Note, in this equation, that if $h = 0$, then $F = 2W$. This says that when the weight is released while in contact with the spring but is not exerting any force on the spring, the largest force is double the weight.

Most systems are not as ideal as those explored here, so be wary about using these relations for nonideal systems.

PROBLEMS

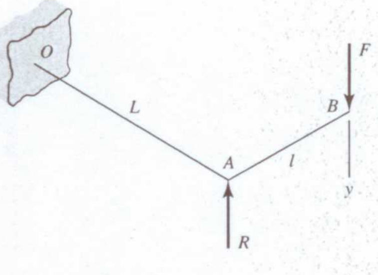
4–1 Structures can often be considered to be composed of a combination of tension and torsion members and beams. Each of these members can be analyzed separately to determine its force-deflection relationship and its spring rate. It is possible, then, to obtain the deflection of a structure by considering it as an assembly of springs having various series and parallel relationships.

(*a*) What is the overall spring rate of three springs in series?

(*b*) What is the overall spring rate of three springs in parallel?

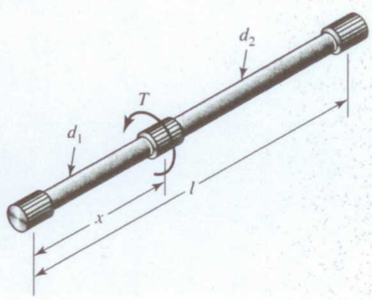
(*c*) What is the overall spring rate of a single spring in series with a pair of parallel springs?

4–2 The figure shows a torsion bar OA fixed at O, simply supported at A, and connected to a cantilever AB. The spring rate of the torsion bar is k_T, in newton-meters per radian, and that of the cantilever is k_C, in newtons per meter. What is the overall spring rate based on the deflection y at point B?



Problem 4–2

4–3 A torsion-bar spring consists of a prismatic bar, usually of round cross section, that is twisted at one end and held fast at the other to form a stiff spring. An engineer needs a stiffer one than usual and so considers building in both ends and applying the torque somewhere in the central portion of the span, as shown in the figure. If the bar is uniform in diameter, that is, if $d = d_1 = d_2$, investigate how the allowable angle of twist, the largest torque, and the spring rate depend on the location x at which the torque is applied. *Hint:* Consider two springs in parallel.



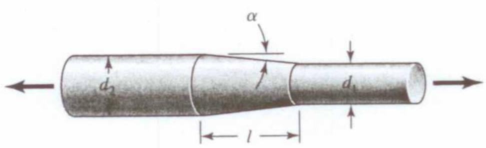
Problem 4–3

4–4 An engineer is forced by geometric considerations to apply the torque on the spring of Prob. 4–3 at the location $x = 0.2l$. For a uniform-diameter spring, this would cause the long leg of the span to be underutilized when both legs have the same diameter. If the diameter of the long leg is reduced sufficiently, the shear stress in the two legs can be made equal. How would this change affect the allowable angle of twist, the largest torque, and the spring rate?

4–5 A bar in tension has a circular cross section and includes a conical portion of length l, as shown. The task is to find the spring rate of the entire bar. Equation (4–4) is useful for the outer portions of diameters d_1 and d_2, but a new relation must be derived for the tapered section. If α is the apex half-angle, as shown, show that the spring rate of the tapered portion of the shaft is

$$k = \frac{EA_1}{l}\left(1 + \frac{2l}{d_1}\tan\alpha\right)$$

Problem 4–5



4–6 When a hoisting cable is long, the weight of the cable itself contributes to the elongation. If a cable has a weight per unit length of w, a length of l, and a load P attached to the free end, show that the cable elongation is

$$\delta = \frac{Pl}{AE} + \frac{wl^2}{2AE}$$

4–7 Use integration to verify the deflection equation given for the uniformly loaded cantilever beam of appendix Table A–9–3.

4–8 Use integration to verify the deflection equation given for the end moment loaded cantilever beam of appendix Table A–9–4.

