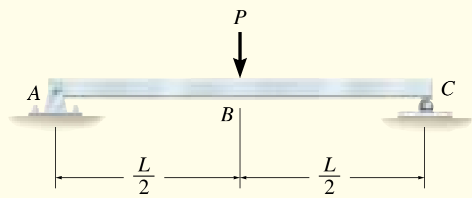
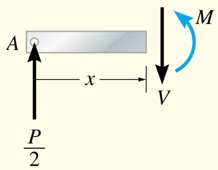
EXAMPLE 6.1

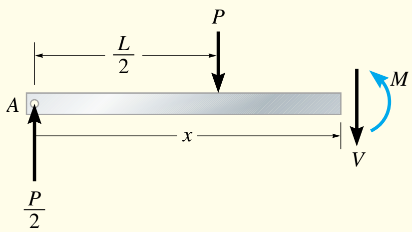
Draw the shear and moment diagrams for the beam shown in Fig. 6–4*a*.



(a)



(b)



(c)

Solution

Support Reactions. The support reactions have been determined, Fig. 6–4*d*.

Shear and Moment Functions. The beam is sectioned at an arbitrary distance x from the support A, extending within region AB, and the free-body diagram of the left segment is shown in Fig. 6–4*b*. The unknowns **V** and **M** are indicated acting in the *positive sense* on the right-hand face of the segment according to the established sign convention. Applying the equilibrium equations yields

$$+\uparrow \Sigma F_y = 0; \qquad V = \frac{P}{2} \tag{1}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M = \frac{P}{2}x \tag{2}$$

A free-body diagram for a left segment of the beam extending a distance x within region BC is shown in Fig. 6–4*c*. As always, **V** and **M** are shown acting in the positive sense. Hence,

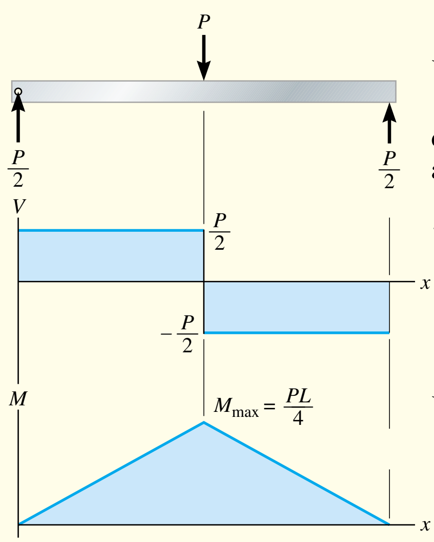
$$+\uparrow \Sigma F_y = 0; \qquad \frac{P}{2} - P - V = 0$$

$$V = -\frac{P}{2} \tag{3}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + P\left(x - \frac{L}{2}\right) - \frac{P}{2}x = 0$$

$$M = \frac{P}{2}(L - x) \tag{4}$$

The shear diagram represents a plot of Eqs. 1 and 3, and the moment diagram represents a plot of Eqs. 2 and 4, Fig. 6–4*d*. These equations can be checked in part by noting that $dV/dx = -w$ and $dM/dx = V$ in each case. (These relationships are developed in the next section as Eqs. 6–1 and 6–2.)

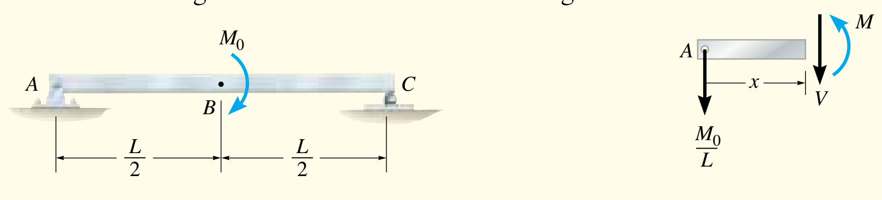


(d)

Fig. 6–4

EXAMPLE 6.2

Draw the shear and moment diagrams for the beam shown in Fig. 6–5*a*.



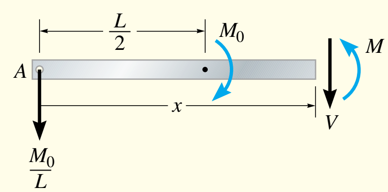
Solution

Support Reactions. The support reactions have been determined in Fig. 6–5*d*.

Shear and Moment Functions. This problem is similar to the previous example, where two x coordinates must be used to express the shear and moment in the beam throughout its length. For the segment within region AB, Fig. 6–5*b*, we have

$$+\uparrow \Sigma F_y = 0; \qquad V = -\frac{M_0}{L}$$

$$\circlearrowright + \Sigma M = 0; \qquad M = -\frac{M_0}{L}x$$



(c)

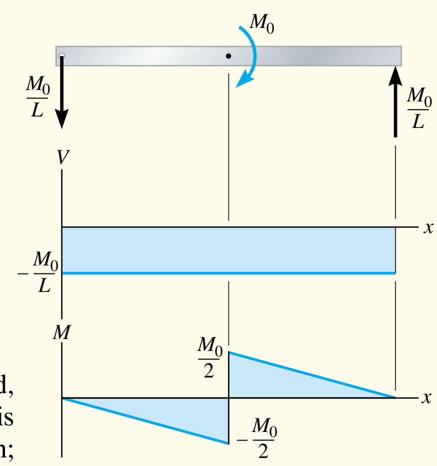
And for the segment within region BC, Fig. 6–5*c*,

$$+\uparrow \Sigma F y = 0; \qquad V = -\frac{M_0}{L}$$

$$\circlearrowright + \Sigma M = 0; \qquad M = M_0 - \frac{M_0}{L}x$$

$$M = M_0\left(1 - \frac{x}{L}\right)$$

Shear and Moment Diagrams. When the above functions are plotted, the shear and moment diagrams shown in Fig. 6–5*d* are obtained. In this case, notice that the shear is constant over the entire length of the beam; i.e., it is not affected by the couple moment $\mathbf{M}_0$ acting at the center of the beam. Just as a force creates a jump in the shear diagram, Example 6–1, a couple moment creates a jump in the moment diagram.



(d)

Fig. 6–5

EXAMPLE 6.3

Draw the shear and moment diagrams for the beam shown in Fig. 6–6*a*.

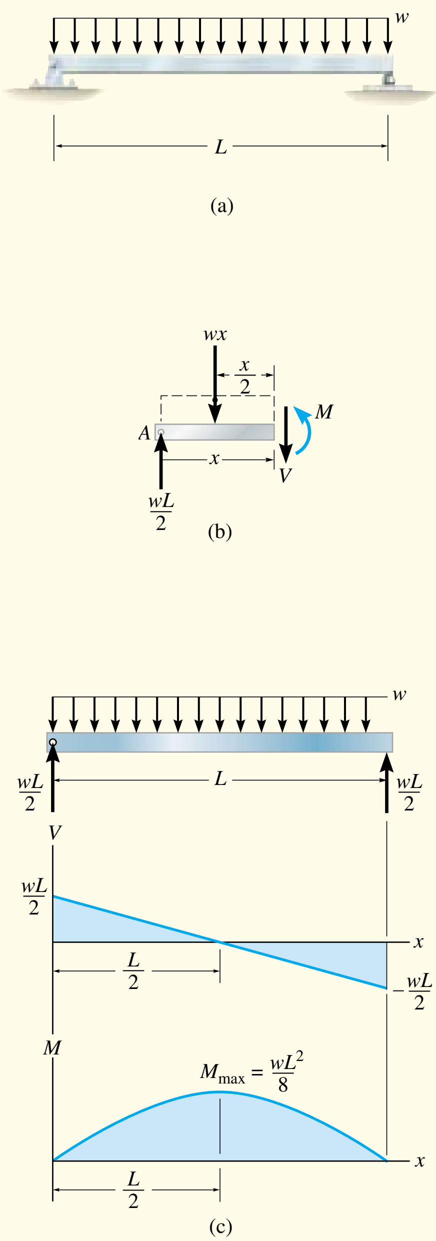


Fig. 6–6

Solution

Support Reactions. The support reactions have been computed in Fig. 6–6*c*.

Shear and Moment Functions. A free-body diagram of the left segment of the beam is shown in Fig. 6–6*b*. The distributed loading on this segment is represented by its resultant force only *after* the segment is isolated as a free-body diagram. Since the segment has a length x, the *magnitude* of the *resultant force* is wx. This force acts through the centroid of the area comprising the distributed loading, a distance of $x/2$ from the right end. Applying the two equations of equilibrium yields

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \tag{1}$$

$$\circlearrowright + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \tag{2}$$

These results for V and M can be checked by noting that $dV/dx = -w$. This is indeed correct, since positive w acts downward. Also, notice that $dM/dx = V$.

Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 6–6*c* are obtained by plotting Eqs. 1 and 2. The point of *zero shear* can be found from Eq. 1:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

$$x = \frac{L}{2}$$

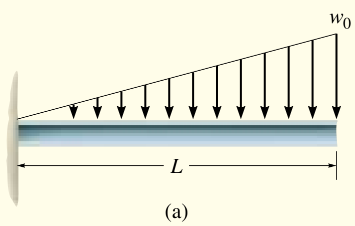
From the moment diagram, this value of x happens to represent the point on the beam where the *maximum moment* occurs, since by Eq. 6–2, the slope $V = 0 = dM/dx$. From Eq. 2, we have

$$M_{max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

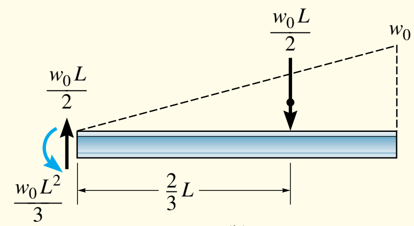
$$= \frac{wL^2}{8}$$

EXAMPLE 6.4

Draw the shear and moment diagrams for the beam shown in Fig. 6–7*a*.



(a)

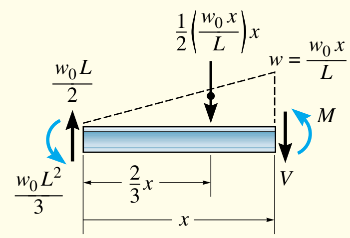


(b)

Solution

Support Reactions. The distributed load is replaced by its resultant force and the reactions have been determined as shown in Fig. 6–7*b*.

Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–7*c*. Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram, Fig. 6–7*c*. Thus,



(c)

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{2} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad \frac{w_0 L^2}{3} - \frac{w_0 L}{2}(x) + \frac{1}{2}\left(\frac{w_0 x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

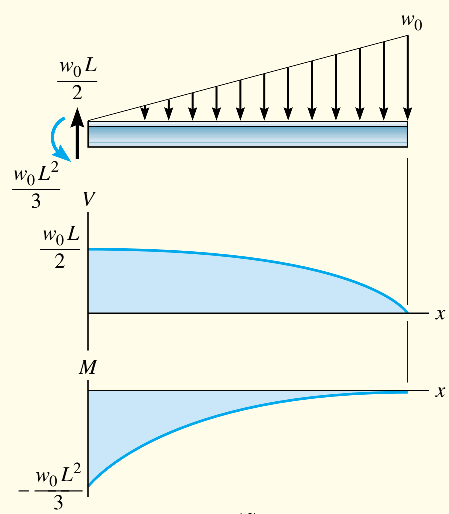
$$M = \frac{w_0}{6L}(-2L^3 + 3L^2 x - x^3) \qquad (2)$$

These results can be checked by applying Eqs. 6–1 and 6–2, that is,

$$w = -\frac{dV}{dx} = -\frac{w_0}{2L}(0 - 2x) = \frac{w_0 x}{L} \qquad \text{OK}$$

$$V = \frac{dM}{dx} = \frac{w_0}{6L}(-0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$

Shear and Moment Diagrams. The graphs of Eqs. 1 and 2 are shown in Fig. 6–7*d*.



(d)

Fig. 6–7

EXAMPLE 6.5

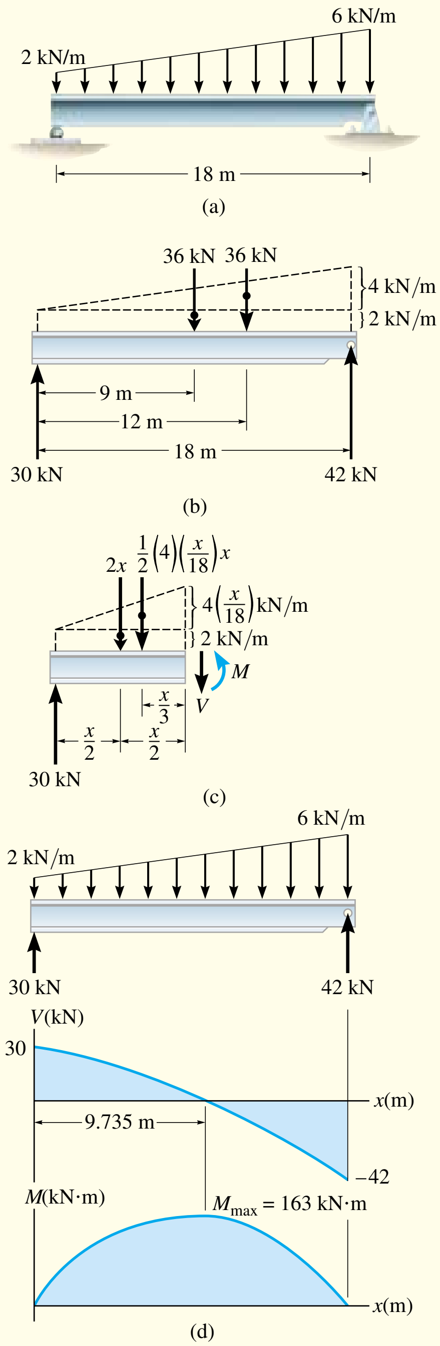


Fig. 6–8

Draw the shear and moment diagrams for the beam shown in Fig. 6–8*a*.

Solution

Support Reactions. The distributed load is divided into triangular and rectangular component loadings and these loadings are then replaced by their resultant forces. The reactions have been determined as shown on the beam's free-body diagram, Fig. 6–8*b*.

Shear and Moment Functions. A free-body diagram of the left segment is shown in Fig. 6–8*c*. As above, the trapezoidal loading is replaced by rectangular and triangular distributions. Note that the intensity of the triangular load at the section is found by proportion. The resultant force and the location of each distributed loading are also shown. Applying the equilibrium equations, we have

$$+\uparrow\Sigma F_y = 0; \quad 30\text{ kN} - (2\text{ kN/m})x - \frac{1}{2}(4\text{ kN/m})\left(\frac{x}{18\text{ m}}\right)x - V = 0$$

$$V = \left(30 - 2x - \frac{x^2}{9}\right)\text{kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0;$$

$$-30\text{ kN}(x) - (2\text{ kN/m})x\left(\frac{x}{2}\right) + \frac{1}{2}(4\text{ kN/m})\left(\frac{x}{18\text{ m}}\right)x\left(\frac{x}{3}\right) + M = 0$$

$$M = \left(30x - x^2 - \frac{x^3}{27}\right)\text{kN}\cdot\text{m} \qquad (2)$$

Equation 2 may be checked by noting that $dM/dx = V$, that is, Eq. 1. Also, $w = -dV/dx = 2 + \frac{2}{9}x$. This equation checks, since when $x = 0$, $w = 2$ kN/m, and when $x = 18$ m, $w = 6$ kN/m, Fig. 6–8*a*.

