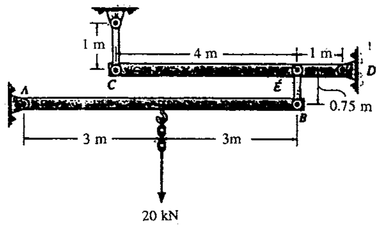
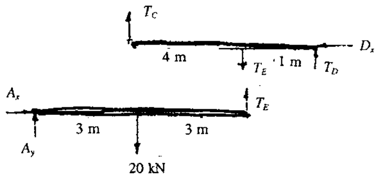
***4-8.** The assembly consists of two rigid bars that are originally horizontal. They are supported by pins and 6-mm-diameter A-36 steel rods. If the vertical load of 20 kN is applied to the bottom bar AB, determine the displacement C, B, and E. ($E_{A\text{-}36} = 210$ GPa.)



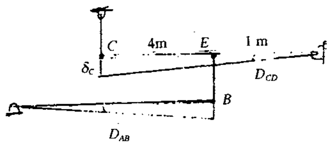


$\circlearrowleft + \Sigma M_A = 0; \quad T_B(6) - 20(3) = 0$

$T_B = 10$ kN

$\circlearrowleft + \Sigma M_D = 0; \quad 10(1) - T_C(5) = 0$

$T_C = 2$ kN



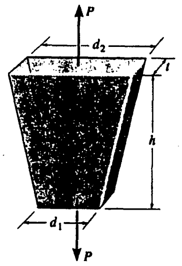
$$\delta_{B/E} = \frac{PL}{AE} = \frac{10(0.75)(10^6)}{\frac{\pi}{4}(6)^2(210)(10^3)} = 1.263 \text{ mm}$$

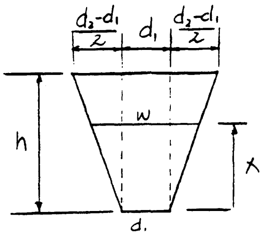
$$\delta_C = \frac{PL}{AE} = \frac{2(1)(10^6)}{\frac{\pi}{4}(6)^2(210)(10^3)} = 0.3368 \text{ mm} \qquad \textbf{Ans}$$

$$\delta_E = \left(\frac{1}{5}\right)\delta_C = \frac{1}{5}(0.3368) = 0.06736 \text{ mm} \qquad \textbf{Ans}$$

$$\delta_B = \delta_E + \delta_{B/E} = 0.06736 + 1.263 = 1.3304 \text{ mm} \qquad \textbf{Ans}$$

4–27 Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.





$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

$$\delta = \int \frac{P(x)\,dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\frac{[d_1 h + (d_2 - d_1)x]t}{h}}$$

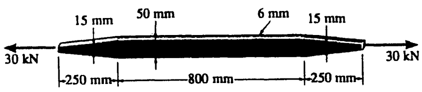
$$= \frac{Ph}{E\,t}\int_0^h \frac{dx}{d_1 h + (d_2 - d_1)x}$$

$$= \frac{Ph}{E\,t\,d_1 h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1 h}x} = \frac{Ph}{E\,t\,d_1 h}\left(\frac{d_1 h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1 h}x\right)\right]\Big|_0^h$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right] \qquad \textbf{Ans}$$

***4–28** Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. E_{al} = 70 GPa. *Hint:* Use the result of Prob. 4–27.

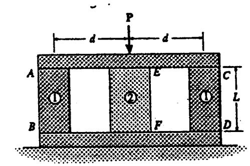


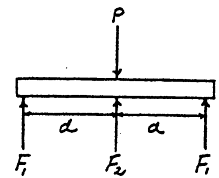
$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05 - 0.015)}\left(\ln\frac{50}{15}\right) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$= 2.37$ mm **Ans**

***4-60.** The assembly consists of two posts AB and CD made from material 1 having a modulus of elasticity of E_1 and each a cross-sectional area A_1, and a central post EF made from material 2 having a modulus of elasticity E_2 and a cross-sectional area A_2. If post EF is to be replaced by one having a material 1, determine the required cross-sectional area of this new post so that both assemblies deform the same amount when loaded.





