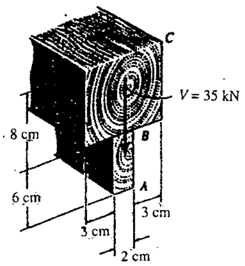
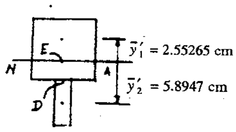
7-11. Sketch the intensity of the shear-stress distribution acting over the beam's cross-sectional area, and determine the resultant shear force acting on the segment AB. The shear acting at the section is $V = 35$ kN. Show that $I_{NA} = 872.49\ \text{cm}^4$.



$$\bar{y} = \frac{(4)(8)(8) + (11)(6)(2)}{8(8) + 6(2)} = 5.1053\ \text{cm}$$

$$I = \frac{1}{12}(8)(8^3) + 8(8)(5.1053 - 4)^2$$

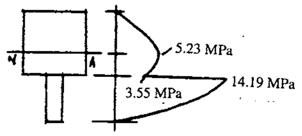
$$+ \frac{1}{12}(2)(6^3) + 2(6)(11 - 5.1053)^2 = 872.49\ \text{cm}^4$$



$$Q_E = \bar{y}_1' A' = (2.55265)(5.1053)(8) = 104.26\ \text{cm}^3$$

$$Q_D = \bar{y}_2' A' = (5.8947)(6)(2) = 70.74\ \text{cm}^3$$

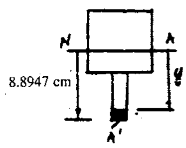
$$\tau = \frac{VQ}{It}$$



$$\tau_E = \frac{35(10^3)(104.26)}{872.49(8)} = 523\ \text{N/cm}^2 = 5.23\ \text{MPa}$$

$$(\tau_D)_{t=2\ \text{cm}} = \frac{35(10^3)(70.74)}{872.49(2)} = 1419\ \text{N/cm}^2 = 14.19\ \text{MPa}$$

$$(\tau_D)_{t=8\ \text{cm}} = \frac{35(10^3)(70.74)}{872.49(8)} = 355\ \text{N/cm}^2 = 3.55\ \text{MPa}$$



$$A' = 2(8.8947 - y)$$

$$\bar{y}' = y + \frac{(8.8947 - y)}{2} = \frac{(8.8947 + y)}{2}$$

$$Q = \bar{y}'A' = 79.1157 - y^2$$

$$\tau = \frac{VQ}{It} = \frac{35(79.1157 - y^2)}{872.49(2)} = 1.586866 - 0.0200575y^2$$

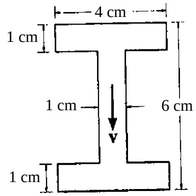
$$V = \int \tau\, dA \qquad dA = 2\, dy$$

$$V = \int (1.586866 - 0.0200575y^2)\, 2\, dy$$

$$= \int_{2.8947}^{8.8947} (3.173732 - 0.040115y^2)\, dy$$

$$= 9.96\ \text{kN}$$ **Ans**

7-18. The beam is made from a polymer and is subjected to a shear of $V = 7$ kN. Determine the maximum shear stress in the beam and plot the shear-stress distribution over the cross section. Report the values of the shear stress every 0.5 cm of beam depth.



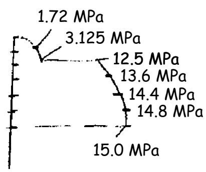
$$I = \frac{1}{12}(1)(4)^3 + 2[\frac{1}{12}(4)(1)^3 + 4(1)(2.5)^2] = 56 \text{ cm}^4$$

$$\tau_1 = \frac{VQ}{It} = \frac{7(2.75)(4)(0.5)}{56(4)} = 1.72 \text{ MPa}$$

$$\tau_{2^+} = \frac{VQ}{It} = \frac{7(2.5)(4)(1)}{56(4)} = 3.125 \text{ MPa}$$

$$\tau_{2^-} = \frac{VQ}{It} = \frac{7(2.5)(4)(1)}{56(1)} = 12.5 \text{ MPa}$$

$$\tau_3 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.75)(1)(0.5)]}{56(1)} = 13.6 \text{ MPa}$$

