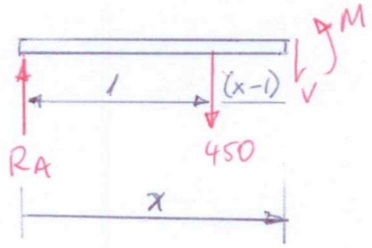


$$\circlearrowleft \Sigma M = 0 \quad : \quad -M + R_A x = 0 \quad \Rightarrow \quad M = R_A \cdot x$$

$$M = \left(450 + \frac{Q}{2}\right) x \qquad \text{———— (3.3)}$$

2p

Section ② $1 \leq x \leq 1.2$



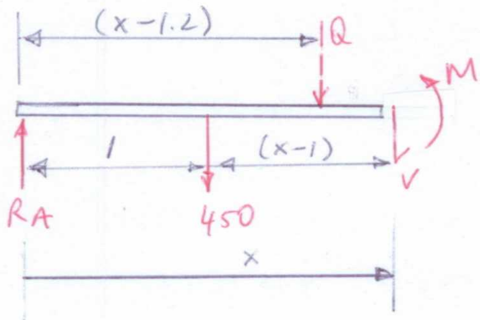
$$\circlearrowleft \Sigma M = 0 \quad : \quad -M + R_A x - 450(x-1) = 0$$

$$M = R_A x - 450(x-1) = \left(450 + \frac{Q}{2}\right)x - 450(x-1)$$

$$M = \frac{Q}{2}x + 450 \qquad \text{———— (3.4)}$$

2p

Section ③ $1.2 \leq x \leq 1.4$



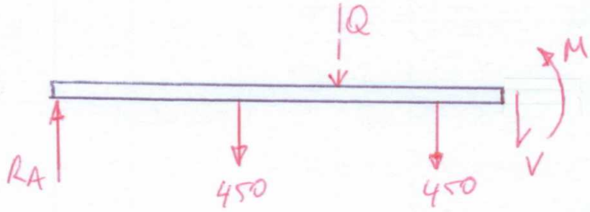
$$\circlearrowleft \Sigma M = 0 \quad : \quad -M + R_A x - 450(x-1) - Q(x-1.2) = 0$$

$$M = \left(450 + \frac{Q}{2}\right)x - 450(x-1) - Q(x-1.2)$$

$$M = -\frac{Q}{2}x + 450 + 1.2Q \qquad \text{———— (3.5)}$$

2p

Section ④ $1.4 \leq x \leq 2.4$



$$\circlearrowleft \Sigma M = 0 : -M + R_A x - 450(x-1) - Q(x-1.2) - 450(x-1.4) = 0$$

$$M = \left(450 + \frac{Q}{2}\right)x - 450x + 450 - Qx + Q(1.2) - 450x + 630$$

$$M = -\frac{Q}{2}x - 450x + 1.2Q + 1080 \qquad \text{———— (3.6)}$$

2p

$$M = \begin{cases} \left(450 + \frac{Q}{2}\right)x & \text{- - - - - - - -} \quad 0 \leq x \leq 1 \\ \frac{Q}{2}x + 450 & \text{— — — —} \quad 1 \leq x \leq 1.2 \\ -\frac{Q}{2}x + 450 + 1.2Q & \text{- - -} \quad 1.2 \leq x \leq 1.4 \\ -\frac{Q}{2}x - 450x + 1.2Q + 1080 & \text{- -} \quad 1.4 \leq x \leq 2.4 \end{cases} \qquad \text{———— (3.7)}$$

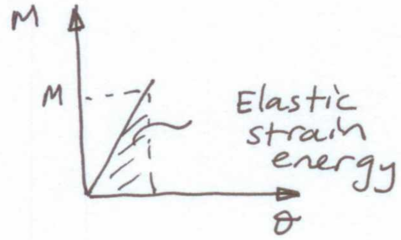
4p

$$\frac{\partial M}{\partial Q} = \begin{cases} \frac{x}{2} & 0 \le x \le 1 \\ \frac{x}{2} & 1 \le x \le 1.2 \\ -\frac{x}{2} + 1.2 & 1.2 \le x \le 1.4 \\ -\frac{x}{2} + 1.2 & 1.4 \le x \le 2.4 \end{cases} \qquad \text{(3.8)}$$