Shear and Moment Diagrams. Equations 1 and 2 are plotted in Fig. 6–8*d*. Since the point of maximum moment occurs when $dM/dx = V = 0$, then, from Eq. 1,

$$V = 0 = 30 - 2x - \frac{x^2}{9}$$

Choosing the positive root,

$$x = 9.735\text{ m}$$

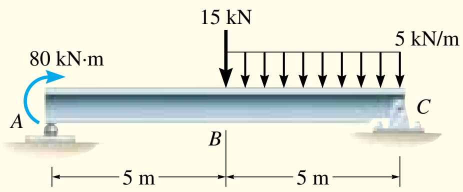
Thus, from Eq. 2,

$$M_{max} = 30(9.735) - (9.735)^2 - \frac{(9.735)^3}{27}$$

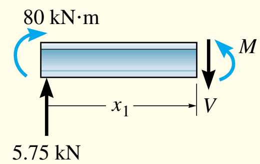
$$= 163\text{ kN}\cdot\text{m}$$

EXAMPLE 6.6

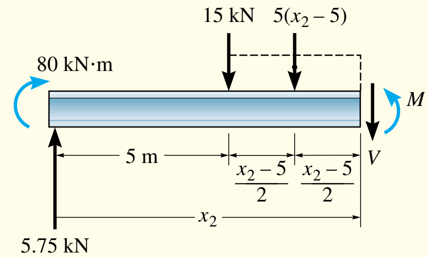
Draw the shear and moment diagrams for the beam shown in Fig. 6–9*a*.



(a)



(b)



(c)

Solution

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. 6–9*d*.

Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of x must be considered in order to describe the shear and moment functions for the entire beam.

$0 \le x_1 < 5$ m, Fig. 6–9*b*:

$$+\uparrow \Sigma F_y = 0; \qquad 5.75 \text{ kN} - V = 0$$

$$V = 5.75 \text{ kN} \tag{1}$$

$$\circlearrowleft + \Sigma M = 0; \qquad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_1 + M = 0$$

$$M = (5.75x_1 + 80) \text{ kN}\cdot\text{m} \tag{2}$$

$5 \text{ m} < x_2 \le 10$ m, Fig. 6–9*c*:

$$+\uparrow \Sigma F_y = 0; \qquad 5.75 \text{ kN} - 15 \text{ kN} - 5 \text{ kN/m}(x_2 - 5 \text{ m}) - V = 0$$

$$V = (15.75 - 5x_2) \text{ kN} \tag{3}$$

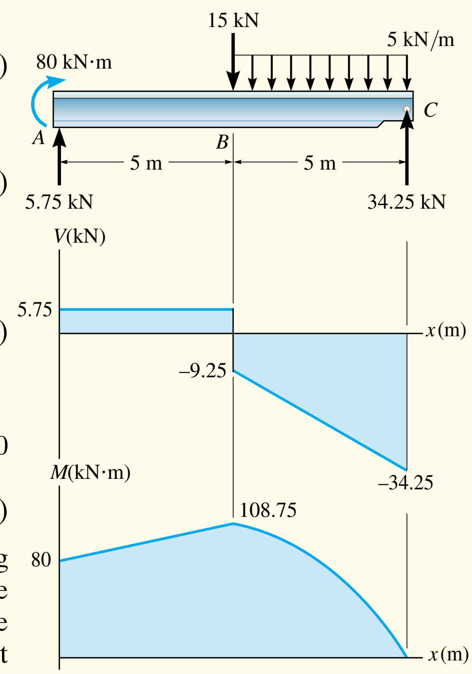
$$\circlearrowleft + \Sigma M = 0; \qquad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_2 + 15 \text{ kN}(x_2 - 5 \text{ m})$$

$$+ 5 \text{ kN/m}(x_2 - 5 \text{ m})\left(\frac{x_2 - 5 \text{ m}}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5) \text{ kN}\cdot\text{m} \tag{4}$$

These results can be checked in part by noting that by applying $w = -dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN·m; when $x_2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–9*d*.

Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. 6–9*d*.

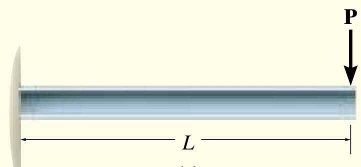


(d)

Fig. 6–9

EXAMPLE 6.7

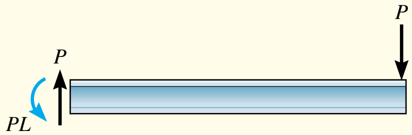
Draw the shear and moment diagrams for the beam in Fig. 6–13*a*.



(a)

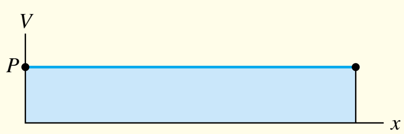
Solution

Support Reactions. The reactions are shown on a free-body diagram, Fig. 6–13*b*.



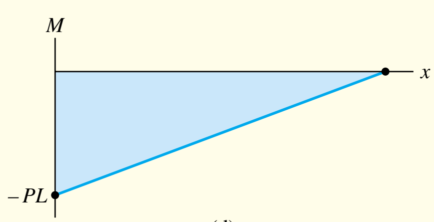
(b)

Shear Diagram. According to the sign convention, Fig. 6–3, at $x = 0$, $V = +P$ and at $x = L$, $V = +P$. These points are plotted in Fig. 6–13*b*. Since $w = 0$, Fig. 6–13*a*, the *slope* of the shear diagram will be zero $(dV/dx = -w = 0)$ at all points, and therefore a horizontal straight line connects the end points.



(c)

Moment Diagram. At $x = 0$, $M = -PL$ and at $x = L$, $M = 0$, Fig. 6–13*d*. The shear diagram indicates that the shear is constant positive and therefore the *slope* of the moment diagram will be *constant positive*, $dM/dx = V = +P$ at all points. Hence, the end points are connected by a straight positive sloped line as shown in Fig. 6–13*d*.

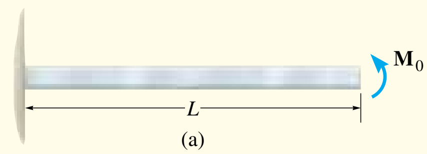


(d)

Fig. 6–13

EXAMPLE 6.8

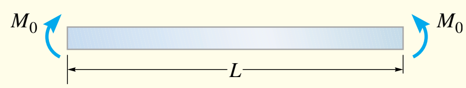
Draw the shear and moment diagrams for the beam shown in Fig. 6–14*a*.



(a)

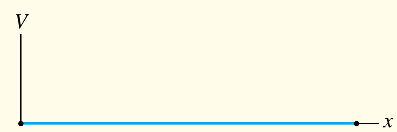
Solution

Support Reactions. The reaction at the fixed support is shown on the free-body diagram, Fig. 6–14*b*.



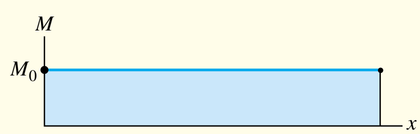
(b)

Shear Diagram. The shear $V = 0$ at each end is plotted first, Fig. 6–14*c*. Since no distributed load exists on the beam the shear diagram will have zero *slope*, at all points. Therefore, a horizontal line connects the end points, which indicates that the shear is zero throughout the beam.



(c)

Moment Diagram. The moment M_0 at the beam's end points is plotted first, Fig. 6–14*d*. From the shear diagram the *slope* of the moment diagram will be zero since $V = 0$. Therefore, a horizontal line connects the end points as shown.

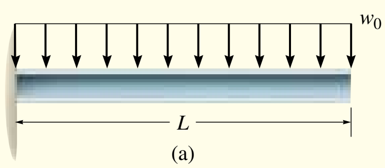


(d)

Fig. 6–14

EXAMPLE 6.9

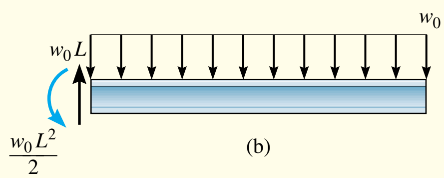
Draw the shear and moment diagrams for the beam shown in Fig. 6–15*a*.



(a)

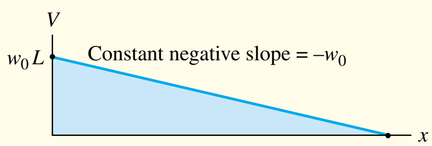
Solution

Support Reactions. The reactions at the fixed support are shown on the free-body diagram, Fig. 6–15*b*.



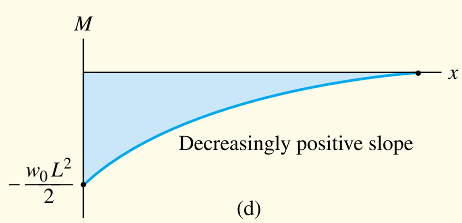
(b)

Shear Diagram. The shear at each end point, is plotted first, Fig. 6–15*c*. The distributed loading on the beam is constant positive, and so the *slope* of the shear diagram will be constant negative $(dV/dx = -w_0)$. This requires a straight negative sloped line that connects the end points.



(c)

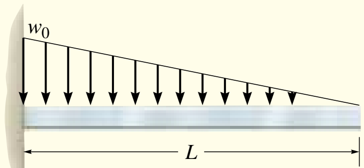
Moment Diagram. The moment at each end point is plotted first, Fig. 6–15*d*. The shear diagram indicates that V is positive and decreases from $w_0 L$ to zero, and so the moment diagram must start with a positive slope of $w_0 L$ and decrease to zero. Specifically, since the shear diagram is a straight sloping line, the moment diagram will be *parabolic*, having a decreasing slope as shown in the figure.



(d)

Fig. 6–15

EXAMPLE 6.10



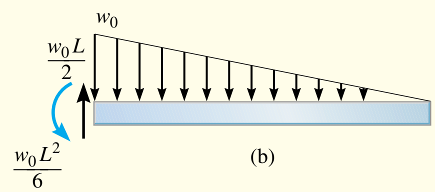
(a)

Fig. 6–16

Draw the shear and moment diagrams for the beam shown in Fig. 6–16*a*.

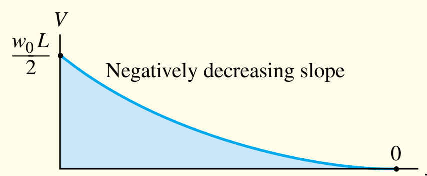
Solution

Support Reactions. The reactions at the fixed support have been calculated and are shown on the free-body diagram, Fig. 6–16*b*.



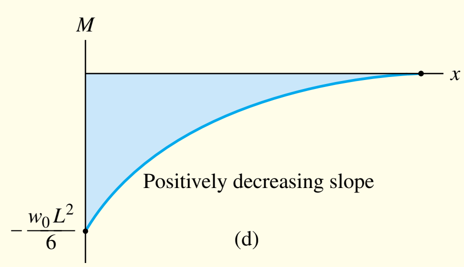
(b)

Shear Diagram. The shear at each end point is plotted first, Fig. 6–16*c*. The distributed loading on the beam is positive yet decreasing. Therefore, the *slope* of the shear diagram will be *negatively decreasing*. At $x = 0$, the slope begins at $-w_0$ and goes to zero at $x = L$. Since the loading is *linear,* the shear diagram is a *parabola* having a negatively decreasing slope.



(c)

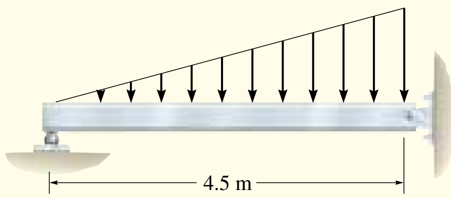
Moment Diagram. The moment at each end is plotted first, Fig. 6–16*d*. From the shear diagram, V is positive but decreases from $w_0 L/2$ at $x = 0$ to zero at $x = L$. The curve of the moment diagram having this slope behavior is a *cubic* function of x, as shown in the figure.



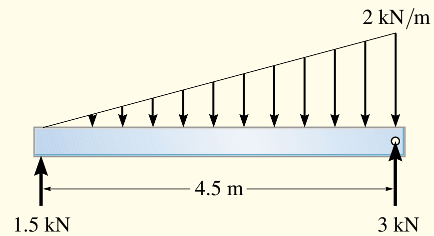
(d)

EXAMPLE 6.11

Draw the shear and moment diagrams for the beam in Fig. 6–17*a*.



(a)



(b)

Fig. 6–17

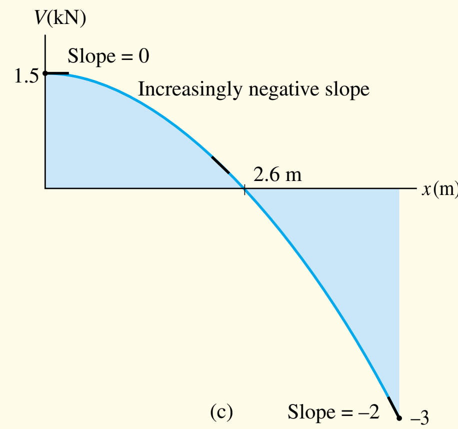
Solution

Support Reactions. The reactions have been determined and are shown on the free-body diagram, Fig. 6–17*b*.

Shear Diagram. The end points $x = 0$, $V = +1.5$, and $x = 4.5$, $V = -3$, are plotted first, Fig. 6–17*c*. From the behavior of the distributed load, the *slope* of the shear diagram will vary from zero at $x = 0$ to -2 at $x = 4.5$. As a result, the shear diagram is a parabola having the shape shown.

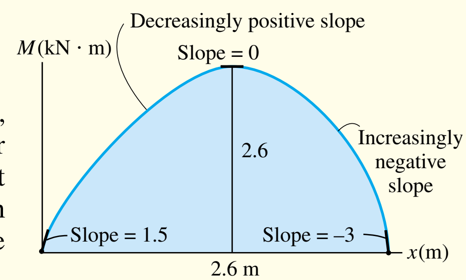
The point of zero shear can be found by using the method of sections for a beam segment of length x, Fig. 6–17*e*. We require that $V = 0$, so that

$$+\uparrow \Sigma F_y = 0; \quad 1.5 \text{ kN} - \frac{1}{2}\left[2 \text{ kN/m}\left(\frac{x}{4.5 \text{ m}}\right)\right]x = 0; \qquad x = 2.6 \text{ m}$$



(c)

Moment Diagram. The end points $x = 0$, $M = 0$ and $x = 45$, $M = 0$ are plotted first, Fig. 6–17*d*. From the behavior of the shear diagram, the slope of the moment diagram will begin at $+1.5$, then it becomes *decreasingly positive* until it reaches zero at 2.6 m. It then becomes *increasingly negative* reaching -3 at $x = 4.5$ m. Here the moment diagram is a cubic function of x. Why?



