TOBB EKONOMİ VE TEKNOLOJİ ÜNİVERSİTESİ

MAK 206 Strength of Materials – 2015 Spring

QUIZ 9

24 March 2015 Ad, Soyad: **SOLUTION**

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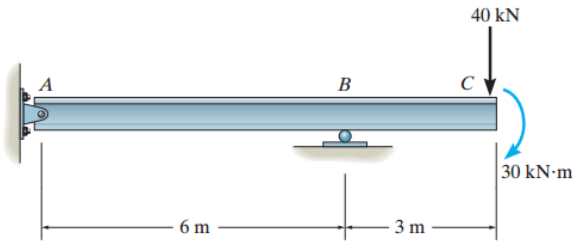
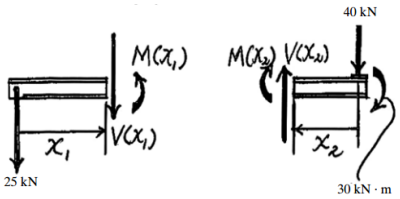
Problem: The dimensions of AC beam are shown. EI is constant.

a) Determine the slope at A point,

b) Determine the deflection at C point.





$\zeta + \Sigma M_o = 0; \quad M(x_1) + 25x_1 = 0 \quad M(x_1) = (-25x_1)\ \text{kN} \cdot \text{m}$

And

$\zeta + \Sigma M_o = 0; \quad -M(x_2) - 40x_2 - 30 = 0 \quad M(x_2) = (-40x_2 - 30)\ \text{kN} \cdot \text{m}$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = (-25x_2)\ \text{kN} \cdot \text{m}$$

$$EI\frac{dv_1}{dx_1} = \left(-\frac{25}{2}x_1^2 + C_1\right)\text{kN} \cdot \text{m}^2 \qquad (1)$$

$$EI\,v_1 = \left(-\frac{25}{6}x_1^3 + C_1x_1 + C_2\right)\text{kN} \cdot \text{m}^3 \qquad (2)$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = (-40x_2 - 30)\ \text{kN} \cdot \text{m}$$

$$EI\frac{dv_2}{dx_2} = (-20x_2^2 - 30x_2 + C_3)\ \text{kN} \cdot \text{m} \qquad (3)$$

$$EI\,v_2 = \left(-\frac{20}{3}x_2^3 - 15x_2^2 + C_3x_2 + C_4\right)\text{kN} \cdot \text{m}^3 \qquad (4)$$

At $x_1 = 0, v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{25}{6}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x_1 = 6$ m, $v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{25}{6}(6^3) + C_1(6) + 0 \qquad C_1 = 150\ \text{kN} \cdot \text{m}^2$$

Also, at $x_2 = 3$ m, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{20}{3}(3^3) - 15(3^2) + C_3(3) + C_4$$

$$3C_3 + C_4 = 315 \qquad (5)$$

At $x_1 = 6$ m and $x_2 = 3$ m, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Then Eq. (1) and (3) gives

$$-\frac{25}{2}(6^2) + 150 = -[-20(3^2) - 30(3) + C_3]$$

$$C_3 = 570\ \text{kN} \cdot \text{m}^2$$

Substitute the value of C_3 into Eq (5),

$$C_4 = -1395\ \text{kN} \cdot \text{m}^3$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{1}{EI}\left(-\frac{25}{6}x_1^2 + 150\right)\text{kN} \cdot \text{m}^2$$

At A, $x_1 = 0$. Thus,

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{150\ \text{kN} \cdot \text{m}^2}{EI} \qquad \textbf{Ans.}$$

Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{1}{EI}\left(-\frac{25}{6}x_1^3 + 150x_1\right)\text{kN} \cdot \text{m}^3 \qquad \textbf{Ans.}$$

$$v_2 = \frac{1}{EI}\left(-\frac{20}{3}x_2^3 - 15x_2^2 + 570x_2 - 1395\right)\text{kN} \cdot \text{m}^3 \qquad \textbf{Ans.}$$

At C, $x_2 = 0$. Thus

$$v_C = v_2|_{x_2=0} = -\frac{1395\ \text{kN} \cdot \text{m}^3}{EI} = \frac{1395\ \text{kN} \cdot \text{m}^3}{EI}\downarrow \qquad \textbf{Ans.}$$