

$$p. 238 \quad n = \frac{K_{Ic}}{K_I} \quad (5-38)$$

where  $K_{Ic}$  is found in Table 5-1 (p. 238)

### Stochastic Analysis

Mean factor of safety defined as  $\bar{n} = \mu_S/\mu_\sigma$  ( $\mu_S$  and  $\mu_\sigma$  are mean strength and stress, respectively)

### Normal-Normal Case

$$p. 241 \quad \bar{n} = \frac{1 \pm \sqrt{1 - (1 - z^2 C_s^2)(1 - z^2 C_\sigma^2)}}{1 - z^2 C_s^2} \quad (5-42)$$

where  $z$  can be found in Table A-10,  $C_s = \hat{\sigma}_S/\mu_S$ , and  $C_\sigma = \hat{\sigma}_\sigma/\mu_\sigma$ .

### Lognormal-Lognormal Case

$$p. 242 \quad \bar{n} = \exp \left[ -z \sqrt{\ln(1 + C_n^2)} + \ln \sqrt{1 + C_n^2} \right] \doteq \exp \left[ C_n \left( -z + \frac{C_n}{2} \right) \right] \quad (5-45)$$

where

$$C_n = \sqrt{\frac{C_s^2 + C_\sigma^2}{1 + C_\sigma^2}}$$

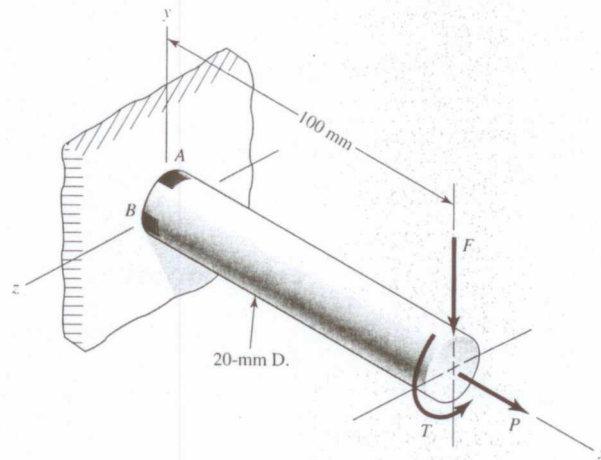
(See other definitions in normal-normal case.)

## PROBLEMS

- 5-1** A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 350 MPa. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:
- $\sigma_x = 84$  MPa,  $\sigma_y = 42$  MPa
  - $\sigma_x = 84$  MPa,  $\tau_{xy} = -56$  MPa
  - $\sigma_x = -42$  MPa,  $\sigma_y = -70$  MPa,  $\tau_{xy} = -35$  MPa
  - $\sigma_x = 84$  MPa,  $\sigma_y = 28$  MPa,  $\tau_{xy} = 7$  MPa
- 5-2** Repeat Prob. 5-1 for:
- $\sigma_A = 84$  MPa,  $\sigma_B = 84$  MPa
  - $\sigma_A = 84$  MPa,  $\sigma_B = 42$  MPa
  - $\sigma_A = 84$  MPa,  $\sigma_B = -84$  MPa
  - $\sigma_A = -42$  MPa,  $\sigma_B = -84$  MPa
- 5-3** Repeat Prob. 5-1 for a bar of AISI 1020 cold-drawn steel and:
- $\sigma_x = 180$  MPa,  $\sigma_y = 100$  MPa
  - $\sigma_x = 180$  MPa,  $\tau_{xy} = 100$  MPa
  - $\sigma_x = -160$  MPa,  $\tau_{xy} = 100$  MPa
  - $\tau_{xy} = 150$  MPa
- 5-4** Repeat Prob. 5-1 for a bar of AISI 1018 hot-rolled steel and:
- $\sigma_A = 100$  MPa,  $\sigma_B = 80$  MPa
  - $\sigma_A = 100$  MPa,  $\sigma_B = 10$  MPa
  - $\sigma_A = 100$  MPa,  $\sigma_B = -80$  MPa
  - $\sigma_A = -80$  MPa,  $\sigma_B = -100$  MPa

