The composite A-36 steel bar shown in Fig. 4–6a is made from two segments, AB and BD, having cross-sectional areas of $A_{AB} = 600 \text{ mm}^2$ and $A_{BD} = 1200 \text{ mm}^2$. Determine the vertical displacement of end A and the displacement of B relative to C.
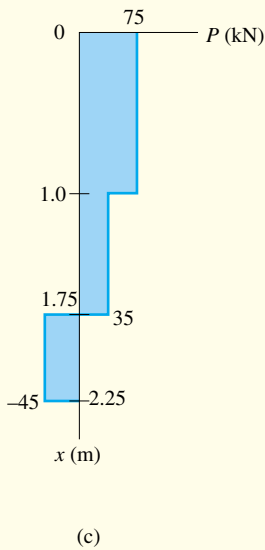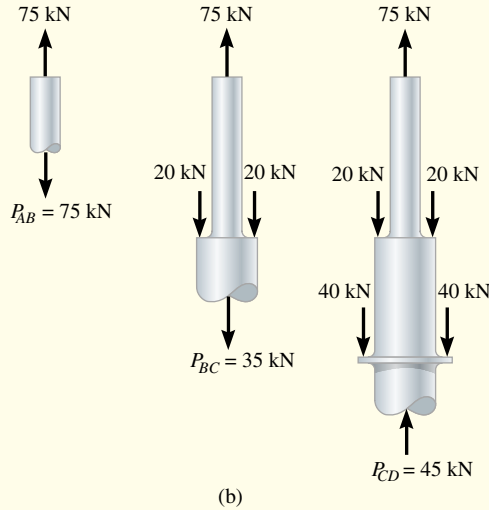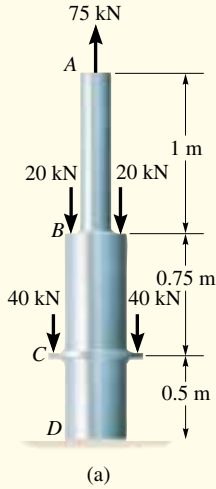
75 kN

A

1 m

20 kN   20 kN

B

0.75 m

40 kN   40 kN

C

0.5 m

D

(a)

75 kN   75 kN   75 kN

20 kN 20 kN   20 kN 20 kN

$P_{AB} = 75 \text{ kN}$

40 kN   40 kN

$P_{BC} = 35 \text{ kN}$

$P_{CD} = 45 \text{ kN}$

(b)

Solution

Internal Force. Due to the application of the external loadings, the internal axial forces in regions AB, BC, and CD will all be different. These forces are obtained by applying the method of sections and the equation of vertical force equilibrium as shown in Fig. 4–6b. This variation is plotted in Fig. 4–6c.

Displacement. From the inside back cover, $E_{st} = 210(10^3)$ MPa. Using the sign convention, i.e., the internal tensile forces are positive and the compressive forces are negative, the vertical displacement of A relative to the *fixed* support D is

$$\delta_A = \Sigma \frac{PL}{AE} = \frac{[+75 \text{ kN}] (1 \text{ m})(10^6)}{[600 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]} + \frac{[+35 \text{ kN}] (0.75 \text{ m})(10^6)}{[1200 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]}$$

$$+ \frac{[-45 \text{ kN}] (0.5 \text{ m})(10^6)}{[1200 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]}$$

$$= +0.61 \text{ mm} \qquad \textit{Ans.}$$

Since the result is *positive*, the bar *elongates* and so the displacement at A is upward.

Applying Eq. 4–2 between points B and C, we obtain,

$$\delta_{B/C} = \frac{P_{BC}L_{BC}}{A_{BC}B} = \frac{[+35 \text{ kN}] (0.75 \text{ m})(10^6)}{[1200 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]} = +0.104 \text{ mm} \quad \textit{Ans.}$$

Here B moves away from C, since the segment elongates.