4p

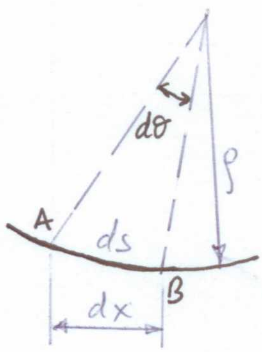
Castigliano theorem

strain energy stored in a beam by bending



$$= \frac{1}{2} M\theta$$

Consider AB section of an elastic curve



strain energy stored in this element

$$dU = \frac{1}{2} M\, d\theta \qquad \text{(3.9)}$$

$$\rho\, d\theta = ds \qquad \text{(3.10)}$$

Using

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{(3.11)}$$

From Equation (3.9), (3.10) & (3.11)

$$dU = \frac{1}{2} M\, d\theta = \frac{1}{2} M \frac{ds}{\rho} = \frac{1}{2} \frac{M^2}{EI} ds$$

For small deflections $ds \cong dx$

$$dU = \frac{1}{2} \frac{M^2}{EI} dx$$

For the entire beam

$$U = \int \frac{M^2 dx}{2EI}$$

From Castigliano's second theorem

$$\delta_E = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

6p

$$\delta_E = \frac{\partial}{\partial Q}\int \frac{M^2 dx}{2EI}\Bigg|_{Q=0} = \int \frac{2M \cdot \frac{\partial M}{\partial Q}dx}{2EI}\Bigg|_{Q=0}$$

$$= \frac{1}{EI}\left\{\int_0^1 M\frac{\partial M}{\partial Q}dx + \int_1^{1.2} M\frac{\partial M}{\partial Q}dx + \int_{1.2}^{1.4} M\frac{\partial M}{\partial Q}dx + \int_{1.4}^{2.4} M\frac{\partial M}{\partial Q}dx\right\}\Bigg|_{Q=0}$$

$$= \frac{1}{EI}\left\{\int_0^1\left(450+\frac{Q}{2}\right)x\cdot\frac{x}{2}dx + \int_1^{1.2}\left(\frac{Q}{2}x+450\right)\frac{x}{2}dx\right.$$

$$+\int_{1.2}^{1.4}\left(-\frac{Q}{2}x+450+1.2Q\right)\left(-\frac{x}{2}+1.2\right)dx$$

$$\left.+\int_{1.4}^{2.4}\left(-\frac{Q}{2}x-450x+1.2Q+1080\right)\left(-\frac{x}{2}+1.2\right)dx\right\}\Bigg|_{Q=0}$$

Setting $Q=0$

$$\delta_E = \frac{1}{EI}\left\{\frac{450}{2}\int_0^1 x^2dx + \frac{450}{2}\int_1^{1.2} x\,dx + \int_{1.2}^{1.4}(450)\left(-\frac{x}{2}+1.2\right)dx\right.$$

$$\left.+\int_{1.4}^{2.4}(-450x+1080)\left(-\frac{x}{2}+1.2\right)dx\right\}$$

$$= \frac{1}{EI}\left\{\frac{450}{2}\frac{x^3}{3}\Big|_0^1 + \frac{450}{4}x^2\Big|_1^{1.2} - \frac{450}{4}x^2\Big|_{1.2}^{1.4} + (450)(1.2)x\Big|_{1.2}^{1.4}\right.$$

$$\left.+\frac{450}{2}\frac{x^3}{3}\Big|_{1.4}^{2.4} - (450)(1.2)\frac{x^2}{2}\Big|_{1.4}^{2.4} - \frac{1080}{4}x^2\Big|_{1.4}^{2.4} + 1080(1.2)x\Big|_{1.4}^{2.4}\right\}$$