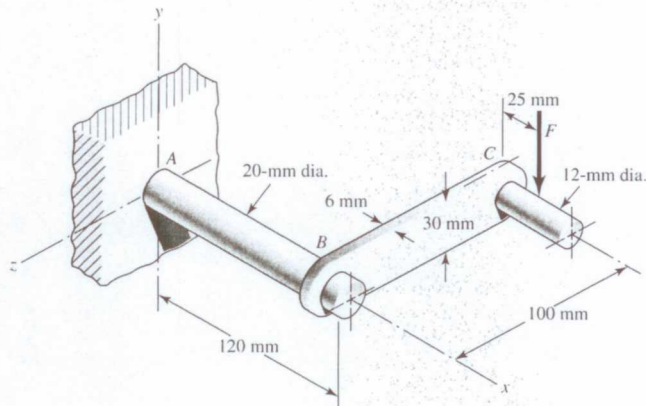
- 5-5** Repeat Prob. 5-3 by first plotting the failure loci in the  $\sigma_A, \sigma_B$  plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.
- 5-6** Repeat Prob. 5-4 by first plotting the failure loci in the  $\sigma_A, \sigma_B$  plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.
- 5-7** An ASTM cast iron has minimum ultimate strengths of 210 MPa in tension and 700 MPa in compression. Find the factors of safety using the MNS, BCM, and MM theories for each of the following stress states. Plot the failure diagrams in the  $\sigma_A, \sigma_B$  plane to scale and locate the coordinates of each stress state.
- (a)  $\sigma_x = 140$  MPa,  $\sigma_y = 42$  MPa  
 (b)  $\sigma_x = 84$  MPa,  $\tau_{xy} = -56$  MPa  
 (c)  $\sigma_x = -42$  MPa,  $\sigma_y = -70$  MPa,  $\tau_{xy} = -35$  MPa  
 (d)  $\sigma_x = -84$  MPa,  $\tau_{xy} = 56$  MPa
- 5-8** For Prob. 5-7, case (d), estimate the factors of safety from the three theories by graphical measurements of the load line.
- 5-9** Among the decisions a designer must make is selection of the failure criteria that is applicable to the material and its static loading. A 1020 hot-rolled steel has the following properties:  $S_y = 294$  MPa,  $S_{ut} = 463$  MPa, and true strain at fracture  $\epsilon_f = 0.90$ . Plot the failure locus and, for the static stress states at the critical locations listed below, plot the load line and estimate the factor of safety analytically and graphically.
- (a)  $\sigma_x = 63$  MPa,  $\sigma_y = -35$  MPa.  
 (b)  $\sigma_x = 84$  MPa,  $\tau_{xy} = 21$  MPa ccw.  
 (c)  $\sigma_x = -28$  MPa,  $\sigma_y = -63$  MPa,  $\tau_{xy} = 35$  MPa cw.  
 (d)  $\sigma_x = 77$  MPa,  $\sigma_y = 28$  MPa,  $\tau_{xy} = 7$  MPa cw.
- 5-10** A 4142 steel Q&T at 30°C exhibits  $S_{yt} = 164.5$  MPa,  $S_{yc} = 192.5$  MPa, and  $\epsilon_f = 0.06$ . Choose and plot the failure locus and, for the static stresses at the critical locations, which are 10 times those in Prob. 5-9, plot the load lines and estimate the factors of safety analytically and graphically.
- 5-11** For grade 20 cast iron, Table A-22 gives  $S_{ut} = 154$  MPa,  $S_{uc} = 581$  MPa. Choose and plot the failure locus and, for the static loadings inducing the stresses at the critical locations of Prob. 5-9, plot the load lines and estimate the factors of safety analytically and graphically.
- 5-12** A cast aluminum 195-T6 has an ultimate strength in tension of  $S_{ut} = 252$  MPa and ultimate strength in compression of  $S_{uc} = 245$  MPa, and it exhibits a true strain at fracture  $\epsilon_f = 0.045$ . Choose and plot the failure locus and, for the static loading inducing the stresses at the critical locations of Prob. 5-9, plot the load lines and estimate the factors of safety analytically and graphically.
- 5-13** An ASTM cast iron, grade 30 (see Table A-22), carries static loading resulting in the stress state listed below at the critical locations. Choose the appropriate failure locus, plot it and the load lines, and estimate the factors of safety analytically and graphically.
- (a)  $\sigma_A = 140$  MPa,  $\sigma_B = 140$  MPa.  
 (b)  $\tau_{xy} = 100$  MPa.  
 (c)  $\sigma_A = \sigma_B = -560$  MPa.  
 (d)  $\sigma_A = 105$  MPa,  $\sigma_B = -175$  MPa.
- 5-14** This problem illustrates that the factor of safety for a machine element depends on the particular point selected for analysis. Here you are to compute factors of safety, based upon the distortion-energy theory, for stress elements at A and B of the member shown in the figure. This bar is made of AISI 1006 cold-drawn steel and is loaded by the forces  $F = 0.55$  kN,  $P = 8.0$  kN, and  $T = 30$  N · m.

