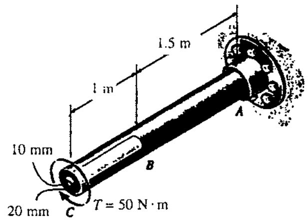
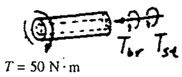
5-79. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50\ \text{N}\cdot\text{m}$ is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear and maximum shear strain in the brass and steel. $G_{st} = 80$ GPa, $G_{br} = 40$ GPa.





Equilibrium :

$$T_{br} + T_{st} - 50 = 0 \qquad [1]$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$

$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{br}(1)(10^3)}{\frac{\pi}{2}(10^4)(40)(10^3)} = \frac{T_{st}(1)(10^3)}{\frac{\pi}{2}[20^4 - 10^4](80)(10^3)}$$

$$T_{br} = 0.03333T_{st} \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$$T_{st} = 48.387\ \text{N}\cdot\text{m}; \qquad T_{br} = 1.613\ \text{N}\cdot\text{m}$$

$$\phi_C = \sum\frac{TL}{JG} = \frac{1.613(10^3)(1)(10^3)}{\frac{\pi}{2}(10^4)(40)(10^3)} + \frac{50(10^3)(1.5)(10^3)}{\frac{\pi}{2}(20^4)(80)(10^3)}$$

$$= 0.006\,297\ \text{rad} = 0.36° \qquad \textbf{Ans}$$

$$(\tau_{st})_{\max AB} = \frac{T_{AB}c}{J} = \frac{50(10^3)(20)}{\frac{\pi}{2}(20^4)} = 3.98\ \text{MPa}$$

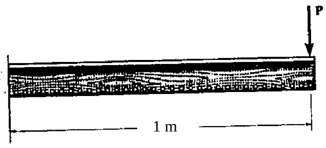
$$(\tau_{st})_{\max BC} = \frac{T_{st}c}{J} = \frac{48.387(10^3)(20)}{\frac{\pi}{2}(20^4 - 10^4)} = 4.11\ \text{MPa (Max)} \qquad \textbf{Ans}$$

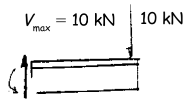
$$(\gamma_{st})_{\max} = \frac{(\tau_{st})_{\max}}{G} = \frac{4.11}{80(10^3)} = 5.14(10^{-6})\ \text{rad} \qquad \textbf{Ans}$$

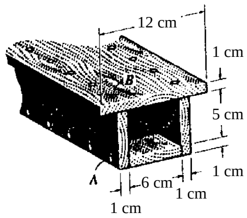
$$(\tau_{br})_{\max} = \frac{T_{br}c}{J} = \frac{1.613(10^3)(10)}{\frac{\pi}{2}(10^4)} = 1.03\ \text{MPa (Max)} \qquad \textbf{Ans}$$

$$(\gamma_{br})_{\max} = \frac{(\tau_{br})_{\max}}{G} = \frac{1.03}{40(10^3)} = 2.58(10^{-5})\ \text{rad} \qquad \textbf{Ans}$$

7-51. The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 cm. If a force $P = 10$ kN is applied to the beam, determine the shear force resisted by each nail at A and B.





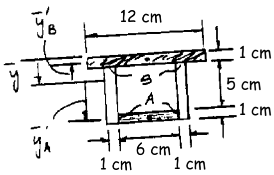


As shown on FBD, $V_{max} = 10$ kN

$$\bar{y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{0.5(12)(1) + 2[4(6)(1)] + 6.5(6)(1)}{12(1) + 2(6)(1) + 6(1)} = 3.10 \text{ cm}$$

$$I = \frac{1}{12}(12)(1^3) + 12(1)(3.10 - 0.5)^2 + 2[\frac{1}{12}(1)(6^3) + (1)(6)(4 - 3.10)^2]$$

$$+ \frac{1}{12}(6)(1^3) + 6(1)(6.5 - 3.10)^2 = 197.7 \text{ cm}^4$$



$Q_B = \bar{y}_B'A' = (3.10 - 0.5)(12)(1) = 31.2 \text{ cm}^3$

$Q_A = \bar{y}_A'A' = (6.5 - 3.10)(6)(1) = 20.4 \text{ cm}^3$

$V = P = 10$ kN

$$q_B = \frac{1}{2}(\frac{VQ_B}{I}) = \frac{1}{2}[\frac{10(31.2)}{197.7}] = 789.07 \text{ N/cm}$$