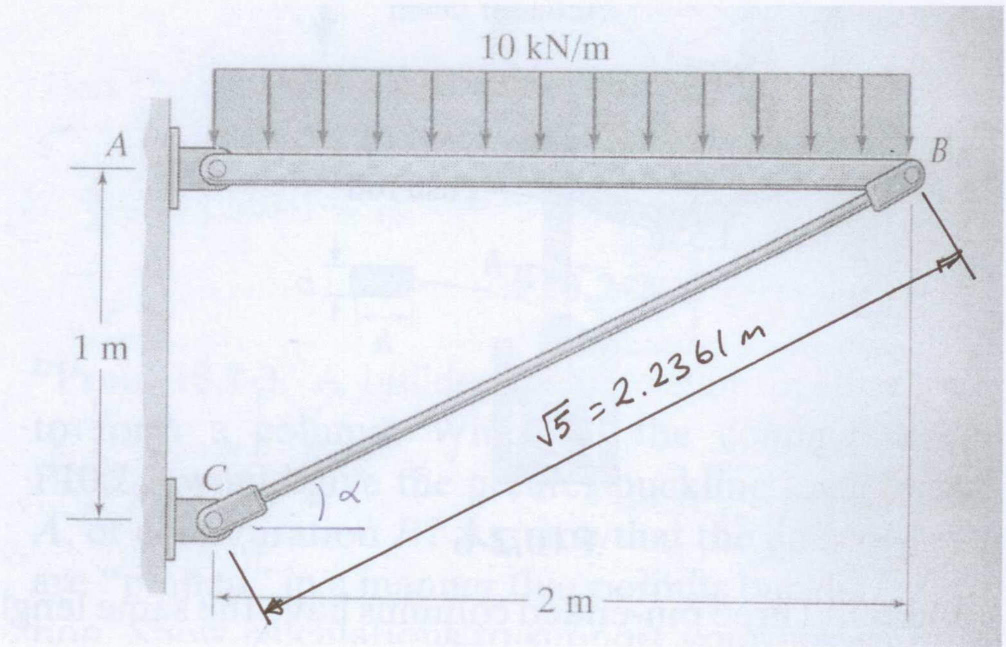
$$= \frac{1}{EI}\left\{75+49.5-58.5+108+831-1026-1026+1296\right\}$$

$$= \frac{249}{EI} = \frac{249\ N}{200\,10^9\,\frac{N}{m^2} * \frac{\pi}{4}\left(0.015^4-0.011^4\right)} = 0.044052\ m \Rightarrow$$

$$\delta_E = 44.1\ mm$$

5p

4. Soru: **(25 Puan)**

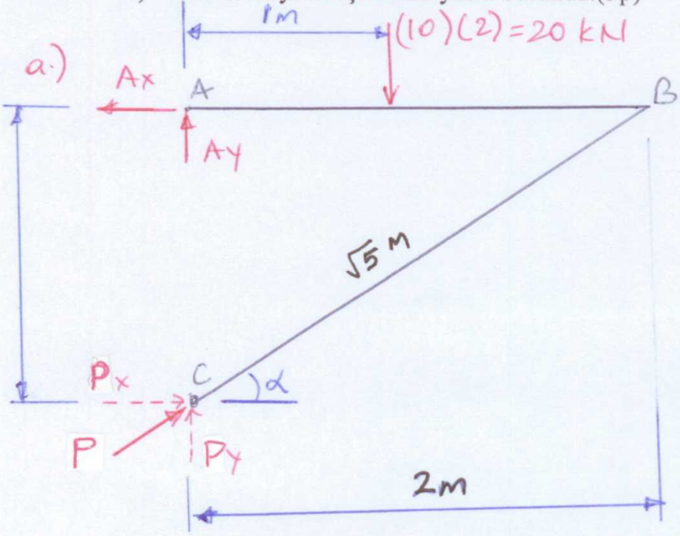


Şekil 4

Şekil 4'te verilen sistemde BC çelik borusu basmaya maruz kalmaktadır. BC borusunun dış çapı $d_0 = 48$ mm ve cidar (et) kalınlığı $t = 5$ mm'dir. AB uzvu üzerinden 10 kN/m büyüklüğünde yayılı yük uygulanmaktadır. Malzemenin elastiklik modülü E=210 GPa ve akma gerilmesi $\sigma_Y = 340$ MPa'dır