0   75
          P (kN)

1.0

1.75
          35

−45   −2.25

x (m)

(c)

Fig. 4–6

The assembly shown in Fig. 4–7a consists of an aluminum tube AB having a cross-sectional area of 400 mm². A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end C of the rod. Take E_{st} = 200 GPa, E_{al} = 70 GPa.
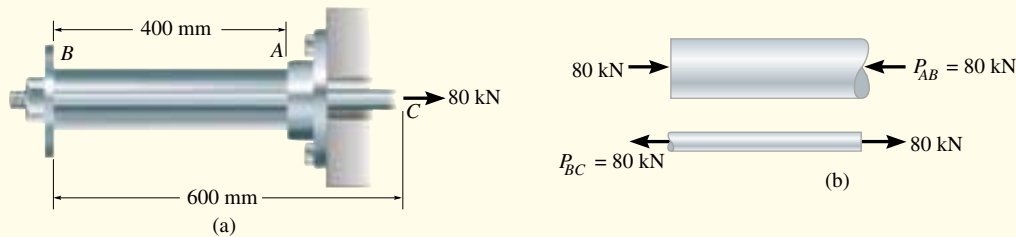

Fig. 4–7

Solution

Internal Force. The free-body diagram of the tube and rod, Fig. 4–7b, shows that the rod is subjected to a tension of 80 kN and the tube is subjected to a compression of 80 kN.

Displacement. We will first determine the displacement of end C with respect to end B. Working in units of newtons and meters, we have

$$\delta_{C/B} = \frac{PL}{AE} = \frac{[+80(10^3)\ \text{N}](0.6\ \text{m})}{\pi(0.005\ \text{m})^2[200(10^9)\ \text{N/m}^2]} = +0.003056\ \text{m} \rightarrow$$

The positive sign indicates that end C moves *to the right* relative to end B, since the bar elongates.

The displacement of end B with respect to the *fixed* end A is

$$\delta_B = \frac{PL}{AE} = \frac{[-80(10^3)\ \text{N}](0.4\ \text{m})}{[400\ \text{mm}^2(10^{-6})\ \text{m}^2/\text{mm}^2][70(10^9)\ \text{N/m}^2]}$$

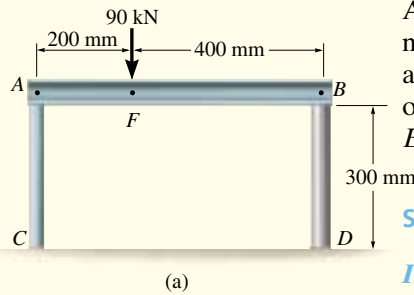$$= -0.001143\ \text{m} = 0.001143\ \text{m} \rightarrow$$

Here the negative sign indicates that the tube shortens, and so B moves to the *right* relative to A.

Since both displacements are to the right, the resultant displacement of C relative to the fixed end A is therefore

$$(\overset{+}{\rightarrow}) \qquad \delta_C = \delta_B + \delta_{C/B} = 0.001143\ \text{m} + 0.003056\ \text{m}$$

$$= 0.00420\ \text{m} = 4.20\ \text{mm} \rightarrow \qquad\qquad Ans.$$

90 kN

200 mm | 400 mm

A ● ● ● ● B
F

300 mm

C | D

(a)

90 kN

200 mm | 400 mm

60 kN | 30 kN

(b)

60 kN | 30 kN

P_{AC} = 60 kN | P_{BD} = 30 kN

(c)

A *rigid beam AB* rests on the two short posts shown in Fig. 4–8a. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take E_{st} = 200 GPa, E_{al} = 70 GPa.
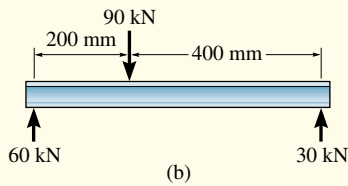
Solution

Internal Force. The compressive forces acting at the top of each post are determined from the equilibrium of member AB, Fig. 4–8b. These forces are equal to the internal forces in each post, Fig. 4–8c.