4–9 When an initially straight beam sags under transverse loading, the ends contract because the neutral surface of zero strain neither extends nor contracts. The length of the deflected neutral surface is the same as the original beam length l. Consider a segment of the initially straight beam Δs. After bending, the x-direction component is shorter than Δs, namely, Δx. The contraction is $\Delta s - \Delta x$, and these summed for the entire beam gives the end contraction λ. Show that

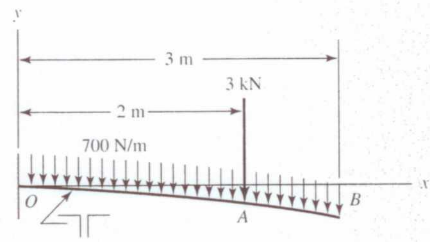
$$\lambda \doteq \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx$$

4–10 Using the results of Prob. 4–9, determine the end contraction of the uniformly loaded cantilever beam of appendix Table A–9–3.

4–11 Using the results of Prob. 4–9, determine the end contraction of the uniformly loaded simply-supported beam of appendix Table A–9–7. Assume the left support cannot deflect in the x direction, whereas the right support can.

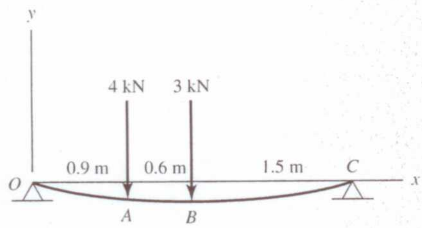
4–12 The figure shows a cantilever consisting of steel angles size 100 × 100 × 12 mm mounted back to back. Using superposition, find the deflection at B and the maximum stress in the beam.

Problem 4–12



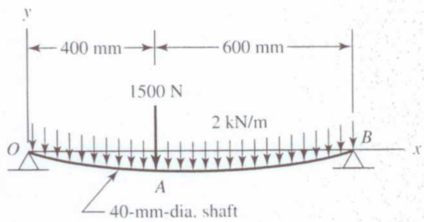
4–13 A simply supported beam loaded by two forces is shown in the figure. Select a pair of structural steel channels mounted back to back to support the loads in such a way that the deflection at midspan will not exceed 1.6 mm and the maximum stress will not exceed 40 MPa. Use superposition.

Problem 4–13



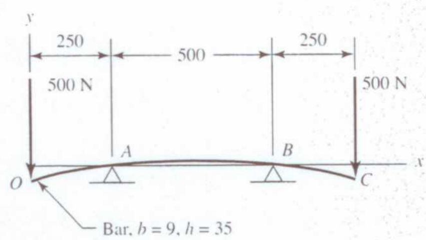
4–14 Using superposition, find the deflection of the steel shaft at A in the figure. Find the deflection at midspan. By what percentage do these two values differ?

Problem 4–14



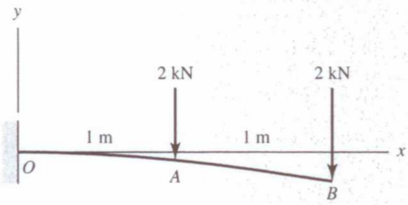
4–15 A rectangular steel bar supports the two overhanging loads shown in the figure. Using superposition, find the deflection at the ends and at the center.

Problem 4–15
Dimensions in millimeters.



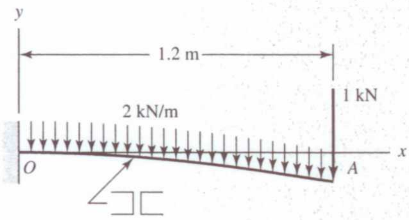
4–16 Using the formulas in Appendix Table A–9 and superposition, find the deflection of the cantilever at B if $I = 5.4 \times 10^6$ mm^4 and $E = 207$ GPa.

Problem 4–16



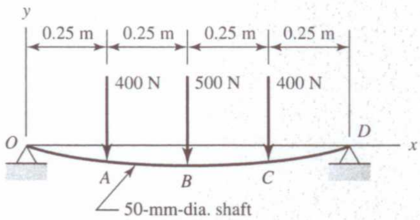
4–17 The cantilever shown in the figure consists of two structural-steel channels size 76 × 38 mm. Using superposition, find the deflection at A.

Problem 4–17



4–18 Using superposition, determine the maximum deflection of the beam shown in the figure. The material is carbon steel.