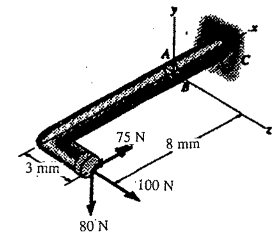
$$q_A = \frac{1}{2}(\frac{VQ_A}{I}) = \frac{1}{2}[\frac{10(20.4)}{197.7}] = 515.95 \text{ N/cm}$$

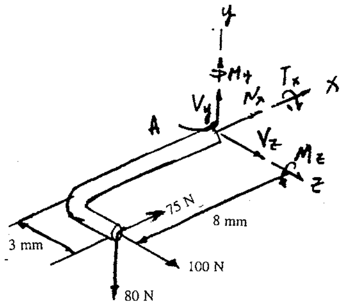
Shear force in nail:

$F_B = q_B s = 789.07(2) = 1578.14$ N **Ans**

$F_A = q_A s = 519.95(2) = 1039.9$ N **Ans**

8-56 The 1-mm-diameter rod is subjected to the loads shown. Determine the state of stress at point A, and show the results on a differential element located at this point.





$\Sigma F_z = 0;\quad V_z + 100 = 0;\quad V_z = -100\text{ N}$

$\Sigma F_x = 0;\quad N_x - 75 = 0;\quad N_x = 75\text{ N}$

$\Sigma F_y = 0;\quad V_y - 80 = 0;\quad V_y = 80\text{ N}$

$\Sigma M_z = 0;\quad M_z + 80(8) = 0;\quad M_z = -640\text{ N}\cdot\text{mm}$

$\Sigma M_x = 0;\quad T_x + 80(3) = 0;\quad T_x = -240\text{ N}\cdot\text{mm}$

$\Sigma M_y = 0;\quad M_y + 100(8) - 75(3) = 0;\quad M_y = -575\text{ N}\cdot\text{mm}$

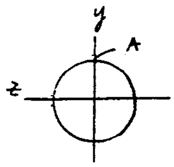
$A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(1^2) = \dfrac{1}{4}\pi\text{ mm}^2$

$J = \dfrac{\pi}{2}c^4 = \dfrac{\pi}{2}(0.5^4) = 0.03125\pi\text{ mm}^4$

$(Q_y)_A = 0$

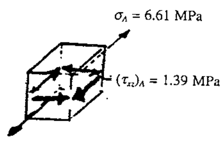
$(Q_z)_A = \bar{y}'A = \dfrac{4(0.5)}{3\pi}\dfrac{1}{2}(\pi)(0.5^2) = 0.08333\text{ mm}^3$

$I_y = I_z = \dfrac{\pi}{4}r^4 = \dfrac{\pi}{4}(0.5^4) = 0.015625\pi\text{ mm}^4$



Normal stress : $\sigma = \dfrac{P}{A} + \dfrac{M_z y}{I_z} + \dfrac{M_y z}{I_y}$

$\sigma_A = \dfrac{75}{\frac{1}{4}\pi} + \dfrac{640(0.5)}{0.0156\pi} + 0 = 6.61\text{ MPa (T)}$ **Ans**



Shear stress :

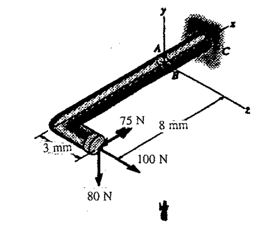
$\tau = \dfrac{VQ}{It} + \dfrac{Tc}{J}$

$(\tau_{xz})_A = \dfrac{100(0.08333)}{0.0156\pi(1)} + \dfrac{240(0.5)}{0.0312\pi}$

$= 1.39\text{ MPa}$ **Ans**

$(\tau_{xy})_A = 0$ **Ans**

8-57 The 1-mm-diameter rod is subjected to the loads shown. Determine the state of stress at point B, and show the results on a differential element located at this point.



