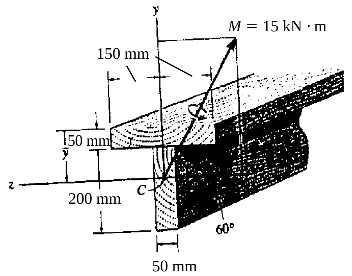
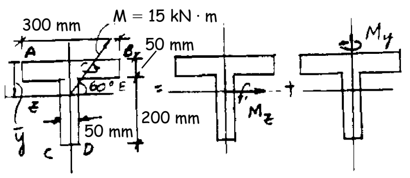
6-105. The T-beam is subjected to a bending moment of $M = 15$ kN · m directed as shown. Determine the maximum bending stress in the beam and the orientation of the neutral axis. The location $\bar{y}$ of the centroid, C, must be determined.





$M_y = 15 \sin 60° = 12.99$ kN · m

$M_z = -15 \sin 60° = -7.5$ kN · m

$$\bar{y} = \frac{(25)(300)(50) + (150)(200)(50)}{(300)(50) + (200)(50)} = 75 \text{ mm}$$

$$I_y = \frac{1}{12}(50)(300)^3 + \frac{1}{12}(200)(50^3) = 114.58 \times 10^6 \text{ mm}^4$$

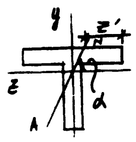
$$I_z = \frac{1}{12}(300)(50)^3 + (300)(50)(50^2) + \frac{1}{12}(50)(200^3) + (50)(200)(75^2)$$
$$= 130.21 \times 10^6 \text{ mm}^4$$

$$\sigma = \frac{M_c y}{I_z} + \frac{M_y c}{I_y}$$

$$\sigma_A = \frac{-(-7.5)(75)(10^6)}{130.21 \times 10^6} + \frac{12.99(10^6)(150)}{114.58 \times 10^6} = 21.33 \text{ MPa}$$ **Ans**

$$\sigma_D = \frac{-(-7.5)(-175)(10^6)}{130.21 \times 10^6} + \frac{12.99(-25)(10^6)}{114.58 \times 10^6} = -12.91 \text{ MPa}$$

$$\sigma_B = \frac{-(-7.5)(75)(10^6)}{130.21 \times 10^6} + \frac{12.99(10^6)(-150)}{114.58 \times 10^6} = -12.69 \text{ MPa}$$



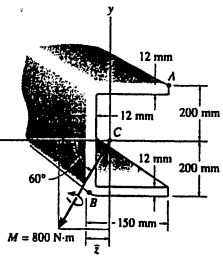
$$\frac{z}{12.69} = \frac{300 - z}{21.33}$$

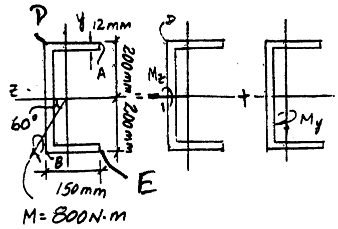
$\therefore \quad z = 111.9$ mm

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta = \frac{130.21 \times 10^6}{114.58 \times 10^6} \tan(-60°)$$

$\therefore \quad \alpha = -63.1°$ **Ans**

6-107. The resultant moment acting on the cross section of the aluminum strut has a magnitude of $M = 800$ N · m and is directed as shown. Determine the maximum bending stress in the strut. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.





$M_z = 800 \cos 60^\circ = 400 \text{ N} \cdot \text{m}$

$M_y = -800 \sin 60^\circ = -692.82 \text{ N} \cdot \text{m}$

$$\bar{z} = \frac{400\,(12)(6) + 2\,(138)(12)(81)}{400\,(12) + 2\,(138)(12)} = 36.6 \text{ mm} \qquad \textbf{Ans}$$

$$I_z = \frac{1}{12}(0.15)(0.4^3) - \frac{1}{12}(0.138)(0.376^3) = 0.18869\,(10^{-3})\text{ m}^4$$

$$I_y = \frac{1}{12}(0.4)(0.012^3) + (0.4)(0.012)(0.03062^2)$$

$$+ 2\left[\frac{1}{12}(0.012)(0.138^3) + (0.138)(0.012)(0.04438^2)\right] = 16.3374\,(10^{-6})\text{ m}^4$$