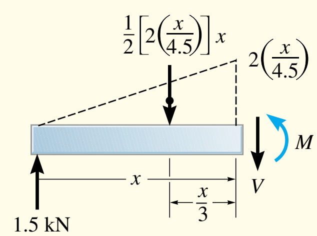
(d)

Notice that the maximum moment is at $x = 2.6$, since $dM/dx = V = 0$ at this point. From the free-body diagram in Fig. 6–17*e* we have

$$\circlearrowright\!+\Sigma M = 0;$$

$$-1.5 \text{ kN}(2.6 \text{ m}) + \frac{1}{2}\left[2 \text{ kN/m}\left(\frac{2.6 \text{ m}}{4.5 \text{ m}}\right)\right](2.6 \text{ m})\left(\frac{2.6 \text{ m}}{3}\right) + M = 0$$

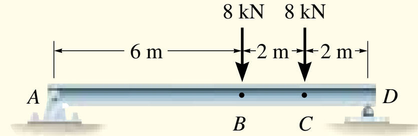
$$M = 2.6 \text{ kN} \cdot \text{m}$$



(e)

EXAMPLE 6.12

Draw the shear and moment diagrams for the beam shown in Fig. 6–18*a*.



(a)

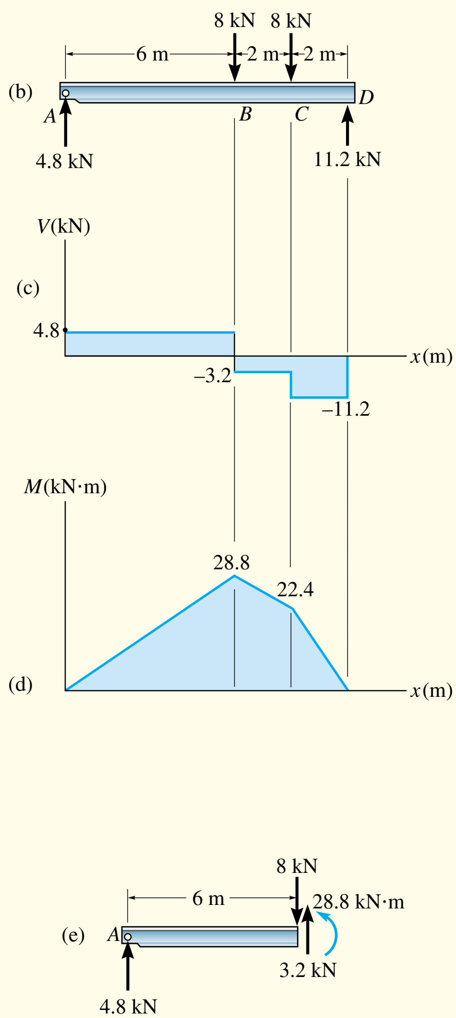


Fig. 6–18

Solution

Support Reactions. The reactions are indicated on the free-body diagram, Fig. 6–18*b*.

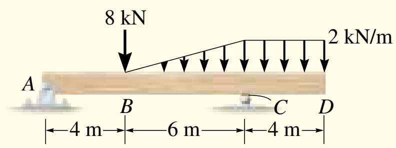
Shear Diagram. At $x = 0$, $V_A = +4.8\text{ kN}$, and at $x = 10$, $V_D = -11.2\text{ kN}$, Fig. 6–18*c*. At intermediate points between each force, the *slope* of the shear diagram will be zero. Why? Hence, the shear retains its value of $+4.8$ up to point B. AT B, the shear is *discontinuous*, since there is a *concentrated force* of 8 kN there. The value of the shear just to the right of B can be found by sectioning the beam at this point, Fig. 6–18*e*, where for equilibrium $V = -3.2\text{ kN}$. Use the method of sections and show that the diagram "jumps" again at C, as shown, then closes to the value of -11.2 kN at D.

It should be noted that based on Eq. 6–5, $\Delta V = -F$, the shear diagram can also be constructed by "following the load" on the free-body diagram. Beginning at A the 4.8-kN force acts upward, so $V_A = +4.8\text{ kN}$. No distributed load acts between A and B, so the shear remains constant $(dV/dx = 0)$. At B the 8-kN force is down, so the shear jumps down 8 kN, from $+4.8\text{ kN}$ to -3.2 kN. Again, the shear is constant from B to C (no distributed load), then at C it jumps down another 8 kN to -11.2 kN. Finally, with no distributed load between C and D, it ends at -11.2 kN.

Moment Diagram. The moment at each end of the beam is zero, Fig. 6–18*d*. The *slope* of the moment diagram from A to B is constant at $+4.8$. Why? The value of the moment at B can be determined by using statics, Fig. 6–18*c*, or by finding the area under the shear diagram between A and B, that is, $\Delta M_{AB} = (4.8\text{ kN})(6\text{ m}) = 28.8\text{ kN}\cdot\text{m}$. Since $M_A = 0$, then $M_B = M_A + \Delta M_{AB} = 0 + 28.8\text{ kN}\cdot\text{m} = 28.8\text{ kN}\cdot\text{m}$. From point B, the slope of the moment diagram is -3.2 until point C is reached. Again, the value of the moment can be obtained by statics or by finding the area under the shear diagram from B to C, that is, $\Delta M_{BC} = (-3.2\text{ kN})(2\text{ m}) = -6.4\text{ kN}\cdot\text{m}$, so that $M_C = 28.8\text{ kN}\cdot\text{m} - 6.4\text{ kN}\cdot\text{m} = 22.4\text{ kN}\cdot\text{m}$. Continuing in this manner, verify that closure occurs at D.

EXAMPLE 6.13

Draw the shear and moment diagrams for the overhanging beam shown in Fig. 6–19*a*.



(a)

Solution

Support Reactions. The free-body diagram with the calculated support reactions is shown in Fig. 6–19*b*.

Shear Diagram. As usual we start by plotting the end shears $V_A = +4.40$ kN, and $V_D = 0$, Fig. 6–19*c*. The shear diagram will have *zero* slope from A to B. It then jumps down 8 kN to -3.60 kN. It then has a slope that is *increasingly negative*. The shear at C can be determined from the area under the load diagram, $V_C = V_B + \Delta V_{BC} = -3.60$ kN $-(1/2)(6\text{ m})(2\text{ kN/m}) = -9.60$ kN. It then jumps up 17.6 kN to 8 kN. Finally, from C to D, the slope of the shear diagram will be *constant yet negative*, until the shear reaches zero at D.

Moment Diagram. The end moments $M_A = 0$ and $M_D = 0$ are plotted first, Fig. 6–19*d*. Study the diagram and note how the slopes and therefore the various curves are established from the shear diagram using $dM/dx = V$. Verify the numerical values for the peaks using the method of sections and statics or by computing the appropriate areas under the shear diagram to find the change in moment between two points. In particular, the point of zero moment can be determined by establishing M as a function of x, where, for convenience, x extends *from* point B into region BC, Fig. 6–19*e*. Hence,

$\circlearrowleft + \Sigma M = 0;$

$$-4.40\text{ kN}(4\text{ m} + x) + 8\text{ kN}(x) + \frac{1}{2}\left(\frac{2\text{ kN/m}}{6\text{ m}}\right)x(x)\left(\frac{x}{3}\right) + M = 0$$

$$M = \left(-\frac{1}{18}x^3 - 3.60x + 17.6\right)\text{kN}\cdot\text{m} = 0$$

$$x = 3.94\text{ m}$$

Reviewing these diagrams, we see that because of the integration process for region AB the load is zero, shear is constant, and moment is linear; for region BC the load is linear, shear is parabolic, and moment is cubic; and for region CD the load is constant, the shear is linear, and the moment is parabolic. It is recommended that Examples 6.1 through 6.6 also be solved using this method.

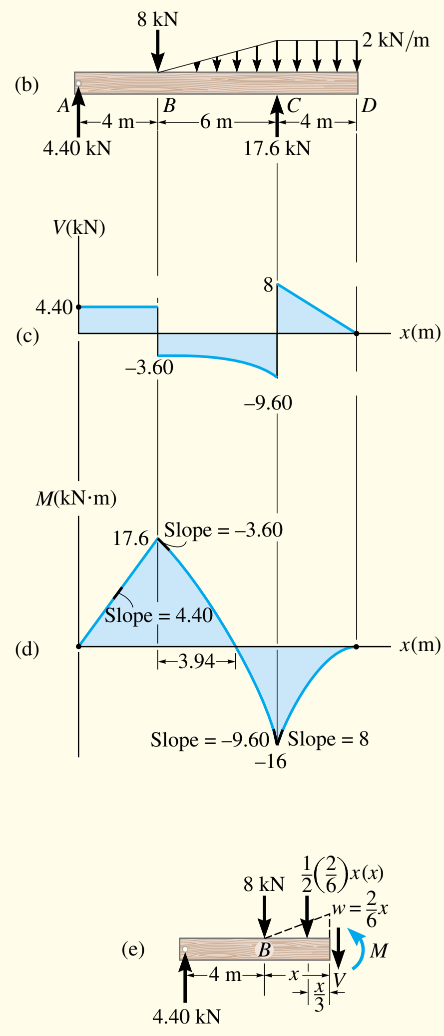


Fig. 6–19

EXAMPLE 6.14

A beam has a rectangular cross section and is subjected to the stress distribution shown in Fig. 6–27*a*. Determine the internal moment **M** at the section caused by the stress distribution (a) using the flexure formula, (b) by finding the resultant of the stress distribution using basic principles.

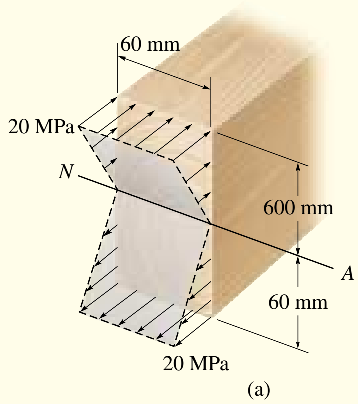


Fig. 6–27

Solution

Part (a). The flexure formula is $\sigma_{max} = Mc/I$. From Fig. 6–27*a*, $c = 60$ mm and $\sigma_{max} = 20$ MPa. The neutral axis is defined as line NA, because the stress is zero along this line. Since the cross section has a rectangular shape, the moment of inertia for the area about NA is determined from the formula for a rectangle given on the inside front cover; i.e.,

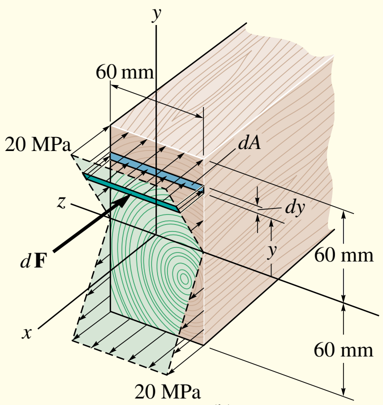
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(60 \text{ mm})(120 \text{ mm})^3 = 864(10^4) \text{ mm}^4$$

Therefore,

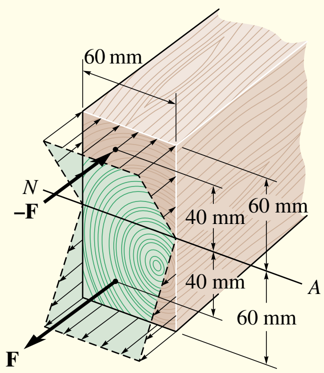
$$\sigma_{max} = \frac{Mc}{I}; \qquad 20 \text{ N/mm}^2 = \frac{M(60 \text{ mm})}{864(10^4) \text{ mm}^4}$$

$$M = 288(10^4) \text{ N} \cdot \text{mm} = 2.88 \text{ kN} \cdot \text{m} \qquad \textit{Ans.}$$

Continued



(b)



(c)

Fig. 6–27

Part (b). First we will show that the resultant force of the stress distribution is zero. As shown in Fig. 6–27*b*, the stress acting on the arbitrary element strip $dA = (60 \text{ mm})\, dy$, located y from the neutral axis, is

$$\sigma = \left(\frac{-y}{60 \text{ mm}}\right)(20 \text{ N/mm}^2)$$

The force created by this stress is $dF = \sigma\, dA$, and thus, for the entire cross section,

$$F_R = \int_A \sigma\, dA = \int_{-60 \text{ mm}}^{+60 \text{ mm}} \left[\left(\frac{-y}{60 \text{ mm}}\right)(20 \text{ N/mm}^2)\right](60 \text{ mm})\, dy$$

$$= (-10 \text{ N/mm}^2)\, y^2 \Big|_{-60 \text{ mm}}^{+60 \text{ mm}} = 0$$

The resultant moment of the stress distribution about the neutral axis (z axis) must equal M. Since the magnitude of the moment of $d\mathbf{F}$ about this axis is $dM = y\, dF$, and $d\mathbf{M}$ is *always positive*, Fig. 6–27*b*, then for the entire area,

$$M = \int_A y\, dF = \int_{-60 \text{ mm}}^{+60 \text{ mm}} y\left[\left(\frac{y}{60 \text{ mm}}\right)(20 \text{ N/mm}^2)\right](60 \text{ mm})\, dy$$

$$= \left(\frac{20}{3} \text{ N/mm}^2\right) y^3 \Big|_{-60 \text{ mm}}^{+60 \text{ mm}}$$

$$= 288(10^4) \text{ N} \cdot \text{mm} = 2.88 \text{ kN} \cdot \text{m}$$ *Ans.*

The above result can *also* be determined without the need for integration. The resultant force for each of the two *triangular* stress distributions in Fig. 6–27*c* is graphically equivalent to the *volume* contained within each stress distribution. Thus, each volume is

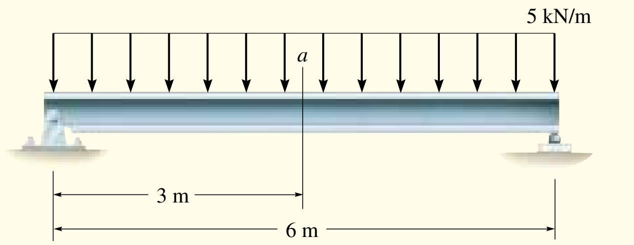
$$F = \frac{1}{2}(60 \text{ mm})(20 \text{ N/mm}^2)(60 \text{ mm}) = 36(10^3) \text{ N} = 36 \text{ kN}$$

These forces, which form a couple, act in the same direction as the stresses within each distribution, Fig. 6–27*c*. Furthermore, they act through the *centroid* of each volume, i.e., $\frac{1}{3}(60 \text{ mm}) = 20 \text{ mm}$ from the top and bottom of the beam. Hence the distance between them is 80 mm as shown. The moment of the couple is therefore

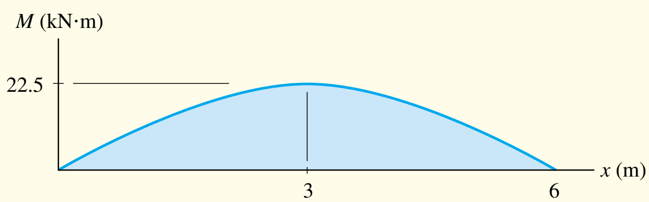
$$M = 36 \text{ kN}\,(80 \text{ mm}) = 2880 \text{ kN} \cdot \text{mm} = 2.88 \text{ kN} \cdot \text{m}$$ *Ans.*

EXAMPLE 6.15

The simply supported beam in Fig. 6–28*a* has the cross-sectional area shown in Fig. 6–28*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



(a)



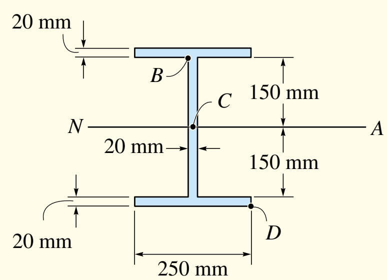
(c)

Fig. 6–28

Solution

Maximum Internal Moment. The maximum internal moment in the beam, $M = 22.5\ \text{kN}\cdot\text{m}$, occurs at the center as shown on the bending moment diagram, Fig. 6–28*c*. See Example 6.3.