Problem 4–18

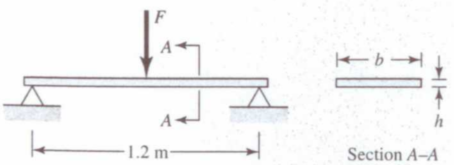


4–19 Illustrated is a rectangular steel bar with simple supports at the ends and loaded by a force F at the middle; the bar is to act as a spring. The ratio of the width to the thickness is to be about $b = 16h$, and the desired spring scale is 400 kN/m.

(*a*) Find a set of cross-section dimensions, using preferred sizes.

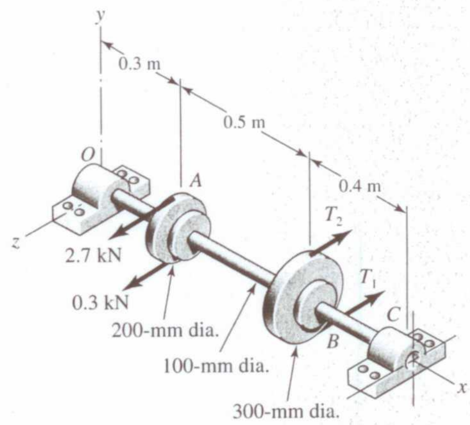
(*b*) What deflection would cause a permanent set in the spring if this is estimated to occur at a normal stress of 620 MPa?

Problem 4–19



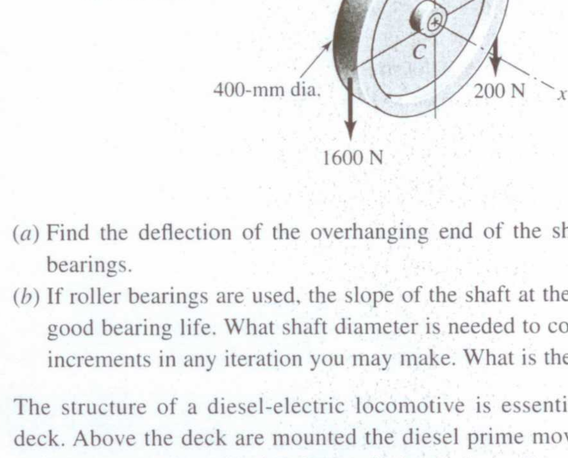
4–20 Illustrated in the figure is a 38-mm-diameter steel countershaft that supports two pulleys. Pulley A delivers power to a machine causing a tension of 2.7 kN in the tight side of the belt and 0.3 kN in the loose side, as indicated. Pulley B receives power from a motor. The belt tensions on pulley B have the relation $T_1 = 0.125T_2$. Find the deflection of the shaft in the z direction at pulleys A and B. Assume that the bearings constitute simple supports.

Problem 4–20



4–21 The figure shows a steel countershaft that supports two pulleys. Pulley C receives power from a motor producing the belt tensions shown. Pulley A transmits this power to another machine through the belt tensions T_1 and T_2 such that $T_1 = 8T_2$.

Problem 4–21



(*a*) Find the deflection of the overhanging end of the shaft, assuming simple supports at the bearings.

(*b*) If roller bearings are used, the slope of the shaft at the bearings should not exceed 0.06° for good bearing life. What shaft diameter is needed to conform to this requirement? Use 3 mm increments in any iteration you may make. What is the deflection at pulley C now?

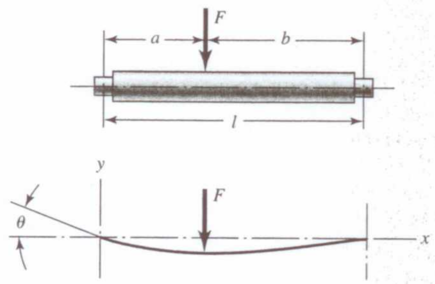
4–22 The structure of a diesel-electric locomotive is essentially a composite beam supporting a deck. Above the deck are mounted the diesel prime mover, generator or alternator, radiators, switch gear, and auxiliaries. Beneath the deck are found fuel and lubricant tanks, air reservoirs, and small auxiliaries. This assembly is supported at bolsters by the trucks that house the traction motors and brakes. This equipment is distributed as uniformly as possible in the span between the bolsters. In an approximate way, the loading can be viewed as uniform between the bolsters and simply supported. Because the hoods that shield the equipment from the weather have many rectangular access doors, which are mass-produced, it is important that the hood structure be level and plumb and sit on a flat deck. Aesthetics plays a role too. The center sill beam has a second moment of area of $I = 2.27 \times 10^9$ mm^4, the bolsters are 10 m apart, and the deck loading is 73 kN/m.