Problem 5-14



**5-15** The figure shows a crank loaded by a force  $F = 800$  N which causes twisting and bending of the 20-mm-diameter shaft fixed to a support at the origin of the reference system. In actuality, the support may be an inertia which we wish to rotate, but for the purposes of a strength analysis we can consider this to be a statics problem. The material of the shaft  $AB$  is hot-rolled AISI 1018 steel (Table A-18). Using the maximum-shear-stress theory, find the factor of safety based on the stress at point A.

Problem 5-15



**5-16** Solve Prob. 5-15 using the distortion energy theory. If you have solved Prob. 5-15, compare the results and discuss the difference.

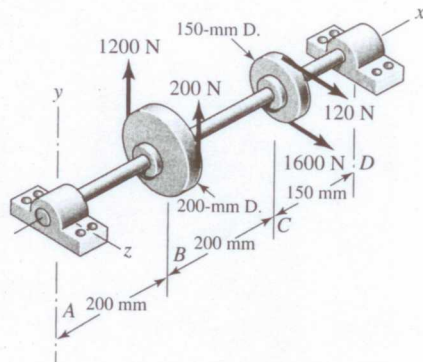
**5-17\*** Design the lever arm  $CD$  of Fig. 5-16 by specifying a suitable size and material.

**5-18** A spherical pressure vessel is formed of 18-gauge (1.25 mm) cold-drawn AISI 1018 sheet steel. If the vessel has a diameter of 200 mm, estimate the pressure necessary to initiate yielding. What is the estimated bursting pressure?