a) Serbest cisim diyagramlarını çizip her bir uzva gelen kuvvetleri bulunuz.(5p)
b) l/k oranını bularak, BC elemanı uzun (long) çubuk mu yoksa orta (intermediate) çubuk mu olduğunu belirleyiniz.(5p)
c) Akma gerilmesine göre güvenlik kat sayısını bulnuz.(5p)
d) Elastik burkulmaya (elastic buckling) göre güvenlik kat sayısını bulunuz.(5p)
e) Burkulmaya karşı kritik yükü bulunuz.(5p)

a.)



$\cos\alpha = \frac{2}{\sqrt{5}}$; $\sin\alpha = \frac{1}{\sqrt{5}}$

$P_x = P\cos\alpha = \frac{2P}{\sqrt{5}}$ ——— (4.1)

$P_y = P\sin\alpha = \frac{P}{\sqrt{5}}$ ——— (4.2)

$\sum M_A = 0$

$20\,kN.(1m) - P_x(1m) = 0$

$P_x = 20\,kN$ ——— (4.3)

using (4.1) & (4.3)

$P_x = \frac{2P}{\sqrt{5}} = 20\,kN \Rightarrow$ $P = 10\sqrt{5}\,kN$

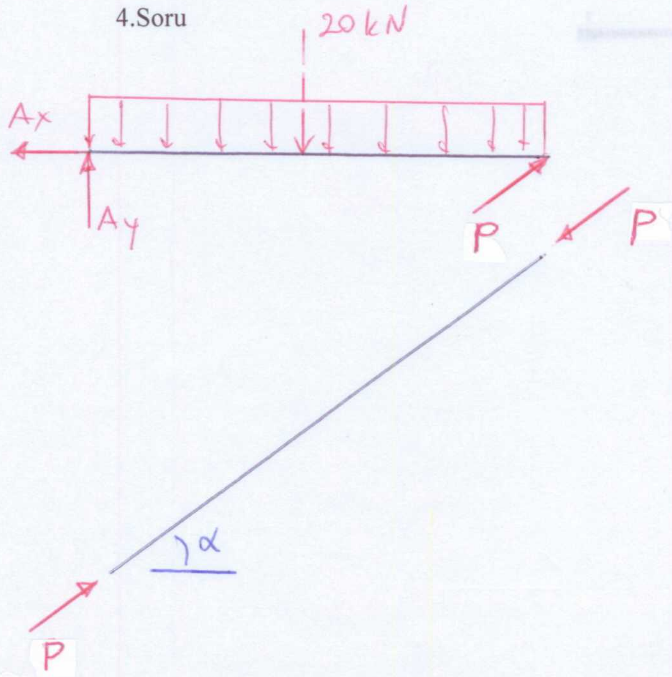
$P = 22.36\,kN$

$P_y = \frac{P}{\sqrt{5}} = 10\,kN$

5p

Öğrenci Adı Soyadı: TOBB ETÜ Tarih: 7 Mart 2010

4.Soru



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$P\cos\alpha - A_x = 0$

$A_x = P_x = P\cos\alpha = \frac{2}{\sqrt{5}} = \frac{(2)10\sqrt{5}}{\sqrt{5}}$

$A_x = 20 \text{ kN}$

$+\uparrow \Sigma F_y = 0$

$A_y - 20 + P\sin\alpha = 0$

$A_y = 20 - P\sin\alpha = 20 - \frac{P}{\sqrt{5}}$

$A_y = 20 - \frac{10\sqrt{5}}{\sqrt{5}}$

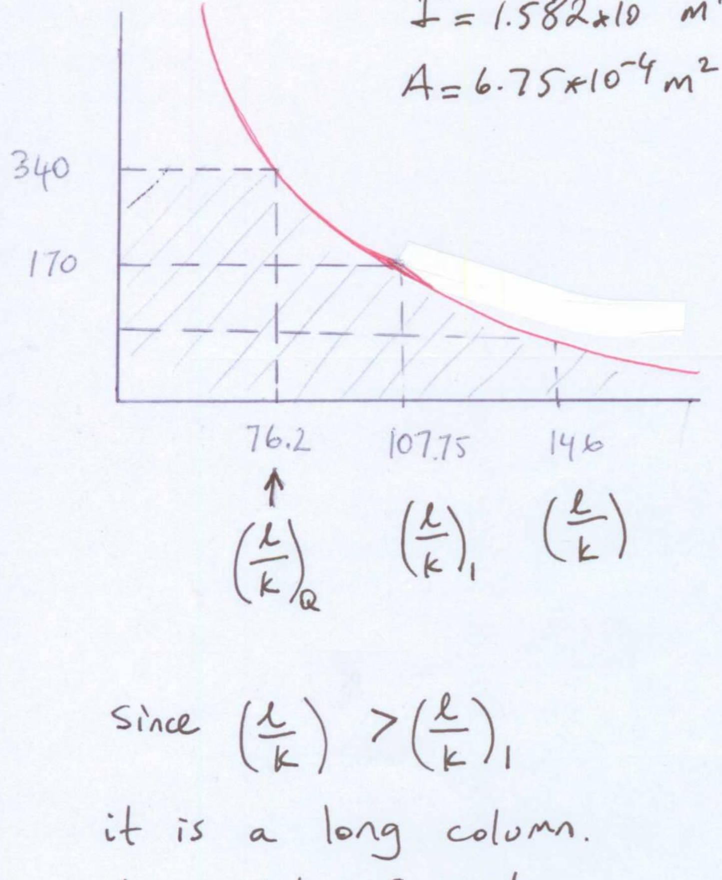
$A_y = 10 \text{ kN}$

b.) since both ends are simply supported $C = 1$