Displacement. The displacement of the top of each post is

Post AC:

$$\delta_A = \frac{P_{AC}L_{AC}}{A_{AC}E_{st}} = \frac{[-60(10^3)\text{ N}](0.300\text{ m})}{\pi(0.010\text{ m})^2[200(10^9)\text{ N/m}^2]} = -286(10^{-6})\text{ m}$$

$$= 0.286\text{ mm} \downarrow$$

Post BD:

$$\delta_B = \frac{P_{BD}L_{BD}}{A_{BD}E_{al}} = \frac{[-30(10^3)\text{ N}](0.300\text{ m})}{\pi(0.020\text{ m})^2[70(10^9)\text{ N/m}^2]} = -102(10^{-6})\text{ m}$$

$$= 0.102\text{ mm} \downarrow$$

A diagram showing the centerline displacements at points A, B, and F on the beam is shown in Fig. 4–8d. By proportion of the shaded triangle, the displacement of point F is therefore

$$\delta_F = 0.102\text{ mm} + (0.184\text{ mm})\left(\frac{400\text{ mm}}{600\text{ mm}}\right) = 0.225\text{ mm} \downarrow \qquad Ans.$$

0.102 mm
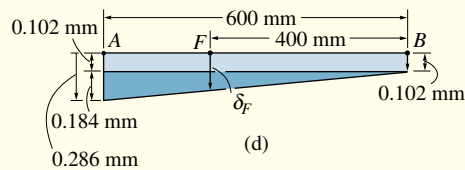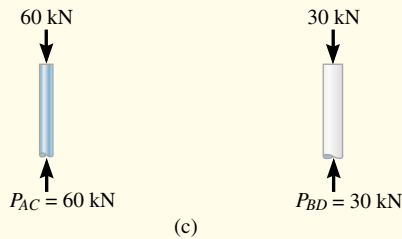
600 mm
A | F | 400 mm | B

δ_F

0.102 mm

0.184 mm

0.286 mm

(d)

Fig. 4–8

A member is made from a material that has a specific weight γ and modulus of elasticity E. If it is formed into a *cone* having the dimensions shown in Fig. 4–9a, determine how far its end is displaced due to gravity when it is suspended in the vertical position.

Solution

Internal Force. The internal axial force varies along the member since it is dependent on the weight $W(y)$ of a segment of the member below any section, Fig. 4–9b. Hence, to calculate the displacement, we must use Eq. 4–1. At the section located at a distance y from its bottom end, the radius x of the cone as a function of y is determined by proportion; i.e.,

$$\frac{x}{y} = \frac{r_0}{L}; \qquad x = \frac{r_0}{L} y$$

The volume of a cone having a base of radius x and height y is

$$V = \frac{\pi}{3} y x^2 = \frac{\pi r_0^2}{3L^2} y^3$$

Since $W = \gamma V$, the internal force at the section becomes

$$+\uparrow \Sigma F_y = 0; \qquad P(y) = \frac{\gamma \pi r_0^2}{3L^2} y^3$$

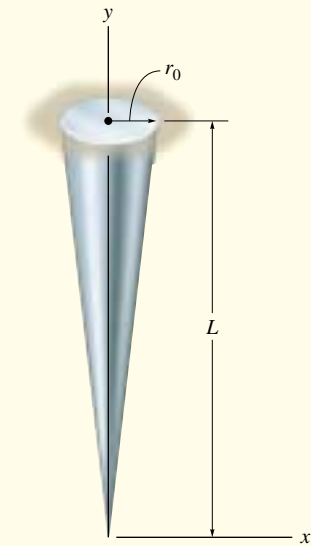*Displacement.* The area of the cross section is also a function of position y, Fig. 4–9b. We have

$$A(y) = \pi x^2 = \frac{\pi r_0^2}{L^2} y^2$$

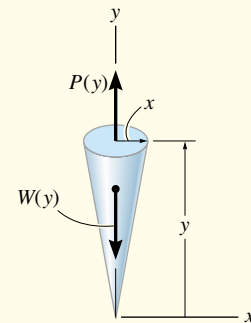Applying Eq. 4–1 between the limits of $y = 0$ and $y = L$ yields