$\Sigma F_z = 0; \quad V_z + 100 = 0; \quad V_z = -100\text{ N}$

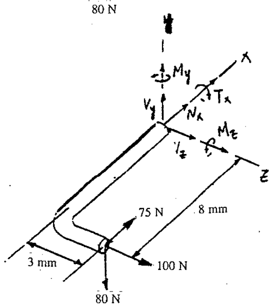
$\Sigma F_x = 0; \quad N_x - 75 = 0; \quad N_x = 75.0\text{ N}$

$\Sigma F_y = 0; \quad V_y - 80 = 0; \quad V_y = 80\text{ N}$

$\Sigma M_z = 0; \quad M_z + 80(8) = 0; \quad M_z = -640\text{ N}\cdot\text{mm}$

$\Sigma M_x = 0; \quad T_x + 80(3) = 0; \quad T_x = -240\text{ N}\cdot\text{mm}$

$\Sigma M_y = 0; \quad M_y + 1000(8) - 75(3) = 0; \quad M_y = -575\text{ N}\cdot\text{mm}$

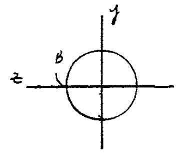


$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1^2) = \frac{\pi}{4}\text{ mm}^2$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.5^4) = 0.03125\pi\text{ mm}^4$$

$$(Q_y)_B = \frac{4(0.5)}{3\pi}\frac{1}{2}\left(\frac{\pi}{4}\right)(1^2) = 0.08333\text{ mm}^3$$

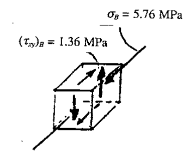
$$I_y = I_z = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.5^4) = 0.015625\pi\text{ mm}^4$$



Normal stress :

$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = \frac{75}{\frac{\pi}{4}} + 0 - \frac{575(0.5)}{0.015625\pi} = -5.76\text{ MPa} = 5.76\text{ MPa (C)}\ \textbf{Ans}$$



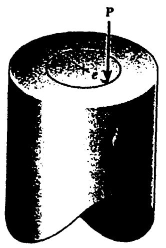
Shear stress :

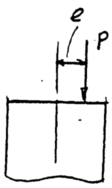
$$\tau = \frac{VQ}{It} \text{ and } \tau = \frac{Tc}{J}$$

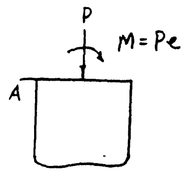
$$(\tau_{xy})_B = \frac{Tc}{J} - \frac{VQ}{It} = \frac{240(0.5)}{0.03125\pi} + \frac{80(0.0833)}{0.015625\pi(1)} = 1.36\text{ MPa} \qquad \textbf{Ans}$$

$$(\tau_{xz})_B = 0 \qquad \textbf{Ans}$$

8-58 The post has a circular cross section of radius c. Determine the maximum radius e at which the load can be applied so that no part of the post experiences a tensile stress. Neglect the weight of the post.





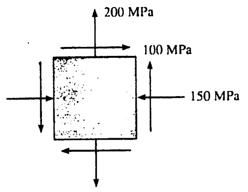


Require $\sigma_A = 0$

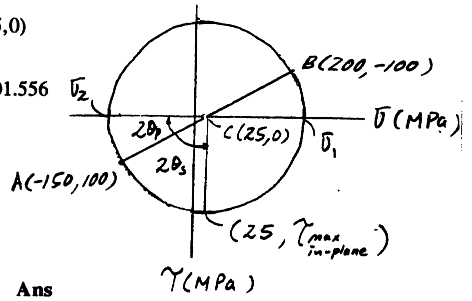
$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}; \qquad 0 = \frac{-P}{\pi c^2} + \frac{(Pe)c}{\frac{\pi}{4}c^4}$$

$$e = \frac{c}{4} \qquad \textbf{Ans}$$

9–70 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-150, 100)$ $\quad B(200, -100)$ $\quad C(25, 0)$