$E = 210 \text{ GPa}$, $S_y = 340 \text{ MPa}$

$I = 1.582 \times 10^{-7} \text{ m}^4$

$A = 6.75 \times 10^{-4} \text{ m}^2$



$I = Ak^2$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}(r_o^4 - r_i^4)}{\pi(r_o^2 - r_i^2)}}$

$k = \sqrt{\frac{0.024^4 - 0.019^4}{4(0.024^2 - 0.019^2)}} = 0.015305 \text{ m}$

$k = 15.305 \text{ mm}$

$\frac{l}{k} = \frac{2236.1}{15.305} = 146.1$ 2p

$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left(\frac{2\pi^2 \cdot 210000 \text{ MPa}}{340 \text{ MPa}}\right)^{1/2}$

2p

$\left(\frac{l}{k}\right)_1 = 110.41$

$\left(\frac{l}{k}\right)_Q = \left(\frac{\pi^2 CE}{S_y}\right)^{1/2} = \left(\frac{\pi^2\, 210000}{340}\right)^{1/2}$

$\left(\frac{l}{k}\right)_Q = 78.08$

Since $\left(\frac{l}{k}\right) > \left(\frac{l}{k}\right)_1$

it is a long column. Use Euler formula for Pcritical 1 c.) Factor of safety against yielding

$$n = \frac{S_y}{\frac{P}{A}} = \frac{340 \text{ MPa}}{\frac{22360.7 \text{ N}}{\pi(24^2 - 19^2) \text{ mm}^2}} = 10.27$$ 5p

e.) $$P_{cr} = \frac{\pi^2 E I}{l^2} = \frac{\pi^2 \cdot 210 * 10^9 \frac{N}{m^2} \frac{\pi}{4} \left[0.024^4 - 0.019^4\right] m^4}{(\sqrt{5})^2 \ m^2}$$

$P_{cr} = 65586.9 \text{ N} = 65.6 \text{ kN}$ 5p

d.) Factor of safety against elastic buckling

$$n = \frac{P_{cr}}{P} = \frac{65.6 \text{ kN}}{10\sqrt{5} \text{ kN}} = 2.93$$ 5p