$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E} = \int_0^L \frac{[(\gamma \pi r_0^2/3L^2)\,y^3]\,dy}{[(\pi r_0^2/L^2)\,y^2]\,E}$$

$$= \frac{\gamma}{3E} \int_0^L y\,dy$$

$$= \frac{\gamma L^2}{6E} \qquad\qquad \text{Ans.}$$

As a partial check of this result, notice how the units of the terms, when canceled, give the displacement in units of length as expected.
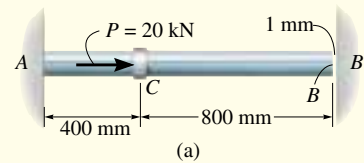

(a)

(b)

Fig. 4–9

The steel rod shown in Fig. 4–12a has a diameter of 5 mm. It is attached to the fixed wall at A, and before it is loaded, there is a gap between the wall at B' and the rod of 1 mm. Determine the reactions at A and B' if the rod is subjected to an axial force of $P = 20$ kN as shown. Neglect the size of the collar at C. Take $E_{st} = 200$ GPa.
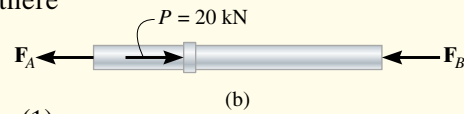
(a)

Solution

Equilibrium. As shown on the free-body diagram, Fig. 4–12b, we will assume that the force P is large enough to cause the rod's end B to contact the wall at B'. The problem is statically indeterminate since there are two unknowns and only one equation of equilibrium.
   Equilibrium of the rod requires

(b)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_A - F_B + 20(10^3)\ \text{N} = 0 \qquad (1)$$

Compatibility. The loading causes point B to move to B', with no further displacement. Therefore the compatibility condition for the rod is

$$\delta_{B/A} = 0.001\ \text{m}$$

   This displacement can be expressed in terms of the unknown reactions by using the load–displacement relationship, Eq. 4–2, applied to segments AC and CB, Fig. 4–12c. Working in units of newtons and meters, we have

(c)

Fig. 4–12
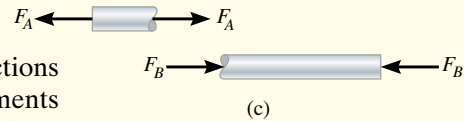
$$\delta_{B/A} = 0.001\ \text{m} = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE}$$

$$0.001\ \text{m} = \frac{F_A(0.4\ \text{m})}{\pi(0.0025\ \text{m})^2[200(10^9)\ \text{N/m}^2]}$$

$$- \frac{F_B(0.8\ \text{m})}{\pi(0.0025\ \text{m})^2[200(10^9)\ \text{N/m}^2]}$$

or

$$F_A(0.4\ \text{m}) - F_B(0.8\ \text{m}) = 3927.0\ \text{N} \cdot \text{m} \qquad (2)$$

Solving Eqs. 1 and 2 yields

$$F_A = 16.6\ \text{kN} \qquad F_B = 3.39\ \text{kN} \qquad \textit{Ans.}$$

Since the answer for F_B is *positive*, indeed the end B contacts the wall at B' as originally assumed. On the other hand, if F_B were a negative quantity, the problem would be statically determinate, so that $F_B = 0$ and $F_A = 20$ kN.

P = 45 kN

50 mm — 25 mm

0.5 m

P = 45 kN

$\mathbf{F}_{br}$

$\mathbf{F}_{al}$

(b)

σ_{br} = 7.64 MPa
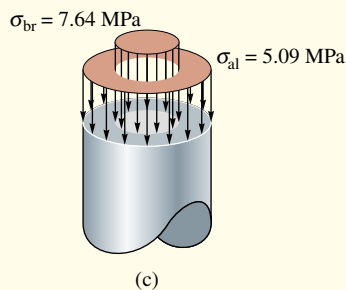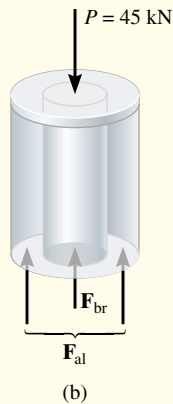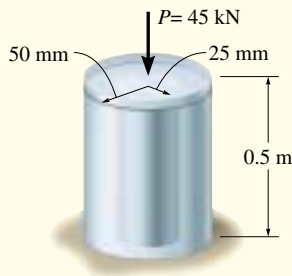
σ_{al} = 5.09 MPa

(c)

Fig. 4–13

The aluminum post shown in Fig. 4–13a is reinforced with a brass core. If this assembly supports a resultant axial compressive load of $P = 45$ kN, applied to the rigid cap, determine the average normal stress in the aluminum and the brass. Take $E_{al} = 70(10^3)$ MPa and $E_{br} = 105(10^3)$ MPa.