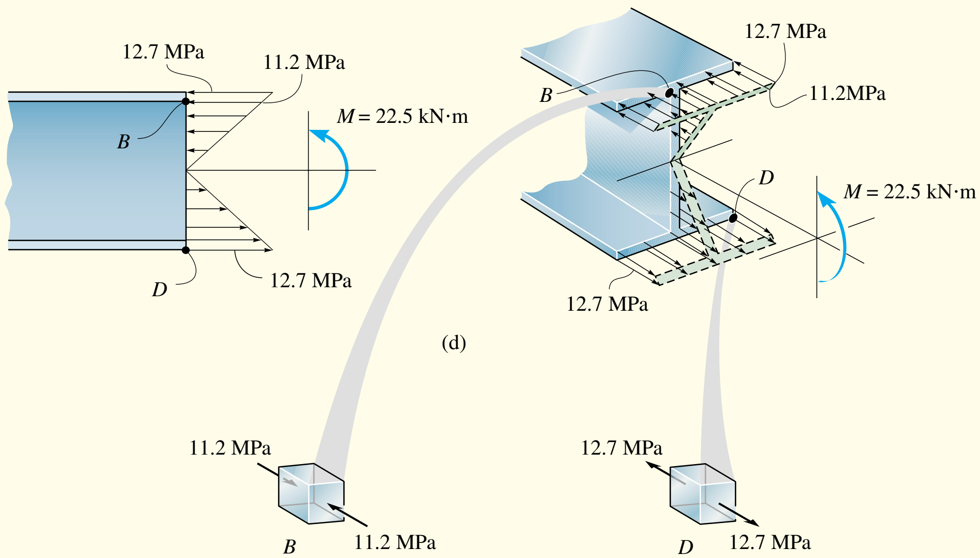
Section Property. By reasons of symmetry, the centroid C and thus the neutral axis pass through the midheight of the beam, Fig. 6–28*b*. The area is subdivided into the three parts shown, and the moment of inertia of each part is computed about the neutral axis using the parallel-axis theorem. (See Eq. A–5 of Appendix A.) Choosing to work in meters, we have



(b)

$$
\begin{aligned}
I &= \Sigma(\bar{I} + Ad^2)\\
&= 2\left[\frac{1}{12}(0.25\ \text{m})(0.020\ \text{m})^3 + (0.25\ \text{m})(0.020\ \text{m})(0.160\ \text{m})^2\right]\\
&\quad + \left[\frac{1}{12}(0.020\ \text{m})(0.300\ \text{m})^3\right]\\
&= 301.3(10^{-6})\ \text{m}^4
\end{aligned}
$$

Continued



(d)

(e)

Bending Stress. Applying the flexure formula, with $c = 170$ mm, the absolute maximum bending stress is

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_{\max} = \frac{22.5 \text{ kN} \cdot \text{m}(0.170 \text{ m})}{301.3(10^{-6}) \text{ m}^4} = 12.7 \text{ MPa} \qquad \textit{Ans.}$$

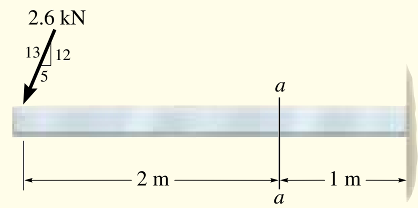
Two-and-three-dimensional views of the stress distribution are shown in Fig. 6–28*d*. Notice how the stress at each point on the cross section develops a force that contributes a moment $d\mathbf{M}$ about the neutral axis such that it has the same direction as $\mathbf{M}$. Specifically, at point B, $y_B = 150$ mm, and so

$$\sigma_B = \frac{My_B}{I}; \quad \sigma_B = \frac{22.5 \text{ kN} \cdot \text{m}(0.150 \text{ m})}{301.3(10^{-6}) \text{ m}^4} = 11.2 \text{ MPa}$$

The normal stress acting on elements of material located at points B and D is shown in Fig. 6–28*e*.

EXAMPLE 6.16

The beam shown in Fig. 6–29*a* has a cross-sectional area in the shape of a channel, Fig. 6–29*b*. Determine the maximum bending stress that occurs in the beam at section *a–a*.



(a)

Solution

Internal Moment. Here the beam's support reactions do not have to be determined. Instead, by the method of sections, the segment to the left of section *a–a* can be used, Fig. 6–29*c*. In particular, note that the resultant internal axial force **N** passes through the centroid of the cross section. Also, realize that *the resultant internal moment must be computed about the beam's neutral axis* at section *a–a*.

To find the location of the neutral axis, the cross-sectional area is subdivided into three composite parts as shown in Fig. 6–29*b*. Since the neutral axis passes through the centroid, then using Eq. A–2 of Appendix A, we have

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.100\text{ m}](0.200\text{ m})(0.015\text{ m}) + [0.010\text{ m}](0.02\text{ m})(0.250\text{ m})}{2(0.200\text{ m})(0.015\text{ m}) + 0.020\text{ m}(0.250\text{ m})}$$

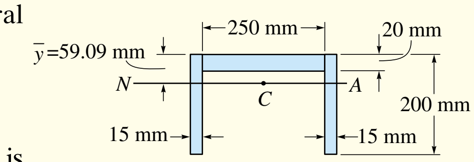
$$= 0.05909\text{ m} = 59.09\text{ mm}$$

This dimension is shown in Fig. 6–29*c*.

Applying the moment equation of equilibrium about the neutral axis, we have

$$\curvearrowleft + \Sigma M_{NA} = 0; \qquad 24\text{ kN}(2\text{ m}) + 1.0\text{ kN}(0.05909\text{ m}) - M = 0$$

$$M = 4.859\text{ kN}\cdot\text{m}$$



(b)

Section Property. The moment of inertia about the neutral axis is determined using the parallel-axis theorem applied to each of the three composite parts of the cross-sectional area. Working in meters, we have

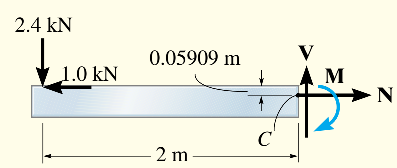
$$I = \left[\frac{1}{12}(0.250\text{ m})(0.020\text{ m})^3 + (0.250\text{ m})(0.020\text{ m})(0.05909\text{ m} - 0.010\text{ m})^2\right]$$

$$+ 2\left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3 + (0.015\text{ m})(0.200\text{ m})(0.100\text{ m} - 0.05909\text{ m})^2\right]$$

$$= 42.26(10^{-6})\text{ m}^4$$

Maximum Bending Stress. The maximum bending stress occurs at points farthest away from the neutral axis. This is at the bottom of the beam, $c = 0.200\text{ m} - 0.05909\text{ m} = 0.1409\text{ m}$. Thus,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{4.859\text{ kN}\cdot\text{m}(0.1409\text{ m})}{42.26(10^{-6})\text{ m}^4} = 16.2\text{ MPa} \qquad \textit{Ans.}$$



(c)

Fig. 6–29

Show that at the top of the beam the bending stress is $\sigma' = 6.79$ MPa. Note that in addition to this effect of bending, the normal force of $N = 1$ kN and shear force $V = 2.4$ kN will also contribute additional stress on the cross section. The superposition of all these effects will be discussed in a later chapter.

EXAMPLE 6.17

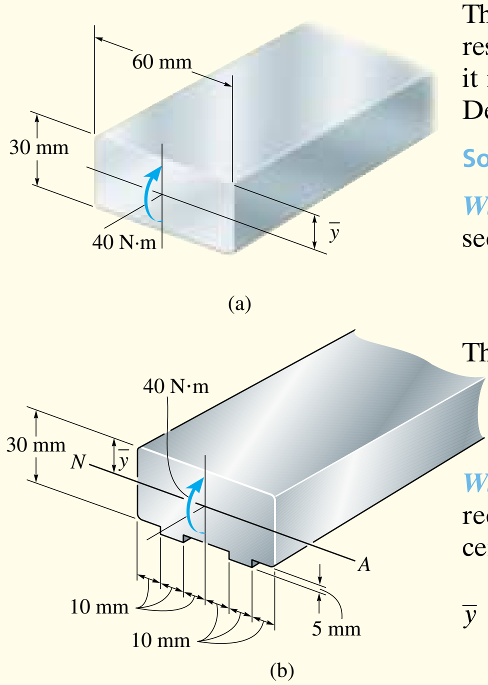


Fig. 6–30

The member having a rectangular cross section, Fig. 6–30*a*, is designed to resist a moment of 40 N · m. In order to increase its strength and rigidity, it is proposed that two small ribs be added at its bottom, Fig. 6–30*b*. Determine the maximum normal stress in the member for both cases.

Solution

Without Ribs. Clearly the neutral axis is at the center of the cross section, Fig. 6–30*a*, so $\bar{y} = c = 15\text{ mm} = 0.015\text{ m}$. Thus,

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.06\text{ m})(0.03\text{ m})^3 = 0.135(10^{-6})\text{ m}^4$$

Therefore the maximum normal stress is

$$\sigma_{\max} = \frac{Mc}{I} = \frac{(40\text{ N}\cdot\text{m})(0.015\text{ m})}{0.135(10^{-6})\text{ m}^4} = 4.44\text{ MPa} \qquad \textit{Ans.}$$

With Ribs. From Fig. 6–30*b*, segmenting the area into the large main rectangle and the bottom two rectangles (ribs), the location $\bar{y}$ of the centroid and the neutral axis is determined as follows:

$$\begin{aligned}\bar{y} &= \frac{\Sigma \bar{y}A}{\Sigma A}\\ &= \frac{[0.015\text{ m}](0.030\text{ m})(0.060\text{ m}) + 2[0.0325\text{ m}](0.005\text{ m})(0.010\text{ m})}{(0.03\text{ m})(0.060\text{ m}) + 2(0.005\text{ m})(0.010\text{ m})}\\ &= 0.01592\text{ m}\end{aligned}$$

This value does not represent c. Instead

$$c = 0.035\text{ m} - 0.01592\text{ m} = 0.01908\text{ m}$$

Using the parallel-axis theorem, the moment of inertia about the neutral axis is

$$\begin{aligned}I &= \left[\frac{1}{12}(0.060\text{ m})(0.030\text{ m})^3 + (0.060\text{ m})(0.030\text{ m})(0.01592\text{ m} - 0.015\text{ m})^2\right]\\ &\quad + 2\left[\frac{1}{12}(0.010\text{ m})(0.005\text{ m})^3 + (0.010\text{ m})(0.005\text{ m})(0.0325\text{ m} - 0.01592\text{ m})^2\right]\\ &= 0.1642(10^{-6})\text{ m}^4\end{aligned}$$

Therefore, the maximum normal stress is

$$\sigma_{\max} = \frac{Mc}{I} = \frac{40\text{ N}\cdot\text{m}(0.01908\text{ m})}{0.1642(10^{-6})\text{ m}^4} = 4.65\text{ MPa} \qquad \textit{Ans.}$$

This surprising result indicates that the addition of the ribs to the cross section will *increase* the normal stress rather than decrease it, and for this reason they should be omitted.

EXAMPLE 6.18

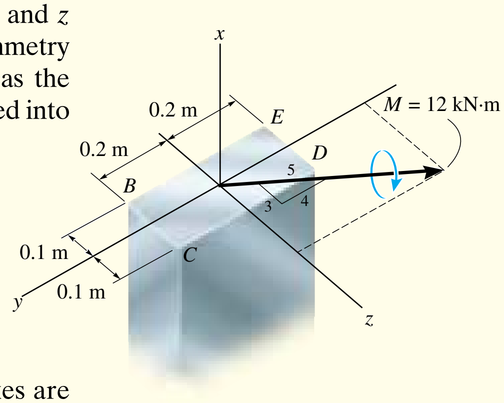
The rectangular cross section shown in Fig. 6–35*a* is subjected to a bending moment of $M = 12\ \text{kN}\cdot\text{m}$. Determine the normal stress developed at each corner of the section, and specify the orientation of the neutral axis.