$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad [1]$$

Compatibility :

$$\delta_{in} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad [2]$$

Solving Eq. [1] and [2] yields :

$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{in} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

Compatibility : When material 2 has been replaced by material 1 for central posts, then

$$\delta_{final} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2' E_1} \qquad F_2 = \left(\frac{A_2'}{A_1}\right) F_1 \qquad [3]$$

Solving for F_1 from Eq. [1] and [3]

$$F_1 = \left(\frac{A_1}{2A_1 + A_2'}\right) P$$

$$\delta_{final} = \frac{F_1 L}{A_1 E_1} = \frac{\left(\frac{A_1}{2A_1 + A_2'}\right) PL}{A_1 E_1} = \frac{PL}{E_1 (2A_1 + A_2')}$$

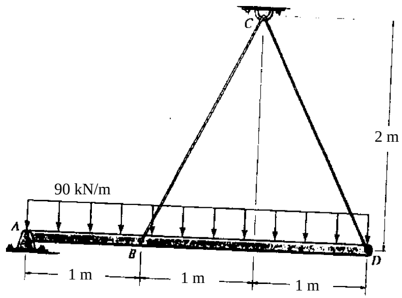
Requires,

$$\delta_{in} = \delta_{final}$$

$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_1 (2A_1 + A_2')}$$

$$A_2' = \left(\frac{E_2}{E_1}\right) A_2 \qquad \textbf{Ans}$$

***4-68.** The rigid bar supports the uniform distributed load of 90 kN/m. Determine the force in each cable if each cable has a cross-sectional area of 30 mm², and $E = 200(10^3)$ MPa.



$$\circlearrowleft + \Sigma M_A = 0, \qquad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(1) - 270(1.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)(3) = 0$$

$$\theta = \tan^{-1}\frac{2}{2} = 45°$$

$$L_{B'C}^2 = (1)^2 + (2.8284)^2 - 2(1)(2.8284)\cos\theta'$$

Also,

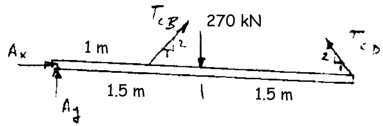
$$L_{D'C}^2 = (3)^2 + (2.8284)^2 - 2(3)(2.8284)\cos\theta' \qquad (2)$$

Thus, eliminating $\cos\theta'$,

$$-L_{B'C}^2(0.1768) + 1.5910 = -L_{DC'}^2(0.0589) + 1.002$$

$$L_{B'C}^2(0.1768) = (0.0589)L_{DC'}^2 + 0.589$$

$$L_{B'C}^2 = 0.333L_{DC'}^2 + 3.331$$



But,

$$L_{B'C} = \sqrt{5} + \delta_{BC}; \qquad L_{D'C} = \sqrt{5} + \delta_{DC}$$

Neglect squares or δ's since small strain occurs,

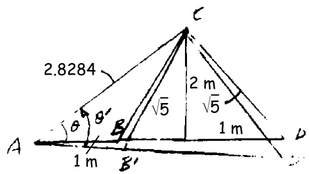
$$L_{B'C}^2 = (\sqrt{5} + \delta_{BC})^2 = 5 + 2\sqrt{5}\delta_{BC}$$

$$L_{D'C}^2 = (\sqrt{5} + \delta_{DC})^2 = 5 + 2\sqrt{5}\delta_{DC}$$

$$5 + 2\sqrt{5}\delta_{BC} = 0.333(5 + 2\sqrt{5}\delta_{DC}) + 3.331$$

$$2\sqrt{5}\delta_{BC} = 0.333(2\sqrt{5})\delta_{BC}$$

$$\delta_{DC} = 3\delta_{BC}$$



Thus,

$$\frac{T_{CD}\sqrt{5}}{AE} = 3\frac{T_{CB}\sqrt{5}}{AE}$$

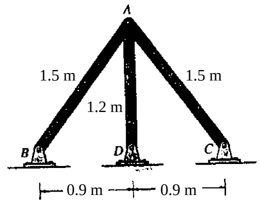
$$T_{CD} = 3T_{CB}$$

From Eq. (1),

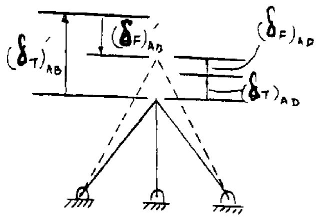
$T_{CD} = 135.84$ kN **Ans**

$T_{CB} = 45.28$ kN **Ans**

4-83. The three bars are made of A-36 steel and form a pin-connected truss. If the truss is constructed when $T_1 = 20°C$, determine the vertical displacement of joint A when $T_2 = 60°C$. Each bar has a cross-sectional area of $1250\ mm^2$. $E = 200$ GPa, $\alpha = 12(10^{-6})/°C$.



$$(\delta_T')_{AB} - (\delta_F')_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD} \qquad (1)$$