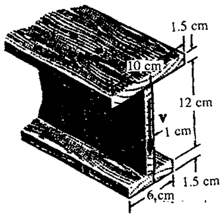


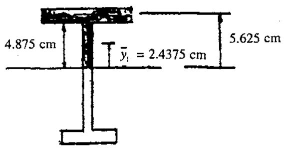
$$\tau_4 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.5)(1)(1)]}{56(1)} = 14.4 \text{ MPa}$$

$$\tau_5 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.25)(1)(1.5)]}{56(1)} = 14.8 \text{ MPa}$$

$$\tau_{max} = \tau_5 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1)(1)(2)]}{56(1)} = 15.0 \text{ MPa}$$ **Ans**

7-37. The beam is constructed from three boards. Determine the maximum shear V that it can sustain if the allowable shear stress for the wood is $\tau_{allow} = 4$ MPa. What is the required spacing s of the nails if each nail can resist a shear force of 800 N?





$$\bar{y} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ cm}$$

$$I = \frac{1}{12}(10)(1.5^3) + 10(1.5)(6.375 - 0.75)^2 + \frac{1}{12}(1)(12^3) + (1)(12)(7.5 - 6.375)^2$$

$$+ \frac{1}{12}(6)(1.5^3) + (1.5)(6)(14.25 - 6.375)^2 = 1196.4375 \text{ cm}^4$$

$$Q_{max} = \Sigma\bar{y}'A' = 5.625(10)(1.5) + 2.4375(4.875)(1) = 96.258 \text{ cm}^3$$

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$4(10^2) = \frac{V(96.258)}{1196.4375(1)}(10^3)$$

$V = 4.97$ kN **Ans**

$$Q_t = \bar{y}_t'A' = 5.625(10)(1.5) = 84.375 \text{ cm}^3$$

$$Q_b = \bar{y}_b'A' = 7.875(6)(1.5) = 70.875 \text{ cm}^3$$

$$q_t = \frac{4.9718(10^3)(84.375)}{1196.4375} = 350.62 \text{ N/cm}$$

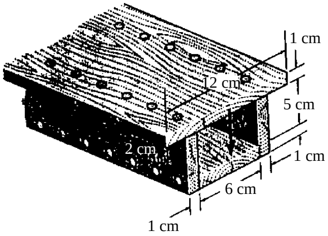
$$q_b = \frac{4.9718(10^3)(70.875)}{1196.4375} = 294.52 \text{ N/cm}$$

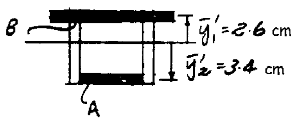
$$s = \frac{F}{q}$$

$s_t = \frac{800}{350.62} = 2.28$ cm **Ans**

$s_b = \frac{800}{294.52} = 2.72$ cm **Ans**

7-47. The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 cm. If each nail can resist a shear of 200 N, determine the greatest shear V that can be applied to the beam without causing failure of the nails.





$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5\,(12)(1) + 2\,(4)(6)(1) + (6.5)(6)(1)}{12(1) + 2(6)(1) + (6)(1)} = 3.1 \text{ cm}$$

$$I = \frac{1}{12}(12)(1^3) + 12(1)(3.1 - 0.5)^2$$

$$+ 2(\frac{1}{12})(1)(6^3) + 2(1)(6)(4 - 3.1)^2$$

$$+ \frac{1}{12}(6)(1^3) + 6(1)(6.5 - 3.1)^2 = 197.7 \text{ cm}^4$$

$$Q_B = \bar{y}_1' A' = 2.6(12)(1) = 31.2 \text{ cm}^3$$

$$q_B = \frac{1}{2}(\frac{VQ_B}{I}) = \frac{V(31.2)}{2(197.7)} = 0.0789\,V$$

$$q_B\, s = 0.0789V\,(2) = 200$$

$V = 1267.43$ N (controls) **Ans**

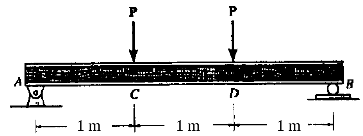
$$Q_A = \bar{y}_2' A' = 3.4(6)(1) = 20.4 \text{ cm}^3$$