Solution

Equilibrium. The free-body diagram of the post is shown in Fig. 4–13b. Here the resultant axial force at the base is represented by the unknown components carried by the aluminum, $\mathbf{F}_{al}$, and brass, $\mathbf{F}_{br}$. The problem is statically indeterminate. Why?

Vertical force equilibrium requires

$$+\uparrow \ \Sigma F_y = 0; \qquad -45 \text{ kN} + F_{al} + F_{br} = 0 \qquad (1)$$

Compatibility. The rigid cap at the top of the post causes both the aluminum and brass to displace the same amount. Therefore,

$$\delta_{al} = \delta_{br}$$

Using the load–displacement relationships,

$$\frac{F_{al}L}{A_{al}E_{al}} = \frac{F_{br}L}{A_{br}E_{br}}$$

$$F_{al} = F_{br}\left(\frac{A_{al}}{A_{br}}\right)\left(\frac{E_{al}}{E_{br}}\right)$$

$$F_{al} = F_{br}\left[\frac{\pi[(0.05 \text{ m})^2 - (0.025 \text{ m})^2]}{\pi(0.025 \text{ m}^2)}\right]\left[\frac{70(10)^3 \text{ MPa}}{105(10^3) \text{ MPa}}\right]$$

$$F_{al} = 2F_{br} \qquad (2)$$

Solving Eqs. 1 and 2 simultaneously yields

$$F_{al} = 30 \text{ kN} \qquad F_{br} = 15 \text{ kN}$$

Since the results are positive, indeed the stress will be compressive.

The average normal stress in the aluminum and brass is therefore

$$\sigma_{al} = \frac{30 \text{ kN}}{\pi[(0.05 \text{ m})^2 - (0.025 \text{ m})^2]} = 5.09 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{br} = \frac{15 \text{ kN}}{\pi[(0.025 \text{ m})^2]} = 7.64 \text{ MPa} \qquad \textit{Ans.}$$

The stress distributions are shown in Fig. 4–13c.

The three A-36 steel bars shown in Fig. 4–14a are pin connected to a *rigid* member. If the applied load on the member is 15 kN, determine the force developed in each bar. Bars *AB* and *EF* each have a cross-sectional area of 25 mm², and bar *CD* has a cross-sectional area of 15 mm².

Solution

Equilibrium. The free-body diagram of the rigid member is shown in Fig. 4–14b. This problem is statically indeterminate since there are three unknowns and only two available equilibrium equations. These equations are

$$+\uparrow \Sigma F_y = 0; \qquad F_A + F_C + F_E - 15 \text{ kN} = 0 \qquad (1)$$

$$\downarrow + \Sigma M_C = 0; \quad -F_A(0.4 \text{ m}) + 15 \text{ kN}(0.2 \text{ m}) + F_E(0.4 \text{ m}) = 0 \qquad (2)$$

Compatibility. The applied load will cause the horizontal line *ACE* shown in Fig. 4–14c to move to the inclined line *A'C'E'*. The displacements of points *A*, *C*, and *E* can be related by proportional triangles. Thus, the compatibility equation for these displacements is

$$\frac{\delta_A - \delta_E}{0.8 \text{ m}} = \frac{\delta_C - \delta_E}{0.4 \text{ m}}$$

$$\delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E$$

Using the load–displacement relationship, Eq. 4–2, we have

$$\frac{F_C L}{(15 \text{ mm}^2) E_{st}} = \frac{1}{2}\left[\frac{F_A L}{(25 \text{ mm}^2) E_{st}}\right] + \frac{1}{2}\left[\frac{F_E L}{(25 \text{ mm}^2) E_{st}}\right]$$

