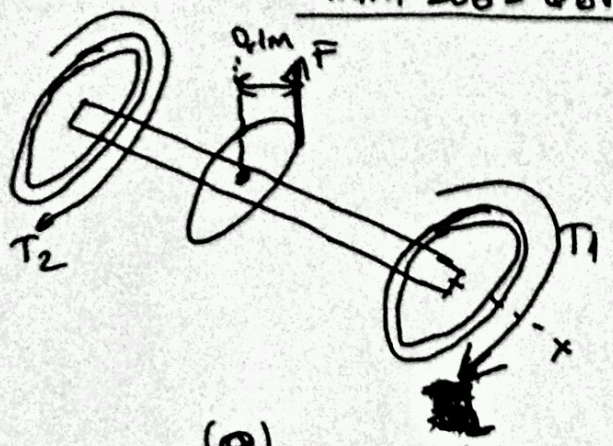
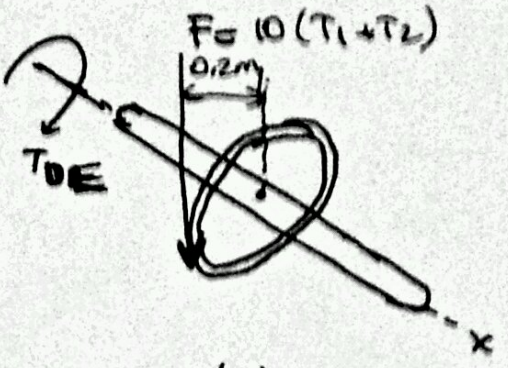
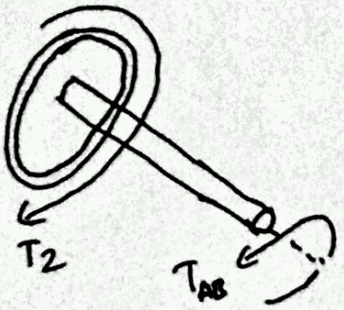
# MAK 206 - QUIZ 4 - SOLUTION



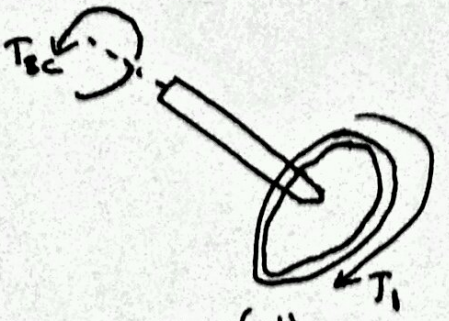

(a)




(b)




(c)




(d)

**Equilibrium**: Referring to FBD of shaft ABC shown in Fig. (a),

$\Sigma M_x = 0 \Rightarrow F.(0,1) - T_1 - T_2 = 0 \Rightarrow F = 10(T_1+T_2)$

**Internal Loading**: Referring to FBD of gear D in fig. b,

$\Sigma M_x = 0 \Rightarrow 10(T_1+T_2).(0,2) - T_{DE} = 0 \Rightarrow T_{DE} = 2(T_1+T_2)$

* Gear A in Fig c.

$\Sigma M_x = 0 \Rightarrow T_{AB} - T_2 = 0 \Rightarrow T_{AB} = T_2$

* Gear C in Fig. d.

$\Sigma M_x = 0 \Rightarrow T_{BC} - T_1 = 0 \Rightarrow T_{BC} = T_1$

**Angle of Twist**

$J_{AB} = J_{BC} = \frac{\pi}{2}.(0,025^4) = 6,135^{\times 10^{-7}} m^4$, $J_{DE} = \frac{\pi}{2}(0,03^4) = 1,272 \times 10^{-6} m^4$

$\phi_D = \frac{T_{DE}.L_{DE}}{J_{DE}.G_{A-36}} = \frac{2(T_1+T_2).(0,5)}{1,272\times10^{-6}\times80\times10^9} = 9,82^\circ(T_1+T_2)\times10^{-6}$

Then, using the gear ratio,

$$\phi_B = \phi_D \left(\frac{r_D}{r_B}\right) = 9,82 \times 10^{-6} (T_1 + T_2) \cdot \left(\frac{400}{100}\right) \Rightarrow \phi_B = 19,64 \times 10^{-6} (T_1 + T_2)$$

Also,

$$\phi_{C/B} = \frac{T_{BC} \cdot L_{BC}}{J_{BC} \cdot G_{A36}} = \frac{T_1 \times (0,8)}{6,135 \times 10^{-7} \times 80 \times 10^9} = 1,629 \times 10^{-5} T_1$$

$$\phi_{A/B} = \frac{T_{AB} \cdot L_{AB}}{J_{AB} \cdot G_{A36}} = \frac{T_2 \times (0,5)}{6,135 \times 10^{-7} \times 80 \times 10^9} = 1,018 \times 10^{-5} T_2$$

* It is required that $\phi_A = \phi_C = 0,04$ rad,

$$\phi_A = \phi_B + \phi_{A/B} \Rightarrow 0,04 = 19,64 \times 10^{-6} (T_1 + T_2) + 1,018 \times 10^{-5} T_2$$

$$(1,964\, T_1 + 2,982\, T_2) \times 10^{-5} = 0,04$$

- $1,964\, T_1 + 2,982\, T_2 = 4000$ ①

$$\phi_C = \phi_B + \phi_{C/B} \Rightarrow 0,04 = 1,964 \times 10^{-5} (T_1 + T_2) + 1,629 \times 10^{-5} T_1$$

- $3,593\, T_1 + 1,964\, T_2 = 4000$ ②

* Solving Eq ① and Eq. ②

$T_1 = 513,45$ N.m , $T_2 = 1003,2$ N.m