(*a*) What is the camber of the curve to which the deck will be built in order that the service-ready locomotive will have a flat deck?

(*b*) What equation would you give to locate points on the curve of part (*a*)?

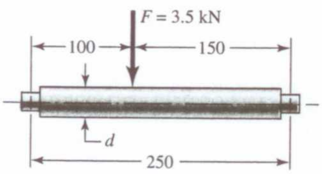
4–23 The designer of a shaft usually has a slope constraint imposed by the bearings used. This limit will be denoted as ξ. If the shaft shown in the figure is to have a uniform diameter d except in the locality of the bearing mounting, it can be approximated as a uniform beam with simple supports. Show that the minimum diameters to meet the slope constraints at the left and right bearings are, respectively,

$$d_L = \left| \frac{32Fb(l^2 - b^2)}{3\pi E l \xi} \right|^{1/4} \qquad d_R = \left| \frac{32Fa(l^2 - a^2)}{3\pi E l \xi} \right|^{1/4}$$



Problem 4–23

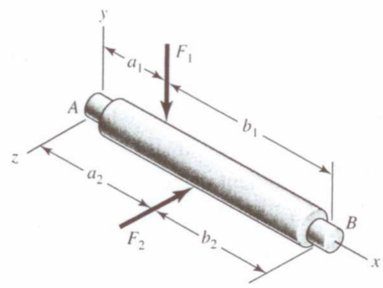
4–24 A shaft is to be designed so that it is supported by roller bearings. The basic geometry is shown in the figure. The allowable slope at the bearings is 0.001 mm/mm without bearing life penalty. For a design factor of 1.28, what uniform-diameter shaft will support the 3.5-kN load 100 mm from the left bearing without penalty? Use $E = 207$ GPa.



Problem 4–24
nsions in millimeters.

4–25 Determine the maximum deflection of the shaft of Prob. 4–24.

4–26 For the shaft shown in the figure, let $a_1 = 100$ mm, $b_1 = 300$ mm, $a_2 = 250$ mm, $F_1 = 400$ N, $F_2 = 1200$ N, and $E = 207$ GPa. The shaft is to be sized so that the maximum slope at either bearing A or bearing B does not exceed 0.001 rad. Determine a suitable diameter d.

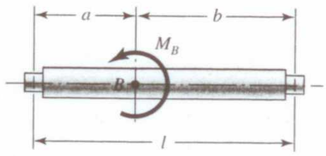


Problem 4–26

4–27 If the diameter of the beam for Prob. 4–26 is 35 mm, determine the deflection of the beam at $x = 0.2$ m.

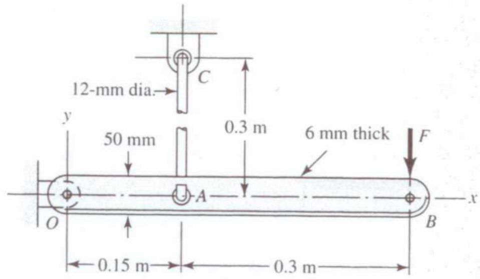
4–28 See Prob. 4–26 and the accompanying figure. The loads and dimensions are $F_1 = 3.5$ kN, $F_2 = 2.7$ kN, $a_1 = 100$ mm, $b_1 = 150$ mm, and $a_2 = 175$ mm. Find the uniform shaft diameter necessary to limit the slope at the bearings to 0.001 rad. Use a design factor of $n_d = 1.5$ and $E = 207$ Gpa.

4–29 Shown in the figure is a uniform-diameter shaft with bearing shoulders at the ends; the shaft is subjected to a concentrated moment $M = 130$ N·m. The shaft is of carbon steel and has $a = 120$ mm and $l = 0.2$ m. The slope at the ends must be limited to 0.002 rad. Find a suitable diameter d.