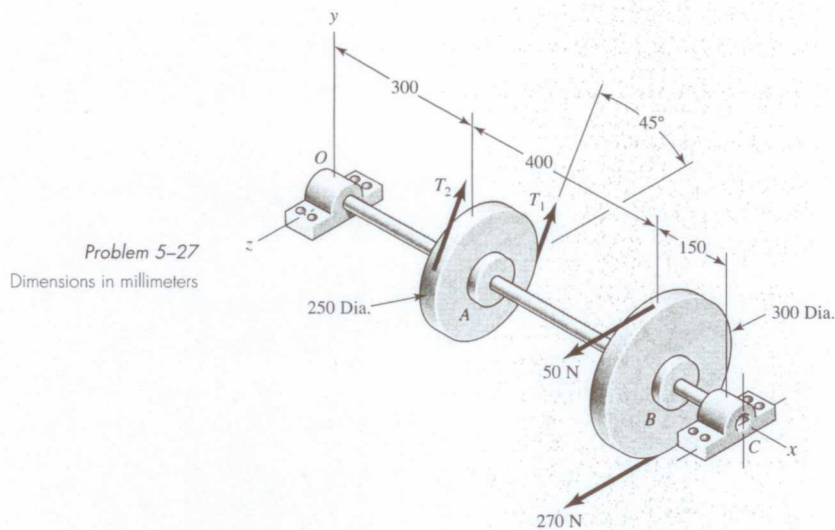
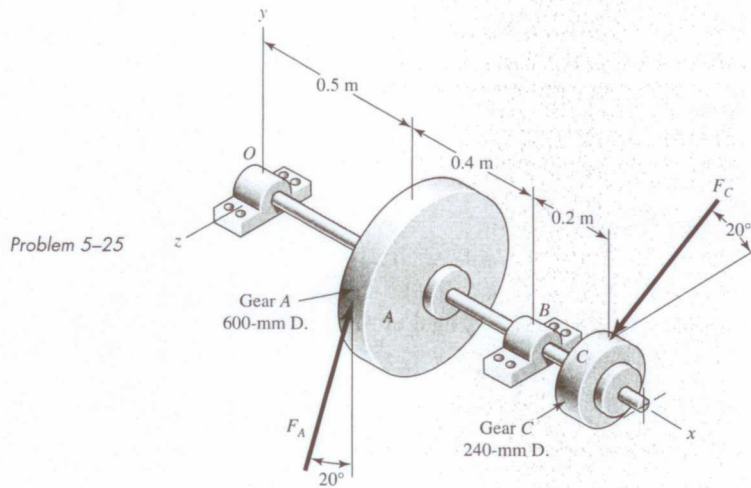
\*The asterisk indicates a problem that may not have a unique result or may be a particularly challenging problem.

- 5-19** This problem illustrates that the strength of a machine part can sometimes be measured in units other than those of force or moment. For example, the maximum speed that a flywheel can reach without yielding or fracturing is a measure of its strength. In this problem you have a rotating ring made of hot-forged AISI 1020 steel; the ring has a 150-mm inside diameter and a 250 mm outside diameter and is 40 mm thick. What speed in revolutions per minute would cause the ring to yield? At what radius would yielding begin? [Note: The maximum radial stress occurs at  $r = (r_o r_i)^{1/2}$ ; see Eq. (3-55).]
- 5-20** A light pressure vessel is made of 2024-T3 aluminum alloy tubing with suitable end closures. This cylinder has a 100-mm OD, a 1.5-mm wall thickness, and  $\nu = 0.334$ . The purchase order specifies a minimum yield strength of 320 MPa. What is the factor of safety if the pressure-release valve is set at 3.5 MPa?
- 5-21** A cold-drawn AISI 1015 steel tube is 300 mm OD by 200 mm ID and is to be subjected to an external pressure caused by a shrink fit. What maximum pressure would cause the material of the tube to yield?
- 5-22** What speed would cause fracture of the ring of Prob. 5-19 if it were made of grade 30 cast iron?
- 5-23** The figure shows a shaft mounted in bearings at *A* and *D* and having pulleys at *B* and *C*. The forces shown acting on the pulley surfaces represent the belt tensions. The shaft is to be made of ASTM grade 25 cast iron using a design factor  $n_d = 2.8$ . What diameter should be used for the shaft?