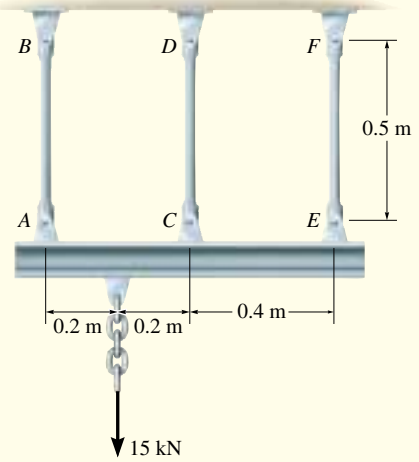$$F_C = 0.3 F_A + 0.3 F_E \qquad (3)$$
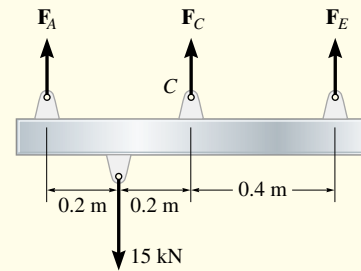
Solving Eqs. 1–3 simultaneously yields

$$F_A = 9.52 \text{ kN} \qquad \qquad \textit{Ans.}$$
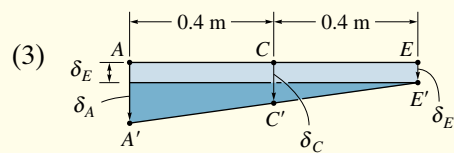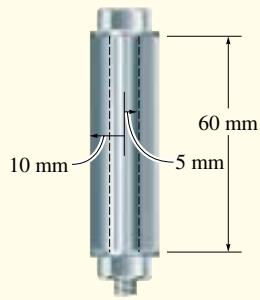$$F_C = 3.46 \text{ kN} \qquad \qquad \textit{Ans.}$$
$$F_E = 2.02 \text{ kN} \qquad \qquad \textit{Ans.}$$

(a)

(b)

(c)

Fig. 4–14

(a)

$\mathbf{F}_t$

$\mathbf{F}_b$

(b)

Final position

δ_b

δ_t 0.5 mm

Initial position
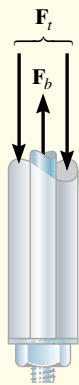
(c)

Fig. 4–15
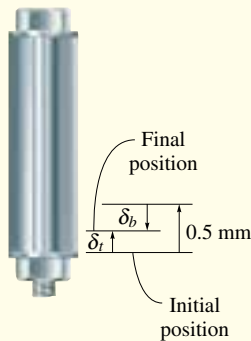
The bolt shown in Fig. 4–15a is made of 2014-T6 aluminum alloy and is tightened so it compresses a cylindrical tube made of Am 1004-T61 magnesium alloy. The tube has an outer radius of 10 mm, and it is assumed that both the inner radius of the tube and the radius of the bolt are 5 mm. The washers at the top and bottom of the tube are considered to be rigid and have a negligible thickness. Initially the nut is hand-tightened slightly; then, using a wrench, the nut is further tightened one-half turn. If the bolt has 20 threads per 20 mm, determine the stress in the bolt.

Solution

Equilibrium. The free-body diagram of a section of the bolt and the tube, Fig. 4–15b, is considered in order to relate the force in the bolt F_b to that in the tube, F_t. Equilibrium requires

$$+\uparrow \Sigma F_y = 0; \qquad\qquad F_b - F_t = 0 \qquad\qquad (1)$$

The problem is statically indeterminate since there are two unknowns in this equation.