Problem 4–29

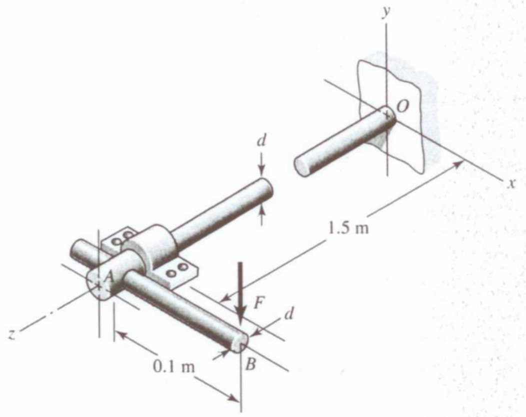
4–30 The rectangular member OAB, shown in the figure, is held horizontal by the round hooked bar AC. The modulus of elasticity of both parts is 70 GPa. Use superposition to find the deflection at B due to a force $F = 350$ N.



Problem 4–30

4–31 The figure illustrates a torsion-bar spring OA having a diameter $d = 12$ mm. The actuating cantilever AB also has $d = 12$ mm. Both parts are of carbon steel. Use superposition and find the spring rate k corresponding to a force F acting at B.

Problem 4–31



4–32 Consider the simply supported beam with an intermediate load in Appendix A–9–6. Determine the deflection equation if the stiffness of the left and right supports are k_1 and k_2, respectively.

4–33 Consider the simply supported beam with a uniform load in Appendix A–9–7. Determine the deflection equation if the stiffness of the left and right supports are k_1 and k_2, respectively.

4–34 Prove that for a uniform-cross-section beam with simple supports at the ends loaded by a single concentrated load, the location of the maximum deflection will never be outside the range of $0.423l \leq x \leq 0.577l$ regardless of the location of the load along the beam. The importance of this is that you can always get a quick estimate of $y_{\max}$ by using $x = l/2$.

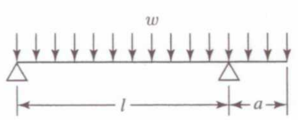
4–35 Solve Prob. 4–12 using singularity functions. Use statics to determine the reactions.

4–36 Solve Prob. 4–13 using singularity functions. Use statics to determine the reactions.

4–37 Solve Prob. 4–14 using singularity functions. Use statics to determine the reactions.

4–38 Consider the uniformly loaded simply supported beam with an overhang as shown. Use singularity functions to determine the deflection equation of the beam. Use statics to determine the reactions.

Problem 4–38

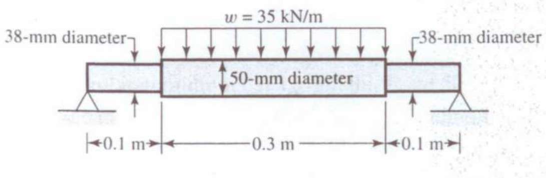


4–39 Solve Prob. 4–15 using singularity functions. Since the beam is symmetric, only write the equation for half the beam and use the slope at the beam center as a boundary condition. Use statics to determine the reactions.

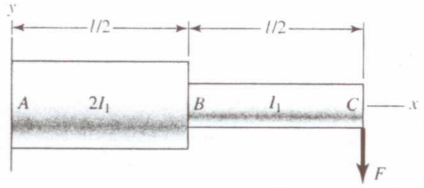
4–40 Solve Prob. 4–30 using singularity functions. Use statics to determine the reactions.

4–41 Determine the deflection equation for the steel beam shown using singularity functions. Since the beam is symmetric, write the equation for only half the beam and use the slope at the beam center as a boundary condition. Use statics to determine the reactions.

Problem 4–41



4–42 Determine the deflection equation for the cantilever beam shown using singularity functions. Evaluate the deflections at B and C and compare your results with Example 4–11.



Problem 4–42

4–43 Examine the expression for the deflection of the cantilever beam, end-loaded, shown in Appendix Table A–9–1 for some intermediate point, $x = a$, as

$$y|_{x=a} = \frac{F_1 a^2}{6EI}(a - 3l)$$

In Table A–9–2, for a cantilever with intermediate load, the deflection at the end is

$$y|_{x=l} = \frac{F_2 a^2}{6EI}(a - 3l)$$

These expressions are remarkably similar and become identical when $F_1 = F_2 = 1$. In other words, the deflection at $x = a$ (station 1) due to a unit load at $x = l$ (station 2) is the same as the deflection at station 2 due to a unit load at station 1. Prove that this is true generally for an elastic body even when the lines of action of the loads are not parallel. This is known as a special case of *Maxwell's reciprocal theorem*. (*Hint:* Consider the potential energy of strain when the body is loaded by two forces in either order of application.)