Problem 5-23

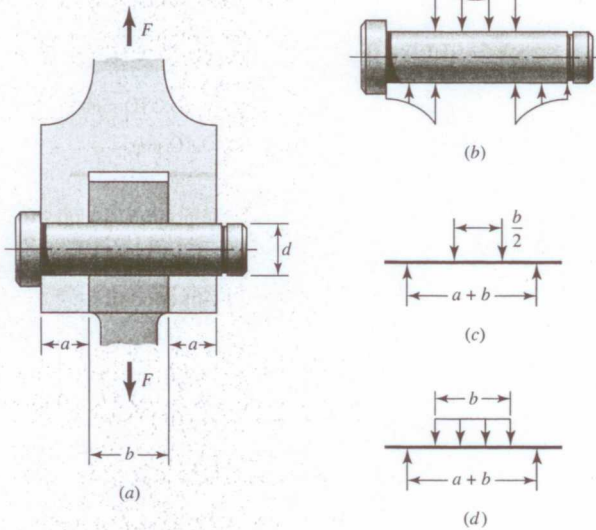


- 5-24** By modern standards, the shaft design of Prob. 5-23 is poor because it is so long. Suppose it is redesigned by halving the length dimensions. Using the same material and design factor as in Prob. 5-23, find the new shaft diameter.
- 5-25** The gear forces shown act in planes parallel to the  $yz$  plane. The force on gear *A* is 1.2 kN. Consider the bearings at *O* and *B* to be simple supports. For a static analysis and a factor of safety of 3.5, use distortion energy to determine the minimum safe diameter of the shaft. Consider the material to have a yield strength of 42 MPa.
- 5-26** Repeat Prob. 5-25 using maximum-shear-stress.
- 5-27** The figure is a schematic drawing of a countershaft that supports two V-belt pulleys. For each pulley, the belt tensions are parallel. For pulley *A* consider the loose belt tension is 15 percent of the tension on the tight side. A cold-drawn UNS G10180 steel shaft of uniform diameter is to be selected for this application. For a static analysis with a factor of safety of 3.0, determine the minimum preferred size diameter. Use the distortion-energy theory.

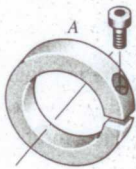


- 5-28** Repeat Prob. 5-27 using maximum shear stress.
- 5-29** The clevis pin shown in the figure is 12 mm in diameter and has the dimensions  $a = 12$  mm and  $b = 18$  mm. The pin is machined from AISI 1018 hot-rolled steel (Table A-18) and is to be loaded to no more than 4.4 kN. Determine whether or not the assumed loading of figure *c* yields a factor of safety any different from that of figure *d*. Use the maximum-shear-stress theory.
- 5-30** Repeat Prob. 5-29, but this time use the distortion-energy theory.
- 5-31** A split-ring clamp-type shaft collar is shown in the figure. The collar is 50 mm OD by 25 mm ID by 12 mm wide. The screw is designated as M6. The relation between the screw tightening torque  $T$ , the nominal screw diameter  $d$ , and the tension in the screw  $F_t$  is approximately  $T = 0.2 F_t d$ . The shaft is sized to obtain a close running fit. Find the axial holding force  $F_x$  of the collar as a function of the coefficient of friction and the screw torque.

Problem 5-29



Problem 5-31

**5-32**

Suppose the collar of Prob. 5-31 is tightened by using a screw torque of 20 N·m. The collar material is AISI 1040 steel heat-treated to a minimum tensile yield strength of 430 MPa.

- Estimate the tension in the screw.
- By relating the tangential stress to the hoop tension, find the internal pressure of the shaft on the ring.
- Find the tangential and radial stresses in the ring at the inner surface.
- Determine the maximum shear stress and the von Mises stress.
- What are the factors of safety based on the maximum-shear-stress hypothesis and the distortion-energy theory?

**5-33**

In Prob. 5-31, the role of the screw was to induce the hoop tension that produces the clamping. The screw should be placed so that no moment is induced in the ring. Just where should the screw be located?

**5-34**

A tube has another tube shrunk over it. The specifications are:

|    | Inner Member     | Outer Member     |
|----|------------------|------------------|
| ID | $25 \pm 0.05$ mm | $49 \pm 0.01$ mm |
| OD | $50 \pm 0.01$ mm | $75 \pm 0.1$ mm  |

Both tubes are made of a plain carbon steel.

- Find the nominal shrink-fit pressure and the von Mises stresses at the fit surface.
- If the inner tube is changed to solid shafting with the same outside dimensions, find the nominal shrink-fit pressure and the von Mises stresses at the fit surface.

- 5-35** Steel tubes with a Young's modulus of 207 GPa have the specifications:

|    | Inner Tube        | Outer Tube           |
|----|-------------------|----------------------|
| ID | $25 \pm 0.050$ mm | $49.98 \pm 0.010$ mm |
| OD | $50 \pm 0.010$ mm | $75 \pm 0.10$ mm     |

These are shrink-fitted together. Find the nominal shrink-fit pressure and the von Mises stress in each body at the fit surface.