Compatibility. When the nut is tightened on the bolt, the tube will shorten δ_t, and the bolt will *elongate* δ_b, Fig. 4–15c. Since the nut undergoes one-half turn, it advances a distance of $\frac{1}{2} \left(\frac{20}{20} \text{ mm} \right) = 0.5$ mm along the bolt. Thus, the compatibility of these displacements requires

$$(+\uparrow) \qquad\qquad \delta_t = 0.5 \text{ mm} - \delta_b$$

Taking the modulus of elasticity $E_{Am} = 45$ GPa, $E_{al} = 75$ GPa, and applying Eq. 4–2, yields

$$\frac{F_t (60 \text{ mm})}{\pi[(10 \text{ mm})^2 - (5 \text{ mm})^2][45(10^3) \text{ MPa}]} = 0.5 \text{ mm} - \frac{F_b (60 \text{ mm})}{\pi(5 \text{ mm})^2[75(10^3) \text{ MPa}]}$$

$$5F_t = 125\pi(1125) - 9F_b \qquad\qquad (2)$$

Solving Eqs. 1 and 2 simultaneously, we get

$$F_b = F_t = 31556 \text{ N} = 31.56 \text{ kN}$$

The stresses in the bolt and tube are therefore

$$\sigma_b = \frac{F_b}{A_b} = \frac{31556 \text{ N}}{\pi(5 \text{ mm})^2} = 401.8 \text{ N/mm}^2 = 401.8 \text{ MPa} \qquad\qquad \textit{Ans.}$$

$$\sigma_s = \frac{F_t}{A_t} = \frac{31556 \text{ N}}{\pi[(10 \text{ mm})^2 - (5 \text{ mm})^2]} = 133.9 \text{ N/mm}^2 = 133.9 \text{ MPa}$$

These stresses are less than the reported yield stress for each material, $(\sigma_Y)_{al} = 414$ MPa and $(\sigma_Y)_{mg} = 152$ MPa (see the inside back cover), and therefore this "elastic" analysis is valid.
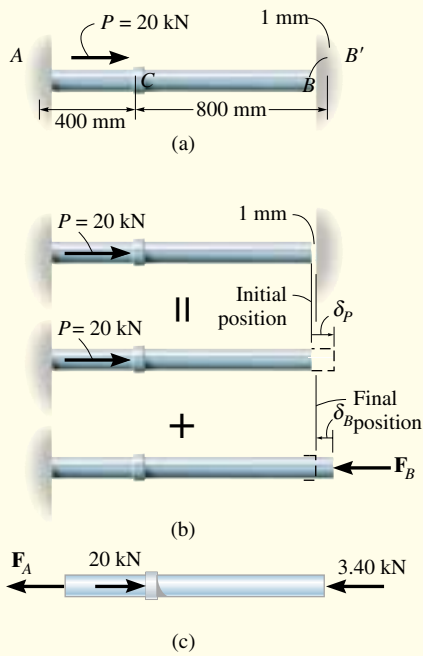
(a)

(b)

(c)

Fig. 4–17

The A-36 steel rod shown in Fig. 4–17a has a diameter of 5 mm. It is attached to the fixed wall at A, and before it is loaded there is a gap between the wall at B' and the rod of 1 mm. Determine the reactions at A and B'.

Solution

Compatibility. Here we will consider the support at B' as redundant. Using the principle of superposition, Fig. 4–17b, we have

$$(\overset{+}{\rightarrow}) \qquad\qquad 0.001 \text{ m} = \delta_P - \delta_B \qquad\qquad (1)$$

The deflections δ_P and δ_B are determined from Eq. 4–2.

$$\delta_P = \frac{PL_{AC}}{AE} = \frac{[20(10^3)\text{ N}](0.4\text{ m})}{\pi(0.0025\text{ m})^2[200(10^9)\text{ N/m}^2]} = 0.002037 \text{ m}$$

$$\delta_B = \frac{F_B L_{AB}}{AE} = \frac{F_B(1.20\text{ m})}{\pi(0.0025\text{ m})^2[200(10^9)\text{ N/m}^2]} = 0.3056(10^{-6})F_B$$

Substituting into Eq. 1, we get

$$0.001 \text{ m} = 0.002037 \text{ m} - 0.3056(10^{-6})F_B$$
$$F_B = 3.40(10^3) \text{ N} = 3.40 \text{ kN} \qquad\qquad \textit{Ans.}$$

Equilibrium. From the free-body diagram, Fig. 4–17c,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_A + 20 \text{ kN} - 3.40 \text{ kN} = 0 \quad F_A = 16.6 \text{ kN} \qquad \textit{Ans.}$$

The A-36 steel bar shown in Fig. 4–18 is constrained to just fit between two fixed supports when $T_1 = 30°C$. If the temperature is raised to $T_2 = 60°C$, determine the average normal thermal stress developed in the bar.