4–44 A steel shaft of uniform 50-mm diameter has a bearing span l of 600 mm and an overhang of 180 mm on which a coupling is to be mounted. A gear is to be attached 220 mm to the right of the left bearing and will carry a radial load of 2 kN. We require an estimate of the bending deflection at the coupling. Appendix Table A–9–6 is available, but we can't be sure of how to expand the equation to predict the deflection at the coupling.

(*a*) Show how Appendix Table A–9–10 and Maxwell's theorem (see Prob. 4–43) can be used to obtain the needed estimate.

(*b*) Check your work by finding the slope at the right bearing and extending it to the coupling location.

4–45 Use Castigliano's theorem to verify the maximum deflection for the uniformly loaded beam of Appendix Table A–9–7. Neglect shear.

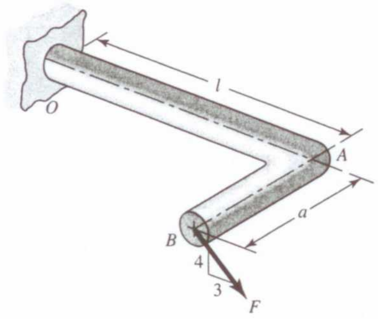
4–46 Solve Prob. 4–17 using Castigliano's theorem. *Hint:* Write the moment equation using a position variable positive to the left starting at the right end of the beam.

4–47 Solve Prob. 4–30 using Castigliano's theorem.

4–48 Solve Prob. 4–31 using Castigliano's theorem.

4–49 Determine the deflection at midspan for the beam of Prob. 4–41 using Castigliano's theorem.

4–50 Using Castigliano's theorem, determine the deflection of point B in the direction of the force F for the bar shown. The solid bar has a uniform diameter, d. Neglect bending shear.



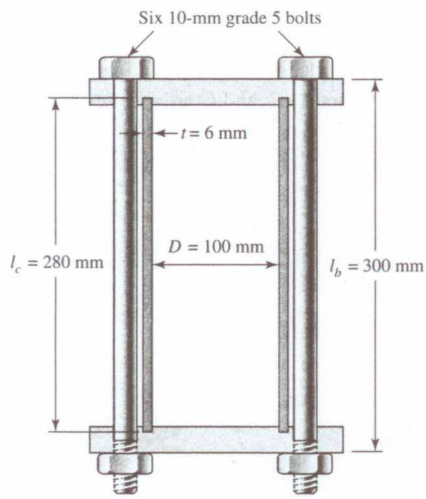
Problem 4–50

4–51 A cable is made using a 1.6-mm steel wire and three strands of 2-mm copper wire. Find the stress in each wire if the cable is subjected to a tension of 1 kN.

4–52 The figure shows a steel pressure cylinder of diameter 100 mm which uses six SAE grade 5 steel bolts having a grip of 300 mm. These bolts have a proof strength (see Chap. 8) of 580 MPa for this size of bolt. Suppose the bolts are tightened to 90 percent of this strength in accordance with some recommendations.

(*a*) Find the tensile stress in the bolts and the compressive stress in the cylinder walls.

(*b*) Repeat part (*a*), but assume now that a fluid under a pressure of 4 MPa is introduced into the cylinder.

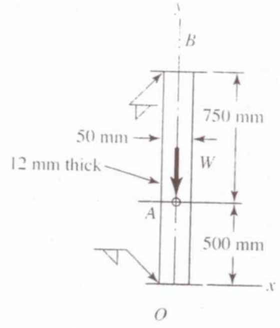


Problem 4–52

4–53 A torsion bar of length L consists of a round core of stiffness $(GJ)_c$ and a shell of stiffness $(GJ)_s$. If a torque T is applied to this composite bar, what percentage of the total torque is carried by the shell?

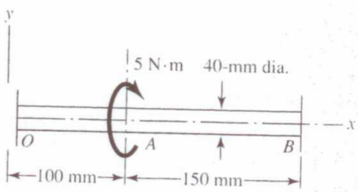
4–54 A rectangular aluminum bar 12 mm thick and 50 mm wide is welded to fixed supports at the ends, and the bar supports a load $W = 3.5$ kN, acting through a pin as shown. Find the reactions at the supports.