Solution

Internal Moment Components. By inspection, it is seen that the y and z axes represent the principal axes of inertia since they are axes of symmetry for the cross section. As required, we have established the z *axis* as the principal axis for *maximum* moment of inertia. The moment is resolved into its y and z components, where

$$M_y = -\frac{4}{5}(12\ \text{kN}\cdot\text{m}) = -9.60\ \text{KN}\cdot\text{m}$$

$$M_z = \frac{3}{5}(12\ \text{kN}\cdot\text{m}) = 7.20\ \text{kN}\cdot\text{m}$$



(a)

Fig. 6–35

Section Properties. The moments of inertia about the y and z axes are

$$I_y = \frac{1}{12}(0.4\ \text{m})(0.2\ \text{m})^3 = 0.2667(10^{-3})\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.2\ \text{m})(0.4\ \text{m})^3 = 1.067(10^{-3})\ \text{m}^4$$

Bending Stress. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = -\frac{7.20(10^3)\ \text{N}\cdot\text{m}(0.2\ \text{m})}{1.067(10^{-3})\ \text{m}^4} + \frac{-9.60(10^3)\ \text{N}\cdot\text{m}(-0.1\ \text{m})}{0.2667(10^{-3})\ \text{m}^4} = 2.25\ \text{MPa} \qquad \textit{Ans.}$$

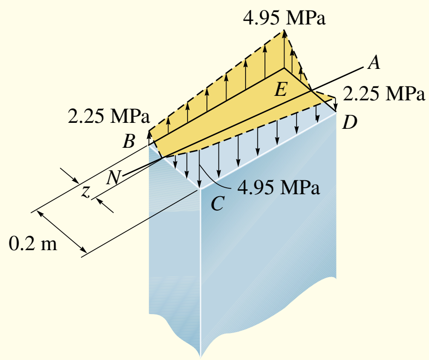
$$\sigma_C = -\frac{7.20(10^3)\ \text{N}\cdot\text{m}(0.2\ \text{m})}{1.067(10^{-3})\ \text{m}^4} + \frac{-9.60(10^3)\ \text{N}\cdot\text{m}(0.1\ \text{m})}{0.2667(10^{-3})\ \text{m}^4} = -4.95\ \text{MPa} \qquad \textit{Ans.}$$

$$\sigma_D = -\frac{7.20(10^3)\ \text{N}\cdot\text{m}(-0.2\ \text{m})}{1.067(10^{-3})\ \text{m}^4} + \frac{-9.60(10^3)\ \text{N}\cdot\text{m}(0.1\ \text{m})}{0.2667(10^{-3})\ \text{m}^4} = -2.25\ \text{MPa} \qquad \textit{Ans.}$$

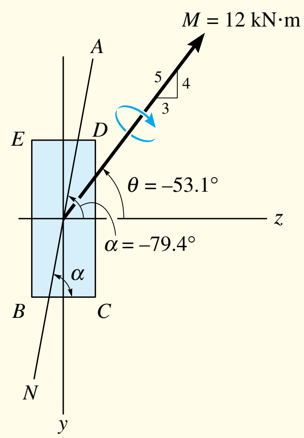
$$\sigma_E = -\frac{7.20(10^3)\ \text{N}\cdot\text{m}(-0.2\ \text{m})}{1.067(10^{-3})\ \text{m}^4} + \frac{-9.60(10^3)\ \text{N}\cdot\text{m}(-0.1\ \text{m})}{0.2667(10^{-3})\ \text{m}^4} = 4.95\ \text{MPa} \qquad \textit{Ans.}$$

The resultant normal-stress distribution has been sketched using these values, Fig. 6–35*b*. Since superposition applies, the distribution is linear as shown.

Continued



(b)



(c)

Orientation of Neutral Axis. The location z of the neutral axis (NA), Fig. 6–35b, can be established by proportion. Along the edge BC, we require

$$\frac{2.25\text{ MPa}}{z} = \frac{4.95\text{ MPa}}{(0.2\text{ m} - z)}$$

$$0.450 - 2.25z = 4.95z$$

$$z = 0.0625\text{ m}$$

In the same manner, this is also the distance from D to the neutral axis in Fig. 6–35b.

We can also establish the orientation of the NA using Eq. 6–19, which is used to specify the angle α that the axis makes with the z or *maximum* principal axis. According to our sign convention, θ must be measured from the $+z$ axis toward the $+y$ axis. By comparison, in Fig. 6–35c, $\theta = -\tan^{-1}\frac{4}{3} = -53.1°$ (or $\theta = +306.9°$). Thus,

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

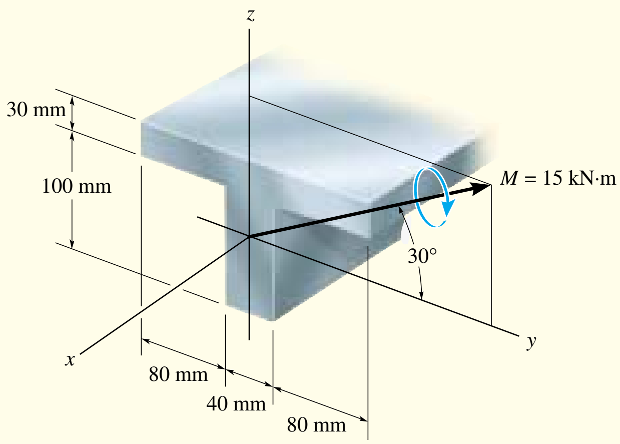
$$\tan\alpha = \frac{1.067(10^{-3})\text{ m}^4}{0.2667(10^{-3})\text{ m}^4}\tan(-53.1°)$$

$$\alpha = -79.4° \qquad \textit{Ans.}$$

This result is shown in Fig. 6–35c. Using the value of z calculated above, verify, using the geometry of the cross section, that one obtains the same answer.

EXAMPLE 6.19

A T-beam is subjected to the bending moment of 15 kN · m as shown in Fig. 6–36*a*. Determine the maximum normal stress in the beam and the orientation of the neutral axis.



(a)

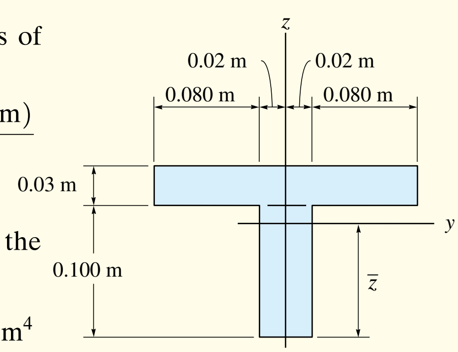
Solution

Internal Moment Components. The y and z axes are principal axes of inertia. Why? From Fig. 6–36*a*, both moment components are positive. We have

$$M_y = (15\text{ kN}\cdot\text{m})\cos 30^\circ = 12.99\text{ kN}\cdot\text{m}$$
$$M_z = (15\text{ kN}\cdot\text{m})\sin 30^\circ = 7.50\text{ kN}\cdot\text{m}$$

Section Properties. With reference to Fig. 6–36*b*, working in units of meters, we have

$$\bar{z} = \frac{\Sigma\tilde{z}A}{\Sigma A} = \frac{[0.05\text{ m}](0.100\text{ m})(0.04\text{ m}) + [0.115\text{ m}](0.03\text{ m})(0.200\text{ m})}{(0.100\text{ m})(0.04\text{ m}) + (0.03\text{ m})(0.200\text{ m})}$$
$$= 0.0890\text{ m}$$



(b)

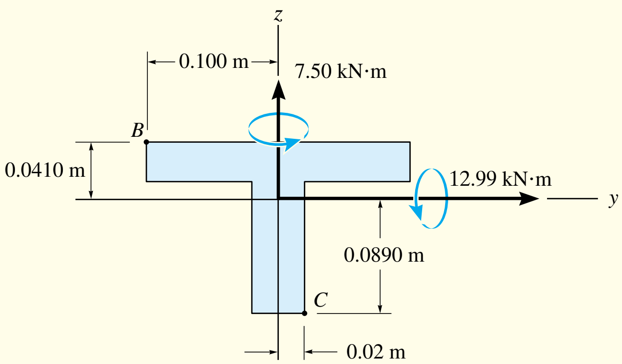
Fig. 6–36

Using the parallel-axis theorem of Appendix A, $I = \bar{I} + Ad^2$, the principal moments of inertia are thus

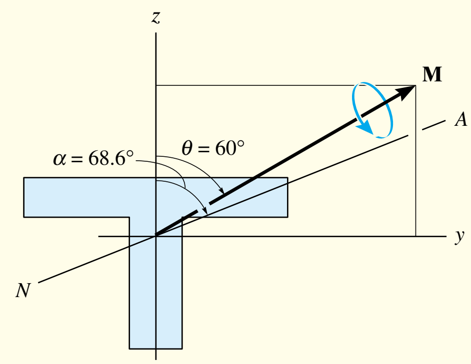
$$I_z = \frac{1}{12}(0.100\text{ m})(0.04\text{ m})^3 + \frac{1}{12}(0.03\text{ m})(0.200\text{ m})^3 = 20.53(10^{-6})\text{ m}^4$$

$$I_y = \left[\frac{1}{12}(0.04\text{ m})(0.100\text{ m})^3 + (0.100\text{ m})(0.04\text{ m})(0.0890\text{ m} - 0.05\text{ m})^2\right]$$
$$+ \left[\frac{1}{12}(0.200\text{ m})(0.03\text{ m})^3 + (0.200\text{ m})(0.03\text{ m})(0.115\text{ m} - 0.0890\text{ m})^2\right]$$
$$= 13.92(10^{-6})\text{ m}^4$$

Continued



(c)



(d)

Maximum Bending Stress. The moment components are shown in Fig. 6–36*c*. By inspection, the largest *tensile* stress occurs at point B, since by superposition both moment components create a tensile stress there. Likewise, the greatest *compressive* stress occurs at point C. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = -\frac{7.50 \text{ kN}\cdot\text{m } (-0.100 \text{ m})}{20.53(10^{-6}) \text{ m}^4} + \frac{12.99 \text{ kN}\cdot\text{m } (0.0410 \text{ m})}{13.92(10^{-6}) \text{ m}^4}$$

$$= 74.8 \text{ MPa}$$

$$\sigma_C = -\frac{7.50 \text{ kN}\cdot\text{m } (0.020 \text{ m})}{20.53(10^{-6}) \text{ m}^4} + \frac{12.99 \text{ kN}\cdot\text{m } (-0.0890 \text{ m})}{13.92(10^{-6}) \text{ m}^4}$$

$$= -90.4 \text{ MPa} \qquad \textit{Ans.}$$

By comparison, the largest normal stress is therefore compressive and occurs at point C.

Orientation of Neutral Axis. When applying Eq. 6–19, it is important to be sure the angles α and θ are defined correctly. As previously stated, y must represent the axis for *minimum* principal moment of inertia, and z must represent the axis for *maximum* principal moment of inertia. These axes are properly positioned here since $I_y < I_z$. Using this setup, θ and α are measured positive from the $+z$ axis toward the $+y$ axis. Hence, from Fig. 6–36*a*, $\theta = +60°$. Thus,

$$\tan \alpha = \left(\frac{20.53(10^{-6}) \text{ m}^4}{13.92(10^{-6}) \text{ m}^4}\right) \tan 60°$$

$$\alpha = 68.6° \qquad \textit{Ans.}$$

The neutral axis is shown in Fig. 6–36*d*. As expected, it lies between the y axis and the line of action of $\mathbf{M}$.

EXAMPLE 6.20

The Z-section shown in Fig. 6–37*a* is subjected to the bending moment of $M = 20\ \text{kN}\cdot\text{m}$. Using the methods of Appendix A (see Example A.4 or A.5), the principal axes y and z are oriented as shown, such that they represent the minimum and maximum principal moments of inertia, $I_y = 0.960(10^{-3})\ \text{m}^4$ and $I_z = 7.54(10^{-3})\ \text{m}^4$, respectively. Determine the normal stress at point P and the orientation of the neutral axis.

Solution

For use of Eq. 6–19, it is important that the z axis be the principal axis for the *maximum* moment of inertia, which it is because most of the area is located furthest from this axis.

Internal Moment Components. From Fig. 6–37*a*,

$$M_y = 20\ \text{kN}\cdot\text{m}\sin 57.1^\circ = 16.79\ \text{kN}\cdot\text{m}$$
$$M_z = 20\ \text{kN}\cdot\text{m}\cos 57.1^\circ = 10.86\ \text{kN}\cdot\text{m}$$

Bending Stress. The y and z coordinates of point P must be determined first. Note that the y', z' coordinates of P are $(-0.2\ \text{m}, 0.35\ \text{m})$. Using the colored and shaded triangles from the construction shown in Fig. 6–37*b*, we have

$$y_P = -0.35\sin 32.9^\circ - 0.2\cos 32.9^\circ = -0.3580\ \text{m}$$
$$z_P = 0.35\cos 32.9^\circ - 0.2\sin 32.9^\circ = 0.1852\ \text{m}$$

Applying Eq. 6–17, we have

$$\sigma_P = -\frac{M_z y_P}{I_z} + \frac{M_y z_P}{I_y}$$
$$= -\frac{(10.86\ \text{kN}\cdot\text{m})(-0.3580\ \text{m})}{7.54(10^{-3})\ \text{m}^4} + \frac{(16.79\ \text{kN}\cdot\text{m})(0.1852\ \text{m})}{0.960(10^{-3})\ \text{m}^4}$$
$$= 3.76\ \text{MPa}$$ *Ans.*

Orientation of Neutral Axis. The angle $\theta = 57.1^\circ$ is shown in Fig. 6–37*a*. Thus,

$$\tan\alpha = \left[\frac{7.54(10^{-3})\ \text{m}^4}{0.960(10^{-3})\ \text{m}^4}\right]\tan 57.1^\circ$$

$$\alpha = 85.3^\circ$$ *Ans.*

The neutral axis is located as shown in Fig. 6–37*b*.

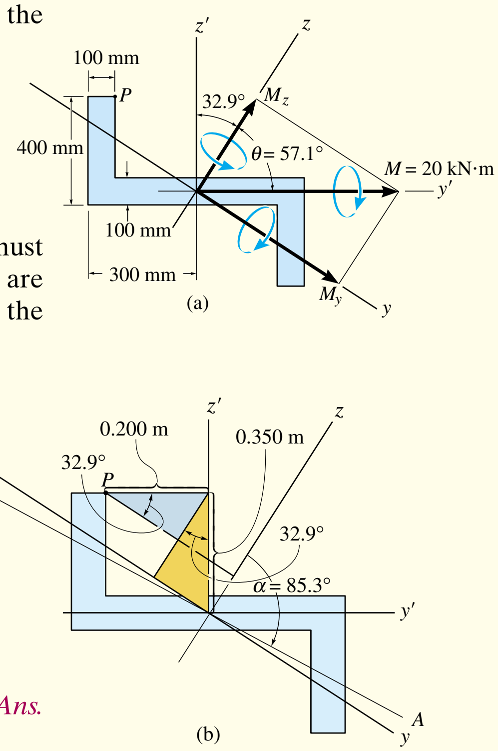


Fig. 6–37

EXAMPLE 6.21

A composite beam is made of wood and reinforced with a steel strap located on its bottom side. It has the cross-sectional area shown in Fig. 6–40*a*. If the beam is subjected to a bending moment of $M = 2\ \mathrm{kN \cdot m}$, determine the normal stress at points B and C. Take $E_w = 12$ GPa GPa and $E_{st} = 200$ GPa. GPa.