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

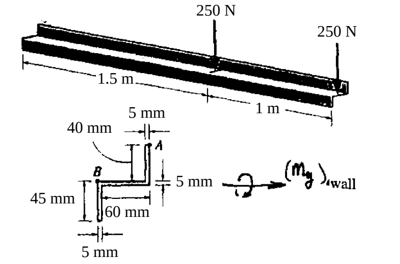
$$\sigma_A = \frac{-(400)(0.2)}{0.18869\,(10^{-3})} + \frac{(-692.82)(-0.11338)}{16.3374\,(10^{-6})} = 4.38 \text{ MPa}$$

$$\sigma_B = \frac{-(400)(-0.2)}{0.18869\,(10^{-3})} + \frac{(-692.82)(0.036621)}{16.3374\,(10^{-6})} = -1.13 \text{ MPa}$$

$$\sigma_D = \frac{-(400)(0.2)}{0.18869\,(10^{-3})} + \frac{(-692.82)(0.036621)}{16.3374\,(10^{-6})} = -1.977 \text{ MPa}$$

$$\sigma_E = -\frac{(400)(-0.2)}{0.18869(10^{-3})} + \frac{(-692.82)(-0.11338)}{16.3374(10^{-6})} = 5.23 \text{ MPa} \qquad \textbf{Ans}$$

***6-112.** The cantilevered beam is made from the Z section having the cross-section shown. If it supports the two loadings, determine the bending stress at the wall in the beam at point A. Use the result of Prob. 6–111.



$(M_y)_{wall} = 250(1.5) + 250(2.5) = 1000\ \text{N}\cdot\text{m}$

$I_y = \frac{1}{12}(65)(5^3) + 2[\frac{1}{12}(5)(40^3) + (5)(40)(22.5)^2] = 256510.42\ \text{mm}^4$

$I_z = \frac{1}{12}(5)(65^3) + 2[\frac{1}{12}(40)(5)^3 + (5)(40)(30^2)] = 475260.48\ \text{mm}^4$

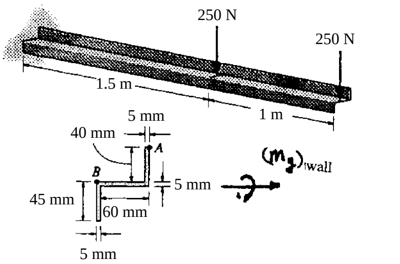
$I_{yz} = 2[(30)(22.5)(40)(5)] = 270000\ \text{mm}^4$

Using the equation developed in Prob. 6 - 111,

$$\sigma = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z$$

$$\sigma_A = \frac{\{-[0 + 1000(10^3)(270000)](32.5) + [1000(10^3)(475260.48 + 0](44.25)\}}{[(256510.42)(47526.48) - 270000^2]} = 250.1\ \text{MPa} \qquad \textbf{Ans}$$

6-113. The cantilevered beam is made from the Z section having the cross-section shown. If it supports the two loadings, determine the bending stress at the wall in the beam at point B. Use the result of Prob. 6–111.



$(M_y)_{wall} = 250(1.5) + 250(2.5) = 1000\ \text{N}\cdot\text{m}$

$I_y = \frac{1}{12}(65)(5^3) + 2[\frac{1}{12}(5)(40^3) + (5)(40)(22.5)^2] = 256510.42\ \text{mm}^4$

$I_z = \frac{1}{12}(5)(65^3) + 2[\frac{1}{12}(40)(5)^3 + (5)(40)(30^2)] = 475260.48\ \text{mm}^4$

$I_{yz} = 2[(30)(22.5)(40)(5)] = 270000\ \text{mm}^4$

Using the equation developed in Prob. 6 - 111,

$$\sigma = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z$$

$$\sigma_B = \frac{-[0 + 1000(10^3)(270000)](-32.5) + [1000(10^3)(475260.48 + 0](2.5)]}{[(256510.42)(47526.48) - 270000^2]}$$

$= 203.33\ \text{MPa}$ **Ans**