Problem 4–54



4–55 The steel shaft shown in the figure is subjected to a torque of 5 N·m applied at point A. Find the torque reactions at O and B.

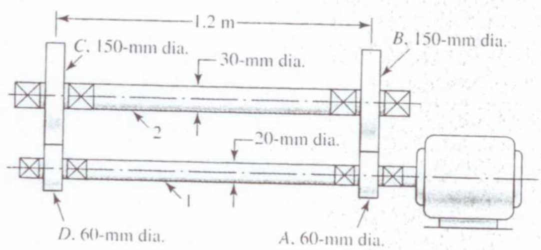
Problem 4–55



4–56 Repeat Prob. 4–55 with the diameters of section OA being 40 mm and section AB being 50 mm.

4–57 In testing the wear life of gear teeth, the gears are assembled by using a pretorsion. In this way, a large torque can exist even though the power input to the tester is small. The arrangement shown in the figure uses this principle. Note the symbol used to indicate the location of the shaft bearings used in the figure. Gears A, B, and C are assembled first, and then gear C is held fixed. Gear D is assembled and meshed with gear C by twisting it through an angle of 4° to provide the pretorsion. Find the maximum shear stress in each shaft resulting from this preload.

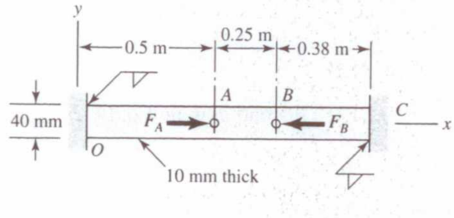
Problem 4–57



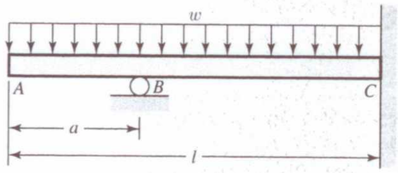
4–58 The figure shows a 10- by 40-mm rectangular steel bar welded to fixed supports at each end. The bar is axially loaded by the forces $F_A = 40$ kN and $F_B = 20$ kN acting on pins at A and B. Assuming that the bar will not buckle laterally, find the reactions at the fixed supports. Use procedure 1 from Sec. 4–10.

4–59 For the beam shown, determine the support reactions using superposition and procedure 1 from Sec. 4–10.

Problem 4–58



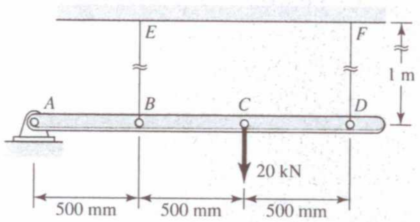
Problem 4–59



4–60 Solve Prob. 4–59 using Castigliano's theorem and procedure 1 from Sec. 4–10.

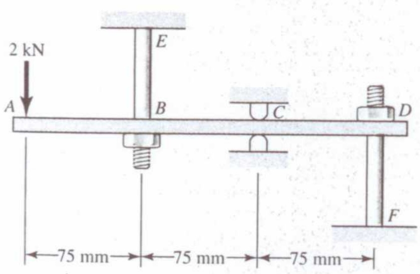
4–61 The steel beam $ABCD$ shown is simply supported at A and supported at B and D by steel cables, each having an effective diameter of 12 mm. The second area moment of the beam is $I = 8(10^5)$ mm^4. A force of 20 kN is applied at point C. Using procedure 2 of Sec. 4–10 determine the stresses in the cables and the deflections of B, C, and D. For steel, let $E = 209$ GPa.

Problem 4–61



4–62 The steel beam $ABCD$ shown is supported at C as shown and supported at B and D by steel bolts each having a diameter of 8 mm. The lengths of BE and DF are 50 and 62 mm, respectively. The beam has a second area moment of 20.8×10^{-9} m^4. Prior to loading, the nuts are just in contact with the horizontal beam. A force of 2 kN is then applied at point A. Using procedure 2 of Sec. 4–10, determine the stresses in the bolts and the deflections of points A, B, and D. For steel, let E = 207 GPa.

Problem 4–62



4–63 The horizontal deflection of the right end of the curved bar of Fig. 4–12 is given by Eq. (4–35) for $R/h > 10$. For the same conditions, determine the vertical deflection.