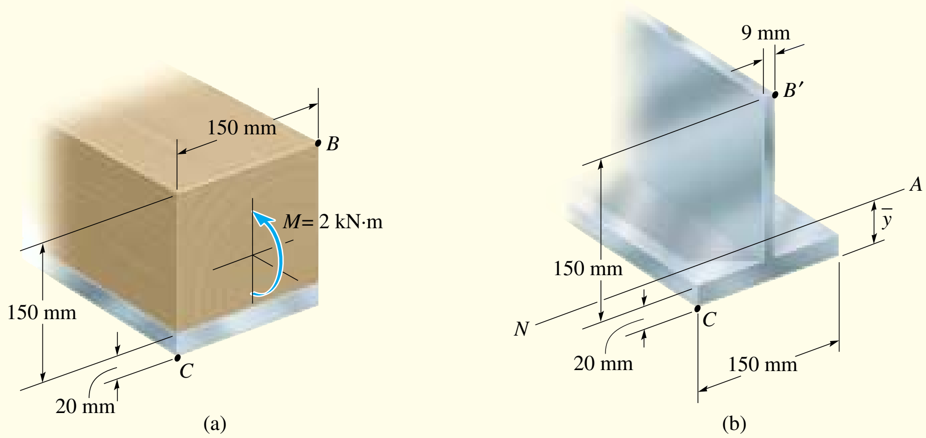


Fig. 6–40

Solution

Section Properties Although the choice is arbitrary, here we will transform the section into one made entirely of steel. Since steel has a greater stiffness than wood ($E_{st} > E_w$), the width of the wood must be *reduced* to an equivalent width for steel. Hence n must be less than one. For this to be the case, $n = E_w/E_{st}$, so that

$$b_{st} = nb_w = \frac{12\ \text{GPa}}{200\ \text{GPa}}(150\ \text{mm}) = 9\ \text{mm}$$

The transformed section is shown in Fig. 6–40*b*.

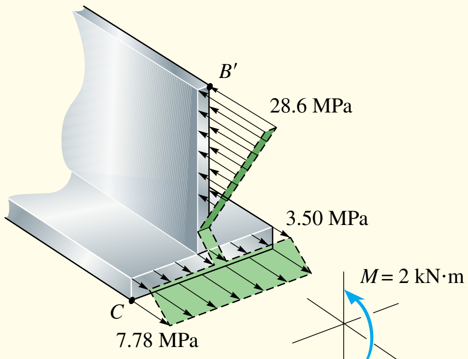
The location of the centroid (neutral axis), computed from a reference axis located at the *bottom* of the section, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{[0.01\ \text{m}](0.02\ \text{m})(0.150\ \text{m}) + [0.095\ \text{m}](0.009\ \text{m})(0.150\ \text{m})}{0.02\ \text{m}(0.150\ \text{m}) + 0.009\ \text{m}(0.150\ \text{m})} = 0.03638\ \text{m}$$

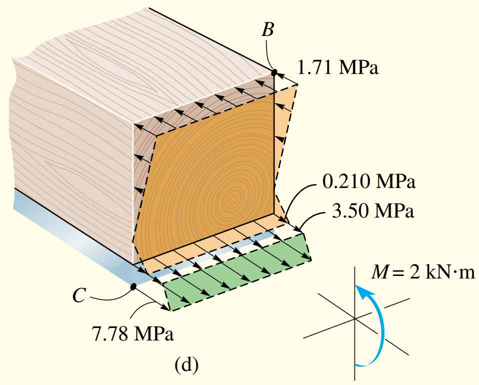
The moment of inertia about the neutral axis is therefore

$$I_{NA} = \left[\frac{1}{12}(0.150\ \text{m})(0.02\ \text{m})^3 + (0.150\ \text{m})(0.02\ \text{m})(0.03638\ \text{m} - 0.01\ \text{m})^2\right]$$
$$+ \left[\frac{1}{12}(0.009\ \text{m})(0.150\ \text{m})^3 + (0.009\ \text{m})(0.150\ \text{m})(0.095\ \text{m} - 0.03638\ \text{m})^2\right]$$
$$= 9.358(10^{-6})\ \text{m}^4$$

Continued



(c)



(d)

Normal Stress. Applying the flexure formula, the normal stress at B' and C is

$$\sigma_{B'} = \frac{2\text{ kN}\cdot\text{m}(0.170\text{ m} - 0.03638\text{ m})}{9.358(10^{-6})\text{ m}^4} = 28.6\text{ MPa}$$

$$\sigma_C = \frac{2\text{ kN}\cdot\text{m}(0.03638\text{ m})}{9.358(10^{-6})\text{ m}^4} = 7.78\text{ MPa} \qquad \textit{Ans.}$$

The normal-stress distribution on the transformed (all steel) section is shown in Fig. 6–40*c*.

The normal stress in the wood, located at B in Fig. 6–40*a*, is determined from Eq. 6–21; that is,

$$\sigma_B = n\sigma_{B'} = \frac{12\text{ GPa}}{200\text{ GPa}}(28.56\text{ MPa}) = 1.71\text{ MPa} \qquad \textit{Ans.}$$

Using these concepts, show that the normal stress in the steel and the wood at the point where they are in contact is $\sigma_{st} = 3.50$ MPa and $\sigma_w = 0.210$ MPa, respectively. The normal-stress distribution in the actual beam is shown in Fig. 6–40*d*.

EXAMPLE 6.22

In order to reinforce the steel beam, an oak board is placed between its flanges as shown in Fig. 6–41*a*. If the allowable normal stress for the steel is $(\sigma_{\text{allow}})_{st} = 168$ MPa, and for the wood $(\sigma_{\text{allow}})_{w} = 21$ MPa, determine the maximum bending moment the beam can support, with and without the wood reinforcement. $E_{st} = 200$ GPa, $E_w = 12$ GPa. The moment of inertia of the steel beam is $I_z = 7.93 \times 10^6\ \text{mm}^4$, and its cross-sectional area is $A = 5493.75\ \text{mm}^2$.

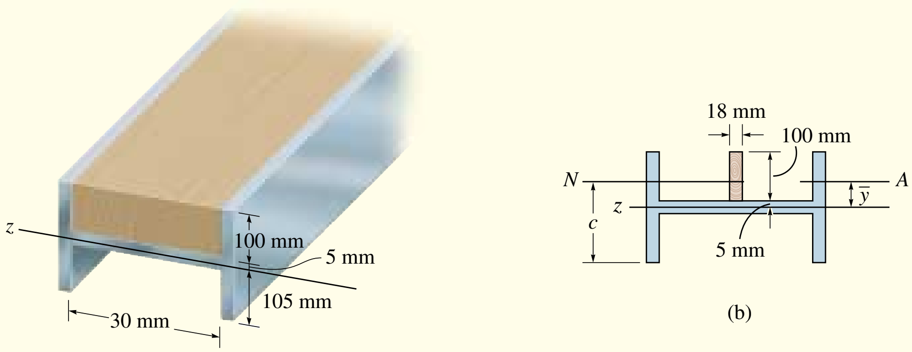


Fig. 6–41

Solution

Without Board. Here the neutral axis coincides with the z axis. Direct application of the flexure formula to the steel beam yields

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{Mc}{I_z}$$

$$168\ \text{N/mm}^2 = \frac{M(105\ \text{mm})}{7.93(10^6)\ \text{mm}^4}$$

$$M = 12.688\ \text{kN}\cdot\text{m}$$ *Ans.*

With Board. Since now we have a composite beam, we must transform the section to a single material. It will be easier to transform the wood to an equivalent amount of steel. To do this, $n = E_{\text{w}}/E_{\text{st}}$. Why? Thus, the width of an equivalent amount of steel is

$$b_{\text{st}} = nb_{\text{w}} = \frac{12(10^3)\ \text{MPa}}{200(10^3)\ \text{MPa}}(300\ \text{mm}) = 18\ \text{mm}$$

Continued

The transformed section is shown in Fig. 6–41b. The neutral axis is at

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{[0](5493.75\text{ mm}^2) + [55\text{ mm}](100\text{ mm})(18\text{ mm})}{5493.75\text{ mm}^2 + 100(18)\text{ mm}^2}$$
$$= 13.57\text{ mm}$$

And the moment of inertia about the neutral axis is

$$I = [7.93(10^6)\text{ mm}^2 + 5493.75\text{ mm}^2(13.57\text{ mm})^2]$$
$$+ \left[\frac{1}{12}(18\text{ mm})(100\text{ mm})^3 + (18\text{ mm})(100\text{ mm})(55\text{ mm} - 13.57\text{ mm})^2\right]$$
$$= 13.53(10^6)\text{ mm}^4$$

The maximum normal stress in the steel will occur at the *bottom* of the beam, Fig. 6–41b. Here $c = 105\text{ mm} + 13.57\text{ mm} = 118.57\text{ mm}$. The maximum moment based on the allowable stress for the steel is therefore

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{Mc}{I}$$
$$168\text{ N/mm}^2 = \frac{M(118.57\text{ mm})}{13.53 \times 10^6\text{ mm}^4}$$
$$M = 19.17\text{ kN}\cdot\text{m}$$

The maximum normal stress in the wood occurs at the top of the beam, Fig. 6–41b. Here $c' = 105\text{ mm} - 13.57\text{ mm} = 91.43\text{ mm}$. Since $\sigma_{\text{w}} = n\sigma_{\text{st}}$, the maximum moment based on the allowable stress for the wood is

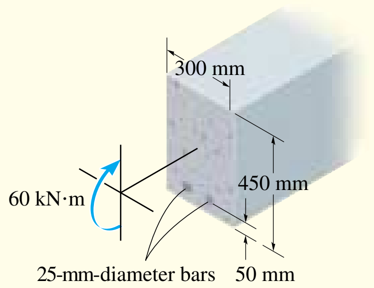
$$(\sigma_{\text{allow}})_{\text{w}} = n\frac{M'c'}{I}$$
$$21\text{ N/mm}^2 = \left[\frac{12(10^3)\text{ MPa}}{200(10^3)\text{ MPa}}\right]\frac{M'(91.43\text{ mm})}{13.53 \times 10^6\text{ mm}^4}$$
$$M' = 51.79\text{ kN}\cdot\text{m}$$

By comparison, the maximum moment is limited by the allowable stress in the steel. Thus,

$$M = 19.17\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

Note also that by using the board as reinforcement, one provides an additional 51% moment capacity for the beam.

EXAMPLE 6.23



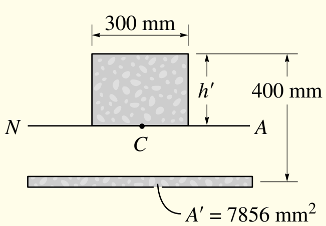
The reinforced concrete beam has the cross-sectional area shown in Fig. 6–43*a*. If it is subjected to a bending moment of M = 60 kN · m, determine the normal stress in each of the steel reinforcing rods and the maximum normal stress in the concrete. Take E_{st} = 200 GPa and E_{conc} = 25 GPa.

Solution

Since the beam is made from concrete, in the following analysis we will neglect its strength in supporting a tensile stress.

Section Properties. The total area of steel, $A_{st} = 2[\pi(12.5 \text{ mm})^2] = 982 \text{ mm}^2$ will be transformed into an equivalent area of concrete, Fig. 6–43*b*. Here

$$A' = nA_{st} = \frac{200(10^3)\text{ MPa}}{25(10^3)\text{ MPa}}(982 \text{ mm}^2) = 7856 \text{ mm}^2$$



(b)

We require the centroid to lie on the neutral axis. Thus $\Sigma\tilde{y}A = 0$, or

$$300 \text{ mm}\,(h')\frac{h'}{2} - 7856 \text{ mm}^2(400 \text{ mm} - h') = 0$$

$$h'^2 + 52.37h' - 20949.33 = 0$$

Solving for the positive root,

$$h' = 120.90 \text{ mm}$$

Using this value for h', the moment of inertia of the transformed section, computed about the neutral axis, is

$$I = \left[\frac{1}{12}(300 \text{ mm})(120.90 \text{ mm})^3 + 300 \text{ mm}(120.90 \text{ mm})\left(\frac{120.9 \text{ mm}}{2}\right)^2 + 7856 \text{ mm}^2(400 \text{ mm} - 120.90 \text{ mm})^2\right.$$

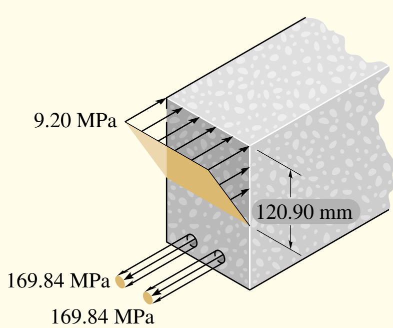
$$= 788.67 \times 10^6 \text{ mm}^4$$

Normal Stress. Applying the flexure formula to the transformed section, the maximum normal stress in the concrete is

$$(\sigma_{\text{conc}})_{\text{max}} = \frac{60 \text{ kN} \cdot \text{m}\,(1000 \text{ mm/m})(120.90 \text{ mm})(1000 \text{ N/kN})}{788.67 \times 10^6 \text{ mm}^4} = 9.20 \text{ MPa} \quad \textit{Ans.}$$

The normal stress resisted by the “concrete” strip, which replaced the steel, is

$$\sigma'_{\text{conc}} = \frac{60 \text{ kN} \cdot \text{m}\,(1000 \text{ mm/m})(1000 \text{ N/kN})(400 \text{ mm} - 120.9 \text{ mm})}{788.67 \times 10^6 \text{ mm}^4} = 21.23 \text{ MPa}$$



(c)

Fig. 6–43

The normal stress in each of the two reinforcing rods is therefore

$$\sigma_{\text{st}} = n\sigma'_{\text{conc}} = \left(\frac{200(10^3)\text{ MPa}}{25(10^3)\text{ MPa}}\right)21.23 \text{ MPa} = 169.84 \text{ MPa} \quad \textit{Ans.}$$

The normal-stress distribution is shown graphically in Fig. 6–43*c*.

EXAMPLE 6.24

A steel bar having a rectangular cross section is shaped into a circular arc as shown in Fig. 6–45*a*. If the allowable normal stress is $\sigma_{\text{allow}} = 140$ MPa, determine the maximum bending moment M that can be applied to the bar. What would this moment be if the bar was straight?

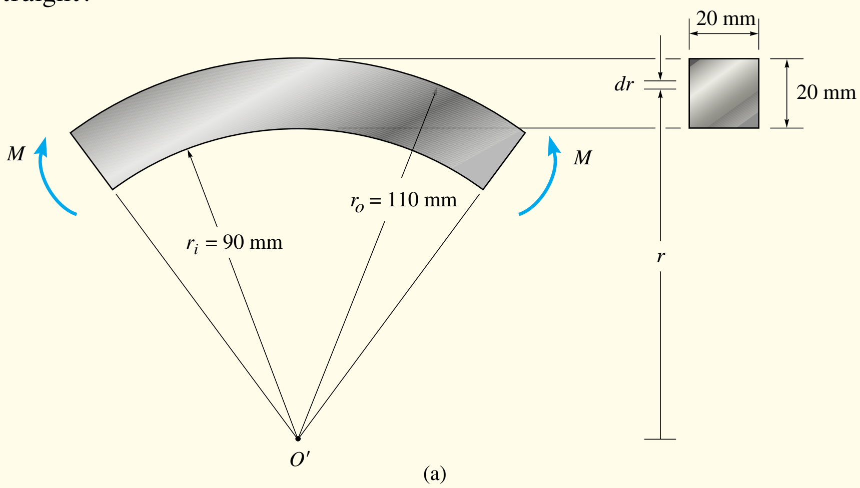


Fig. 6–45

Solution

Internal Moment. Since M tends to increase the bar's radius of curvature, it is positive.