- 5-36** Repeat Prob. 5-35 for maximum shrink-fit conditions.

- 5-37** A 50 mm-diameter solid steel shaft has a gear with ASTM grade 20 cast-iron hub ( $E = 100$  GPa) shrink-fitted to it. The specifications for the shaft are

$$50 \begin{array}{l} + 0.0000 \\ - 0.01 \end{array} \text{ mm}$$

The hole in the hub is sized at  $49 \pm 0.01$  mm with an OD of  $100 \pm 0.8$  mm. Using the midrange values and the modified Mohr theory, estimate the factor of safety guarding against fracture in the gear hub due to the shrink fit.

- 5-38** Two steel tubes are shrink-fitted together where the nominal diameters are 38, 44, and 50 mm. Careful measurement before fitting revealed that the diametral interference between the tubes to be 0.06 mm. After the fit, the assembly is subjected to a torque of 900 N·m and a bending-moment of 700 N·m. Assuming no slipping between the cylinders, analyze the outer cylinder at the inner and outer radius. Determine the factor of safety using distortion energy with  $S_y = 420$  MPa.

- 5-39** Repeat Prob. 5-38 for the inner tube.

- 5-40** For Eqs. (5-36) show that the principal stresses are given by

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \right)$$

$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \right)$$

$$\sigma_3 = \begin{cases} 0 & \text{(plane stress)} \\ \sqrt{\frac{2}{\pi r}} \nu K_I \cos \frac{\theta}{2} & \text{(plane strain)} \end{cases}$$

- 5-41** Use the results of Prob. 5-40 for plane strain near the tip with  $\theta = 0$  and  $\nu = \frac{1}{3}$ . If the yield strength of the plate is  $S_y$ , what is  $\sigma_1$  when yield occurs?

(a) Use the distortion-energy theory.

(b) Use the maximum-shear-stress theory. Using Mohr's circles, explain your answer.

- 5-42** A plate 100 mm wide, 200 mm long, and 12 mm thick is loaded in tension in the direction of the length. The plate contains a crack as shown in Fig. 5-26 with the crack length of 15.65 mm. The material is steel with  $K_{Ic} = 490$  MPa·√in, and  $S_y = 1.1$  GPa. Determine the maximum possible load that can be applied before the plate (a) yields, and (b) has uncontrollable crack growth.

- 5-43** A cylinder subjected to internal pressure  $p_i$  has an outer diameter of 350 mm and a 25-mm wall thickness. For the cylinder material,  $K_{Ic} = 80$  MPa·√m,  $S_y = 1200$  MPa, and  $S_{ur} = 1350$  MPa.

If the cylinder contains a radial crack in the longitudinal direction of depth 12.5 mm determine the pressure that will cause uncontrollable crack growth.

- 5-44** A carbon steel collar of length 25 mm is to be machined to inside and outside diameters, respectively, of

$$D_i = 20 \pm 0.01 \text{ mm} \quad D_o = 30 \pm 0.05 \text{ mm}$$

This collar is to be shrink-fitted to a hollow steel shaft having inside and outside diameters, respectively, of

$$d_i = 10 \pm 0.05 \text{ mm} \quad d_o = 20.06 \pm 0.01 \text{ mm}$$

These tolerances are assumed to have a normal distribution, to be centered in the spread interval, and to have a total spread of  $\pm 4$  standard deviations. Determine the means and the standard deviations of the tangential stress components for both cylinders at the interface.

- 5-45** Suppose the collar of Prob. 5-44 has a yield strength of  $S_y = N(660, 46)$  MPa. What is the probability that the material will not yield?

- 5-46** A carbon steel tube has an outside diameter of 25 mm and a wall thickness of 3 mm. The tube is to carry an internal hydraulic pressure given as  $p = N(42, 3.5)$  MPa. The material of the tube has a yield strength of  $S_y = N(350, 28)$  MPa. Find the reliability using thin-wall theory.