However, $\delta_{AB} = \delta'_{AB}\cos\theta$;

$$\delta'_{AB} = \frac{\delta_{AB}}{\cos\theta} = \frac{5}{4}\delta_{AB}$$

Substitute into Eq. (1)

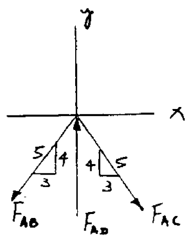
$$\frac{5}{4}(\delta_T)_{AB} - \frac{5}{4}(\delta_F)_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD}$$

$$\frac{5}{4}[12(10^{-6})(60-20)(1.5)(10^3) - \frac{F_{AB}(1.5)(10^3)}{1250(200)(10^3)}$$

$$= 12(10^{-6})(60-20)(1.2)(10^3) + \frac{F_{AD}(1.2)(10^3)}{1250(200)(10^3)}$$

$$900000 - 6F_{AB} = 576000 + 4.8F_{AD}$$

$$4.8F_{AD} + 6F_{AB} = 324000 \qquad (2)$$



$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad \frac{3}{5}F_{AC} - \frac{3}{5}F_{AB} = 0; \qquad F_{AC} = F_{AB}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{AD} - 2(\frac{4}{5}F_{AB}) = 0;$$

$$F_{AD} = 1.6F_{AB} \qquad (3)$$

Solving Eqs. (2) and (3) yields :

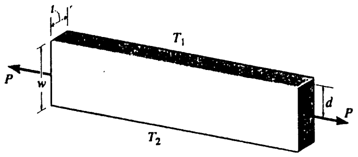
$F_{AB} = 23.68$ kN; $\qquad F_{AD} = 37.89$ kN

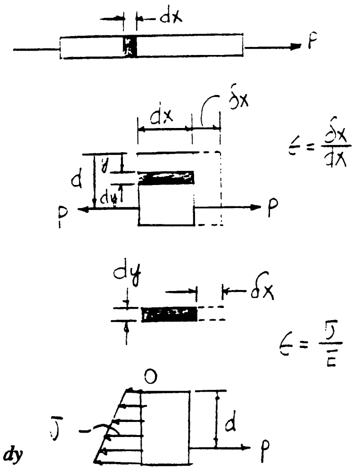
$$(\delta_A)_v = (\delta_T)_{AD} + (\delta_T)_{AD}$$

$$= 12(10^{-6})(60-20)(1.2)(10^3) + \frac{37.89(10^3)(1.2)(10^3)}{1250(200)(10^3)}$$

$$= 0.758\ \text{mm} \uparrow \qquad \textbf{Ans}$$

4–86 The metal strap has a thickness t and width w and is subjected to a temperature gradient T_1 to T_2 ($T_1 < T_2$). This causes the modulus of elasticity for the material to vary linearly from E_1 at the top to a smaller amount E_2 at the bottom. As a result, for any vertical position y, $E = [(E_2 - E_1)/w]\,y + E_1$. Determine the position d where the axial force P must be applied so that the bar stretches uniformly over its cross section.





$$\varepsilon = \text{constant} = \varepsilon_0$$

$$\varepsilon_0 = \frac{\sigma}{E} = \frac{\sigma}{\left(\left(\frac{E_2 - E_1}{w}\right)y + E_1\right)}$$

$$\sigma = \varepsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right)$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P - \int_A \sigma\, dA = 0$$

$$P = \int_0^w \sigma\, t\, dy = \int_0^w \varepsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right) t\, dy$$

$$P = \varepsilon_0\, t\left(\frac{(E_2 - E_1)w}{2} + E_1 w\right) = \varepsilon_0 t\left(\frac{E_2 + E_1}{2}\right)w$$

$$\curvearrowleft + \Sigma M_0 = 0; \qquad P(d) - \int_A y\,\sigma dA = 0$$

$$\varepsilon_0 t\left(\frac{E_2 + E_1}{2}\right)wd = \int_0^w \varepsilon_0\left(\left(\frac{E_2 - E_1}{w}\right)y^2 + E_1 y\right) t\, dy$$

$$\varepsilon_0\, t\left(\frac{E_2 + E_1}{2}\right)wd = \varepsilon_0 t\left(\frac{E_2 - E_1}{3}w^2 + \frac{E_1}{2}w^2\right)$$

$$\left(\frac{E_2 + E_1}{2}\right)d = \frac{1}{6}(2E_2 + E_1)w$$

$$d = \left(\frac{2E_2 + E_1}{3(E_2 + E_1)}\right)w \qquad \textbf{Ans}$$