Section Properties. The location of the neutral axis is determined using Eq. 6–23. From Fig. 6–45*a*, we have

$$\int_A \frac{dA}{r} = \int_{90\text{ mm}}^{110\text{ mm}} \frac{(20\text{ mm})\,dr}{r} = (20\text{ mm})\ln r \Big|_{90\text{ mm}}^{110\text{ mm}} = 4.0134\text{ mm}$$

This same result can of course be obtained directly from Table 6–2. Thus,

$$R = \frac{A}{\displaystyle\int_A \frac{dA}{r}} = \frac{(20\text{ mm})(20\text{ mm})}{4.0134\text{ mm}} = 99.666\text{ mm}$$

Continued

It should be noted that throughout the above calculations, R must be determined to several significant figures to ensure that $(\bar{r} - R)$ is accurate to at least three significant figures.

It is unknown if the normal stress reaches its maximum at the top or at the bottom of the bar, and so we must compute the moment M for each case separately. Since the normal stress at the bar's top is compressive, $\sigma = -140$ MPa,

$$\sigma = \frac{M(R - r_o)}{Ar_o(\bar{r} - R)}$$

$$-140\ \text{N/mm}^2 = \frac{M(99.666\ \text{mm} - 110\ \text{mm})}{(20\ \text{mm})(20\ \text{mm})(110\ \text{mm})(100\ \text{mm} - 99.666\ \text{mm})}$$

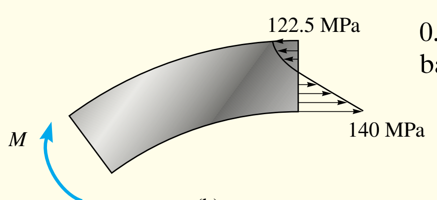
$$M = 199094\ \text{N} \cdot \text{mm} = 0.199\ \text{kN} \cdot \text{m}$$

Likewise, at the bottom of the bar the normal stress will be tensile, so $\sigma = +140$ MPa. Therefore,

$$\sigma = \frac{M(R - r_i)}{Ar_i(\bar{r} - R)}$$

$$140\ \text{N/mm}^2 = \frac{M(99.666\ \text{mm} - 90\ \text{mm})}{(20\ \text{mm})(20\ \text{mm})(90\ \text{mm})(100\ \text{mm} - 99.666\ \text{mm})}$$

$$M = 174153\ \text{N} \cdot \text{mm} = 0.174\ \text{kN} \cdot \text{m}$$ *Ans.*



(b)

By comparison, the maximum moment that can be applied is 0.174 kN · m and so maximum normal stress occurs at the bottom of the bar. The compressive stress at the top of the bar is then

$$\sigma = \frac{174153\ \text{N} \cdot \text{mm}(99.666\ \text{mm} - 110\ \text{mm})}{(20\ \text{mm})(20\ \text{mm})(110\ \text{mm})(100\ \text{mm} - 99.666\ \text{mm})}$$

$$= 122.5\ \text{N/mm}^2$$

The stress distribution is shown in Fig. 6–45*b*.

If the bar was straight, then

$$\sigma = \frac{Mc}{I}$$

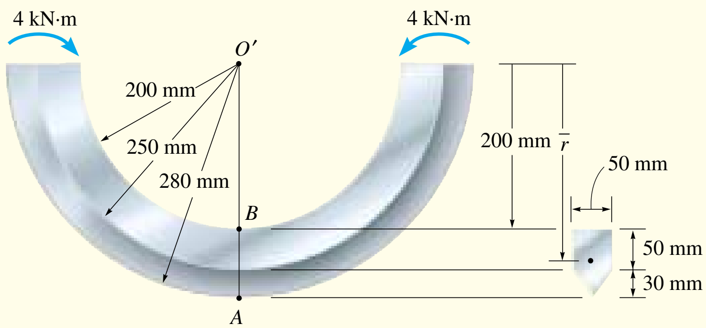
$$140\ \text{N/mm}^2 = \frac{M(10\ \text{mm})}{\frac{1}{12}(20\ \text{mm})(20\ \text{mm})^3}$$

$$M = 186666.7\ \text{N} \cdot \text{mm} = 0.187\ \text{kN} \cdot \text{m}$$ *Ans.*

This represents an error of about 7% from the more exact value determined above.

EXAMPLE 6.25

The curved bar has a cross-sectional area shown in Fig. 6–46*a*. If it is subjected to bending moments of 4 kN·m, determine the maximum normal stress developed in the bar.



(a)

Fig. 6–46

Solution

Internal Moment. Each section of the bar is subjected to the same resultant internal moment of 4 kN·m. Since this moment tends to decrease the bar's radius of curvature, it is negative. Thus, $M = -4\text{ kN}\cdot\text{m}$.

Section Properties. Here we will consider the cross section to be composed of a rectangle and triangle. The total cross-sectional area is

$$\Sigma A = (0.05\text{ m})^2 + \frac{1}{2}(0.05\text{ m})(0.03\text{ m}) = 3.250(10^{-3})\text{ m}^2$$

The location of the centroid is determined with reference to the center of curvature, point O', Fig. 6–46*a*.

$$\bar{r} = \frac{\Sigma \tilde{r} A}{\Sigma A}$$

$$= \frac{[0.225\text{ m}](0.05\text{ m})(0.05\text{ m}) + [0.260\text{ m}]\frac{1}{2}(0.050\text{ m})(0.030\text{ m})}{3.250(10^{-3})\text{ m}^2}$$

$$= 0.23308\text{ m}$$

Continued

We can compute $\int_A dA/r$ for each part using Table 6–2. For the rectangle,

$$\int_A \frac{dA}{r} = 0.05\text{ m}\left(\ln\frac{0.250\text{ m}}{0.200\text{ m}}\right) = 0.011157\text{ m}$$

And for the triangle,

$$\int_A \frac{dA}{r} = \frac{(0.05\text{ m})(0.280\text{ m})}{(0.280\text{ m} - 0.250\text{ m})}\left(\ln\frac{0.280\text{ m}}{0.250\text{ m}}\right) - 0.05\text{ m} = 0.0028867\text{ m}$$

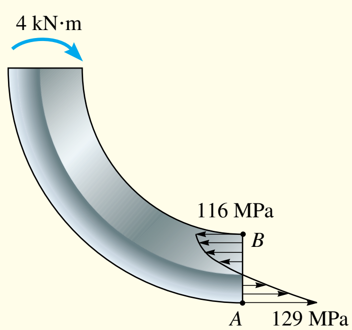
Thus the location of the neutral axis is determined from

$$R = \frac{\Sigma A}{\Sigma \displaystyle\int_A dA/r} = \frac{3.250(10^{-3})\text{ m}^2}{0.011157\text{ m} + 0.0028867\text{ m}} = 0.23142\text{ m}$$

Note that $R < \bar{r}$ as expected. Also, the calculations were performed with sufficient accuracy so that $(\bar{r} - R) = 0.23308\text{ m} - 0.23142\text{ m} = 0.00166\text{ m}$ is now accurate to three significant figures.

Normal Stress. The maximum normal stress occurs either at A or B. Applying the curved-beam formula to calculate the normal stress at B, $r_B = 0.200$ m, we have

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{(-4\text{ kN}\cdot\text{m})(0.23142\text{ m} - 0.200\text{ m})}{3.2500(10^{-3})\text{ m}^2(0.200\text{ m})(0.00166\text{ m})}$$
$$= -116\text{ MPa}$$



(b)

At point A, $r_A = 0.280$ m and the normal stress is

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{(-4\text{ kN}\cdot\text{m})(0.23142\text{ m} - 0.280\text{ m})}{3.2500(10^{-3})\text{ m}^2(0.280\text{ m})0.00166\text{ m})}$$
$$= 129\text{ MPa}$$ *Ans.*

By comparison, the maximum normal stress is at A. A two dimensional representation of the stress distribution is shown in Fig. 6–46*b*.

EXAMPLE 6.26

The transition in the cross-sectional area of the steel bar is achieved using shoulder fillets as shown in Fig. 6–51*a*. If the bar is subjected to a bending moment of 5 kN · m, determine the maximum normal stress developed in the steel. The yield stress is $\sigma_Y = 500$ MPa.

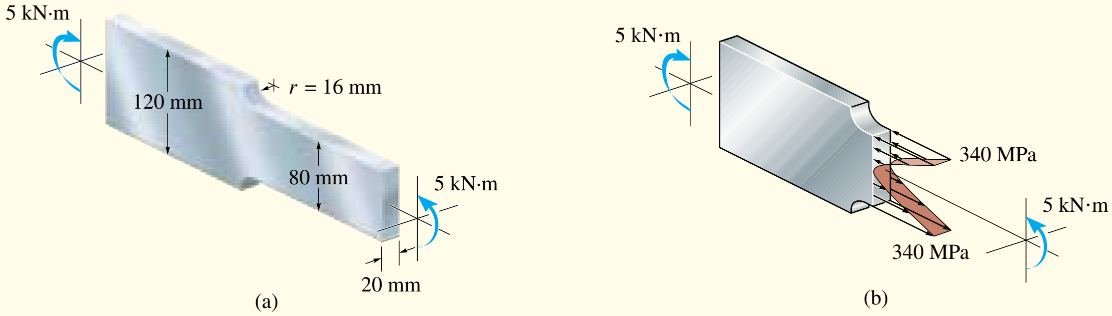


Fig. 6–51

Solution

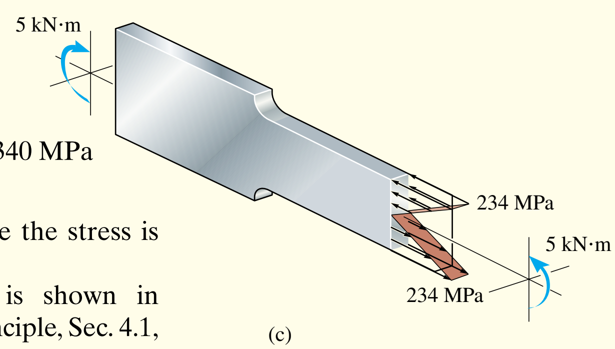
The moment creates the largest stress in the bar at the base of the fillet, where the cross-sectional area is smallest. The stress-concentration factor can be determined by using Fig. 6–48. From the geometry of the bar, we have $r = 16$ mm, $h = 80$ mm, $w = 120$ mm. Thus,

$$\frac{r}{h} = \frac{16\text{ mm}}{80\text{ mm}} = 0.2 \qquad \frac{w}{h} = \frac{120\text{ mm}}{80\text{ mm}} = 1.5$$

These values give $K = 1.45$.

Applying Eq. 6–26, we have

$$\sigma_{\max} = K\frac{M_c}{I} = (1.45)\frac{(5\text{ kN}\cdot\text{m})(0.04\text{ m})}{\left[\frac{1}{12}(0.020\text{ m})(0.08\text{ m})^3\right]} = 340\text{ MPa}$$



This result indicates that the steel remains elastic since the stress is below the yield stress (500 MPa).

The normal-stress distribution is nonlinear and is shown in Fig. 6–51*b*. Realize, however, that by Saint-Venant's principle, Sec. 4.1, these localized stresses smooth out and become linear when one moves (approximately) a distance of 80 mm or more to the right of the transition. In this case, the flexure formula gives $\sigma_{\max} = 234$ MPa, Fig. 6–51*c*. Also note that the choice of a larger-radius fillet will significantly reduce $\sigma_{\max}$, since as r increases in Fig. 6–48, K will decrease.

EXAMPLE 6.27

The steel wide-flange beam has the dimensions shown in Fig. 6–56*a*. If it is made of an elastic perfectly plastic material having a tensile and compressive yield stress of $\sigma_Y = 250$ MPa, determine the shape factor for the beam.

Solution

In order to determine the shape factor, it is first necessary to compute the maximum elastic moment M_Y and the plastic moment M_p.

Maximum Elastic Moment. The normal-stress distribution for the maximum elastic moment is shown in Fig. 6–56*b*. The moment of inertia about the neutral axis is

$$I = \left[\frac{1}{12}(12.5\text{ mm})(225\text{ mm})^3\right] + 2\left[\frac{1}{12}(200\text{ mm})(12.5\text{ mm})^3 + 200\text{ mm}(12.5\text{ mm})(118.75\text{ mm})^2\right]$$

$$= 82.44 \times 10^6\text{ mm}^4$$

Applying the flexure formula, we have

$$\sigma_{\max} = \frac{Mc}{I}; \qquad 250\text{ N/mm}^2 = \frac{M_Y(125\text{ mm})}{82.44 \times 10^6\text{ mm}^4}$$

$$M_Y = 164.88\text{ kN}\cdot\text{m}$$

Plastic Moment. The plastic moment causes the steel over the entire cross section of the beam to yield, so that the normal-stress distribution looks like that shown in Fig. 6–56*c*. Due to symmetry of the cross-sectional area and since the tension and compression stress–strain diagrams are the same, the neutral axis passes through the centroid of the cross section. In order to determine the plastic moment, the stress distribution is divided into four composite rectangular "blocks," and the force produced by each "block" is equal to the volume of the block. Therefore, we have

$$C_1 = T_1 = 250\text{ N/mm}^2(12.5\text{ mm})(112.5\text{ mm}) = 351.56\text{ kN}$$

$$C_2 = T_2 = 250\text{ N/mm}^2(12.5\text{ mm})(200\text{ mm}) = 625\text{ kN}$$

These forces act through the *centroid* of the volume for each block. Computing the moments of these forces about the neutral axis, we obtain the plastic moment:

$$M_p = 2[56.25\text{ mm})(351.56\text{ kN})] + 2[(118.75\text{ mm})(625\text{ kN})] = 188\text{ kN}\cdot\text{m}$$

Shape Factor. Applying Eq. 6–33 gives

$$k = \frac{M_p}{M_Y} = \frac{188\text{ kN}\cdot\text{m}}{164.88\text{ kN}\cdot\text{m}} = 1.14 \qquad \textit{Ans.}$$

This value indicates that a wide-flange beam provides a very efficient section for resisting an *elastic moment*. Most of the moment is developed in the flanges, i.e., in the top and bottom segments, whereas the web or vertical segment contributes very little. In this particular case, only 14% additional moment can be supported by the beam beyond that which can be supported elastically.