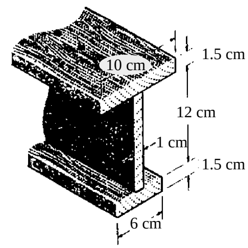
$$q_A = \frac{1}{2}(\frac{VQ_A}{I}) = \frac{V(20.4)}{2(197.7)} = 0.0516V$$

$$q_A\, s = 0.0516V\,(2) = 200$$

$V = 1937.98$ N

7-53. The beam is constructed from three boards. Determine the maximum loads P that it can support if the allowable shear stress for the wood is $\tau_{\text{allow}} = 3$ MPa. What is the required spacing s of the nails used to hold the top and bottom flanges to the web if each nail can resist a shear force of 2000 N?





As shown on FBD

$V_{\max} = P,\ V_{AC} = V_{DB} = P,\ V_{CD} = 0$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ cm}$$

$$I = \frac{1}{12}(10)(1.5^3) + 10(1.5)(6.375 - 0.75)^2 + \frac{1}{12}(1)(12^3)$$

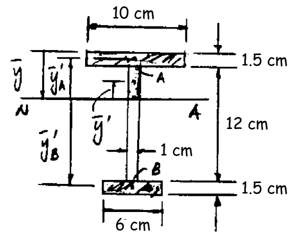
$$+ 1(12)(7.5 - 6.375)^2 + \frac{1}{12}(6)(1.5^3) + 6(1.5)(7.875^2)$$

$$= 1196.4375 \text{ cm}^4$$

$Q_A = \bar{y}_A' A' = 5.625(10)(1.5) = 84.375 \text{ cm}^3$

$Q_B = \bar{y}_B' A' = 7.875(6)(1.5) = 70.875 \text{ cm}^3$

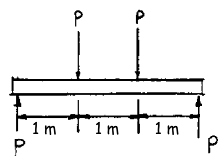
$$Q_{\max} = \Sigma \bar{y}' A' = 5.625(10)(1.5) = \frac{4.875}{2}(4.875)(1) = 96.2578 \text{ cm}^3$$



Maximum shear stress:

$$\tau_{\text{allow}} = \frac{VQ_{\max}}{It}; \qquad 300 = \frac{P(96.2578)}{1196.4375(1)}$$

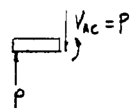
$P = 3728.85 \text{ N} = 3.73 \text{ kN}$ **Ans**



For region AC and BD

$$q_A = \frac{VQ_A}{I} = \frac{3728.85(84.375)}{1196.4375} = 262.97 \text{ N/cm}$$

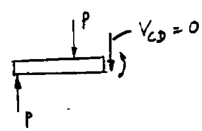
$$q_B = \frac{VQ_B}{I} = \frac{3728.85(70.875)}{1196.4375} = 220.89 \text{ N/cm}$$



Nail spacing at top and bottom flange

$$s_{\text{top}} = \frac{F}{q_A} = \frac{2000}{262.97} = 7.6 \text{ cm}$$ (Regions AC and BD) **Ans**

$$s_{\text{bottom}} = \frac{F}{q_B} = \frac{2000}{220.89} = 9.05 \text{ cm}$$ (Regions AC and BD) **Ans**



For region CD, theoretically no nails are required to hold the flange and web together since $V_{CD} = 0$. However, it is advisible to provide some nails within this region. **Ans**