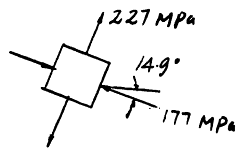
$$R = CA = \sqrt{(150+25)^2 + 100^2} = 201.556$$

$$\tan 2\theta_p = \frac{100}{150+25} = 0.5714$$

$\theta_p = -14.9°$ **Ans**

$\sigma_1 = 25 + 201.556 = 227 \text{ MPa}$ **Ans**

$\sigma_2 = 25 - 201.556 = -177 \text{MPa}$ **Ans**

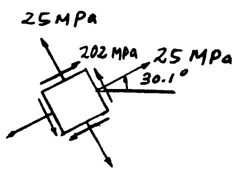


$\tau_{\substack{max \\ in-plane}} = R = 202 \text{ MPa}$ **Ans**

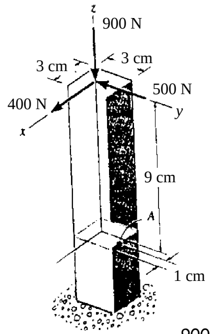
$\sigma_{avg} = 25 \text{ MPa}$ **Ans**

$$\tan 2\theta_s = \frac{150+25}{100} = 1.75$$

$\theta_s = 30.1°$ **Ans**



9–87 The post has a square cross-sectional area. If it is fixed-supported at its base and the loadings are applied at its end as shown, determine (*a*) the maximum in-plane shear stress developed at *A* and (*b*) the principal stresses at *A*.



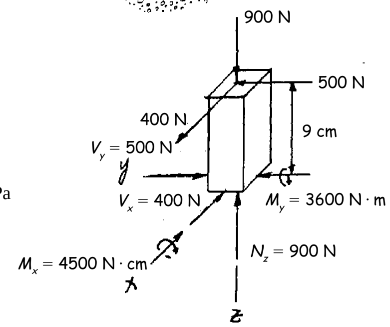
Section properties :

$$I_x = I_y = \frac{1}{12}(3)(3^3) = 6.75\ \text{cm}^4$$

$$A = 3(3) = 9\ \text{cm}^2$$

$$(Q_A)_x = \bar{y}'A' = (1)(1)(3) = 3\ \text{cm}^3$$

$$(Q_A)_y = 0$$

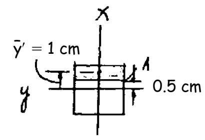


Normal stress : Applying $\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$

$$\sigma_A = -\frac{900}{9} + \frac{4500(1.5)}{6.75} - \frac{3600(0.5)}{6.75} = 633.33\ \text{N/cm}^2 = 6.3333\ \text{MPa}$$

Shear stress : Applying $\tau = \frac{VQ}{It}$

$$\tau_{zx} = \frac{400(3)}{6.75(3)} = 59.259\ \text{N/cm}^2 = 0.59\ \text{MPa}$$

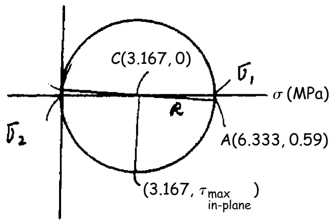


$$\tau_{zy} = 0$$

Mohr's circle :

A(6.333, 0.59) *C*(3.167, 0)

$$R = CA = \sqrt{(6.333 - 3.167)^2 + 0.59^2} = 3.22$$



a)

$\tau_{\substack{max \\ in-plane}} = R = 3.22$ MPa **Ans**

b)

$\sigma_1 = 3.167 + 3.22 = 6.387$ MPa **Ans**

$\sigma_2 = 3.167 - 3.22 = -0.053$ MPa **Ans**