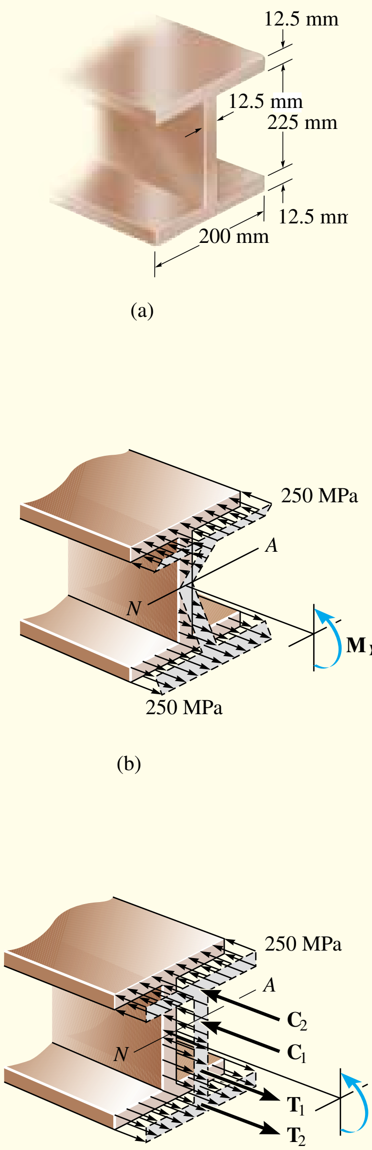


Fig. 6–56

EXAMPLE 6.28

A T-beam has the dimensions shown in Fig. 6–57*a*. If it is made of an elastic perfectly plastic material having a tensile and compressive yield stress of $\sigma_Y = 250$ MPa, determine the plastic moment that can be resisted by the beam.

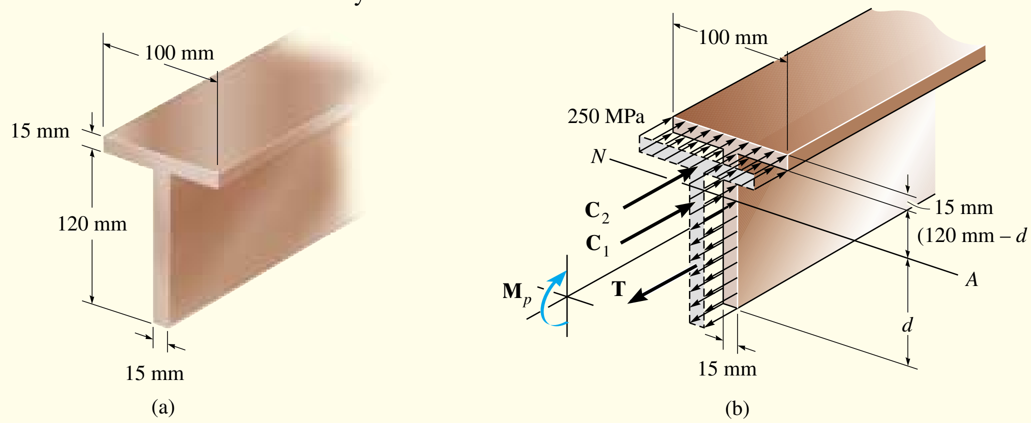


Fig. 6–57

Solution

The "plastic" stress distribution acting over the beam's cross-sectional area is shown in Fig. 6–57*b*. In this case the cross section is not symmetric with respect to a horizontal axis, and consequently, the neutral axis will *not* pass through the centroid of the cross section. To determine the *location* of the neutral axis, d, we require the stress distribution to produce a zero resultant force on the cross section. Assuming that $d \le 120$ mm, we have

$$\int_A \sigma\, dA = 0; \qquad T - C_1 - C_2 = 0$$

$$250 \text{ MPa}(0.015 \text{ m})(d) - 250 \text{ MPa}(0.015 \text{ m})(0.120 \text{ m} - d)$$
$$- 250 \text{ MPa}(0.015 \text{ m})(0.100 \text{ m}) = 0$$
$$d = 0.110 \text{ m} < 0.120 \text{ m} \qquad \text{OK}$$

Using this result, the forces acting on each segment are

$$T = 250 \text{ MN/m}^2(0.015 \text{ m})(0.110 \text{ m}) = 412.5 \text{ kN}$$
$$C_1 = 250 \text{ MN/m}^2(0.015 \text{ m})(0.010 \text{ m}) = 37.5 \text{ kN}$$
$$C_2 = 250 \text{ MN/m}^2(0.015 \text{ m})(0.100 \text{ m}) = 375 \text{ kN}$$

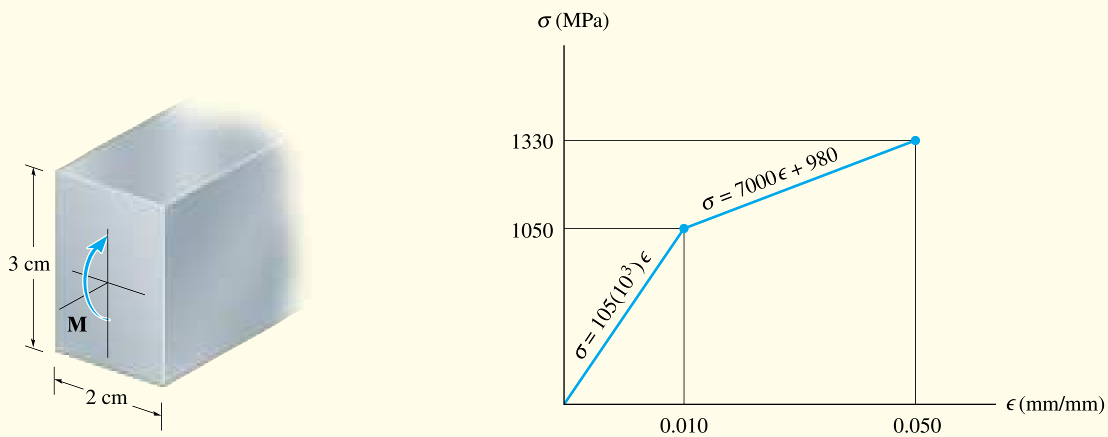
Hence, the resultant plastic moment about the neutral axis is

$$M_p = 412.5 \text{ kN}\left(\frac{0.110 \text{ m}}{2}\right) + 37.5 \text{ kN}\left(\frac{0.01 \text{ m}}{2}\right) + 375 \text{ kN}\left(0.01 \text{ m} + \frac{0.015 \text{ m}}{2}\right)$$

$$M_p = 29.4 \text{ kN}\cdot\text{m}$$ *Ans.*

EXAMPLE 6.29

The beam in Fig. 6–58*a* is made of an alloy of titanium that has a stress–strain diagram that can in part be approximated by two straight lines. If the material behavior is the *same* in both tension and compression, determine the bending moment that can be applied to the beam that will cause the material at the top and bottom of the beam to be subjected to a strain of 0.050 mm/mm.

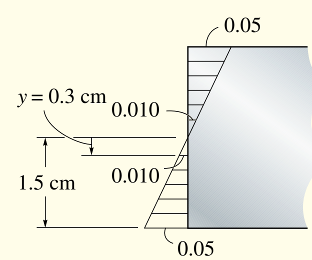


(a)

Solution I

By inspection of the stress–strain diagram, the material is said to exhibit "elastic-plastic behavior with strain hardening." Since the cross section is symmetric and the tension–compression $\sigma-\epsilon$ diagrams are the same, the neutral axis must pass through the centroid of the cross section. The strain distribution, which is always linear, is shown in Fig. 6–58*b*. In particular, the point where maximum elastic strain (0.010 mm/mm) occurs has been determined by proportion, such that $0.05/1.5\text{ cm} = 0.010/y$ or $y = 0.3\text{ cm} = 3\text{ mm}$.

The corresponding normal-stress distribution acting over the cross section is shown in Fig. 6–58*c*. The moment produced by this distribution can be calculated by finding the "volume" of the stress blocks. To do so, we will subdivide this distribution into two triangular blocks and a rectangular block in both the tension and compression regions, Fig. 6–58*d*. Since the beam is 2 cm wide, the resultants and their locations are determined as follows:

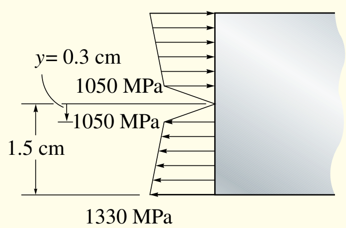


Strain distribution

(b)

Fig. 6–58

Continued



Stress distribution

(c)

$$T_1 = C_1 = \frac{1}{2}(12 \text{ mm})(280 \text{ N/mm}^2)(20 \text{ mm}) = 33600 \text{ N} = 33.6 \text{ kN}$$

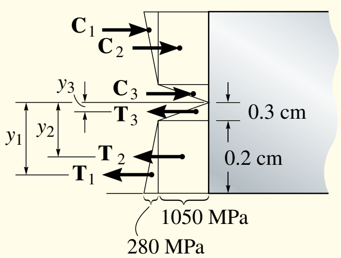
$$y_1 = 0.3\,\text{cm} + \frac{2}{3}(1.2 \text{ cm}) = 1.10 \text{ cm} = 11.0 \text{ mm}$$

$$T_2 = C_2 = (12 \text{ mm})(1050 \text{ N/mm}^2)(20 \text{ mm}) = 25200 \text{ N} = 252 \text{ kN}$$

$$y_2 = 0.3\,\text{cm} + \frac{1}{2}(1.2 \text{ cm}) = 0.90 \text{ cm} = 9 \text{ mm}$$

$$T_3 = C_3 = \frac{1}{2}(3 \text{ mm})(1050 \text{ N/mm}^2)(20 \text{ mm}) = 31500 \text{ N} = 31.5 \text{ kN}$$

$$y_3 = \frac{2}{3}(0.3 \text{ cm}) = 0.2 \text{ cm} = 2 \text{ mm}$$



(d)

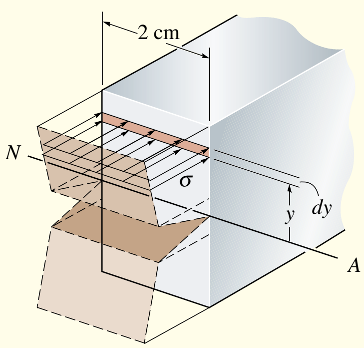
The moment produced by this normal-stress distribution about the neutral axis is therefore

$$M = 2\,[33.6 \text{ kN } (110 \text{ mm}) + 252 \text{ kN } (9 \text{ mm}) + 31.5 \text{ kN } (2 \text{ mm})]$$
$$= 5401.2 \text{ kN}\cdot\text{mm} = 5.40 \text{ kN}\cdot\text{m}$$ *Ans.*

Solution II

Rather than using the above semigraphical technique, it is also possible to compute the moment analytically. To do this, we must express the stress distribution in Fig. 6–58*c* as a function of position y along the beam. Note that $\sigma = f(\epsilon)$ has been given in Fig. 6–58*a*. Also, from Fig. 6–58*b*, the normal strain can be determined as a function of position y by proportional triangles; i.e.,

$$\epsilon = \frac{0.05}{1.5}y \qquad 0 \le y \le 1.5 \text{ cm} = 15 \text{ mm}$$



(e)

Substituting this into the $\sigma - \epsilon$ functions shown in Fig. 6–58*a* gives

$$\sigma = 350y \qquad 0 \le y \le 0.3\,\text{cm} = 3 \text{ mm} \qquad (1)$$
$$\sigma = 23.33y + 980 \qquad 3 \text{ cm} \le y \le 1.5 \text{ cm} = 15 \text{ mm} \qquad (2)$$

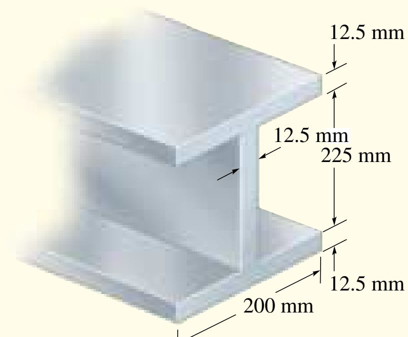
From Fig. 6–58*e*, the moment caused by σ acting on the area strip $dA = 2\,dy$ is

$$dM = y(\sigma\, dA) = y\sigma\,(20\;dy)$$

Using Eqs. 1 and 2, the moment for the entire cross section is thus

$$M = 2\left[20\int_0^3 350y^2\,dy + 20\int_3^{15}(23.33y^2\,dy + 980y)\,dy\right]$$
$$= 5401(10^3) \text{ N}\cdot\text{mm} = 5.40 \text{ kN}\cdot\text{m}$$ *Ans.*

EXAMPLE 6.30



(a)

The steel wide-flange beam shown in Fig. 6–60*a* is subjected to a fully plastic moment of $\mathbf{M}_p$. If this moment is removed, determine the residual-stress distribution in the beam. The material is elastic perfectly plastic and has a yield stress of $\sigma_Y = 250$ MPa.

Solution

The normal-stress distribution in the beam caused by $\mathbf{M}_p$ is shown in Fig. 6–60*b*. When $\mathbf{M}_p$ is removed, the material responds elastically. Removal of $\mathbf{M}_p$ requires applying $\mathbf{M}_p$ in its reverse direction and therefore leads to an assumed elastic stress distribution as shown in Fig. 6–60*c*. The modulus of rupture σ_r is computed from the flexure formula. Using $M_P = 188$ kN · m and $I = 82.44 \times 10^6$ mm^4 from Example 6.27, we have

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_{\text{allow}} = \frac{(188 \times 10^6 \text{ N} \cdot \text{mm})(125 \text{ mm})}{82.44 \times 10^6 \text{ mm}^4}$$

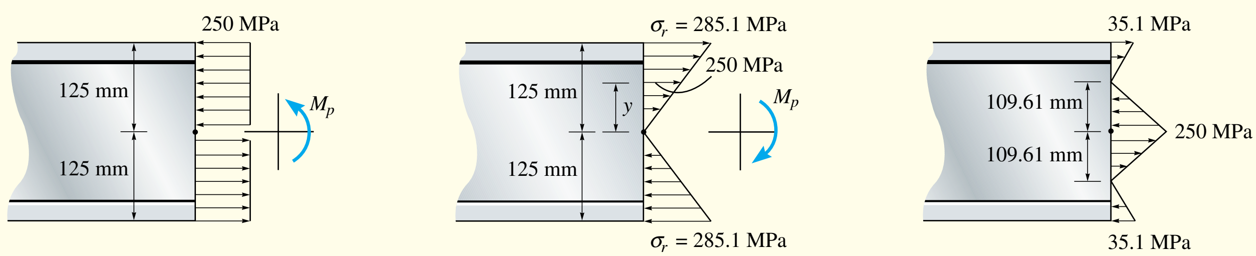
$$= 285.1 \text{ N/mm}^2 = 285.1 \text{ MPa}$$

As expected, $\sigma_r < 2\sigma_Y$.

Superposition of the stresses gives the residual-stress distribution shown in Fig. 6–60*d*. Note that the point of zero normal stress was determined by proportion; i.e., from Fig. 6–60*b* and 6–60*c*, we require that

$$\frac{281.51 \text{ MPa}}{125 \text{ mm}} = \frac{2501 \text{ MPa}}{y}$$

$$y = 109.61 \text{ mm}$$



Plastic moment applied (profile view)

(b)

Plastic moment reversed (profile view)

(c)

Residual stress distribution

(d)

Fig. 6–60