Solution

Equilibrium. The free-body diagram of the bar is shown in Fig. 4–18b. Since there is no external load, the force at A is equal but opposite to the force acting at B; that is,

$$+\uparrow\Sigma F_y = 0; \qquad\qquad F_A = F_B = F$$

The problem is statically indeterminate since this force cannot be determined from equilibrium.

Compatibility. Since $\delta_{A/B} = 0$, the thermal displacement δ_T at A that would occur, Fig. 4–18c, is counteracted by the force **F** that would be required to push the bar δ_F back to its original position. The compatibility condition at A becomes

$$(+\uparrow) \qquad\qquad \delta_{A/B} = 0 = \delta_T - \delta_F$$

Applying the thermal and load–displacement relationships, we have

$$0 = \alpha\Delta T L - \frac{FL}{AL}$$

Thus, from the data on the inside back cover,

$$F = \alpha\Delta T A E$$
$$= [12(10^{-6})/°C]\,(60°C - 30°C)(0.010\ m)^2\,[200(10^6)\ kPa]$$
$$= 7.2\ kN$$

From the magnitude of **F**, it should be apparent that changes in temperature can cause large reaction forces in statically indeterminate members.

Since **F** also represents the internal axial force within the bar, the average normal compressive stress is thus

$$\sigma = \frac{F}{A} = \frac{7.2 \times 10^{-3}\ MN}{(0.01\ m)^2} = 72\ MPa \qquad\qquad \textit{Ans.}$$

10 mm

10 mm

A

1 m

B

(a)

F

F

(b)

δ_T

δ_F

(c)

Fig. 4–18

A 2014-T6 aluminum tube having a cross-sectional area of 600 mm^2 is used as a sleeve for an A-36 steel bolt having a cross-sectional area of 400 mm^2, Fig. 4–19a. When the temperature is $T_1 = 15°C$, the nut holds the assembly in a snug position such that the axial force in the bolt is negligible. If the temperature increases to $T_2 = 80°C$, determine the average normal stress in the bolt and sleeve.
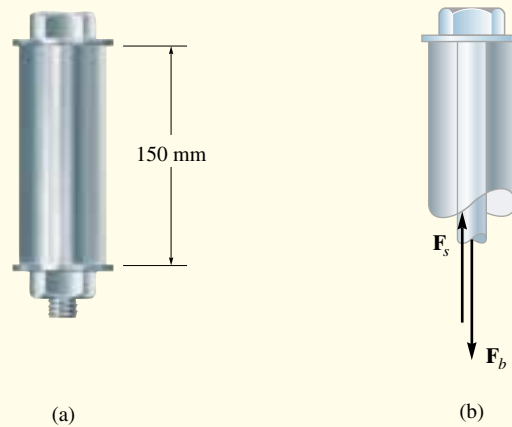

150 mm

F_s

F_b

(a)   (b)

Fig. 4–19

Solution

Equilibrium.   A free-body diagram of a sectioned segment of the assembly is shown in Fig. 4–19b. The forces F_b and F_s are produced since the sleeve has a higher coefficient of thermal expansion than the bolt, and therefore the sleeve will expand more when the temperature is increased. The problem is statically indeterminate since these forces cannot be determined from equilibrium. However, it is required that

$$+\uparrow \Sigma F_y = 0; \qquad\qquad\qquad F_s = F_b \qquad\qquad (1)$$

Compatibility.   The temperature increase causes the sleeve and bolt to expand $(\delta_s)_T$ and $(\delta_b)_T$, Fig. 4–19c. However, the redundant forces F_b and F_s elongate the bolt and shorten the sleeve. Consequently, the end of the assembly reaches a final position, which is not the same as the initial position. Hence, the compatibility condition becomes

$$(+\downarrow) \qquad\qquad \delta = (\delta_b)_T + (\delta_b)_F = (\delta_s)_T - (\delta_s)_F$$

(c)
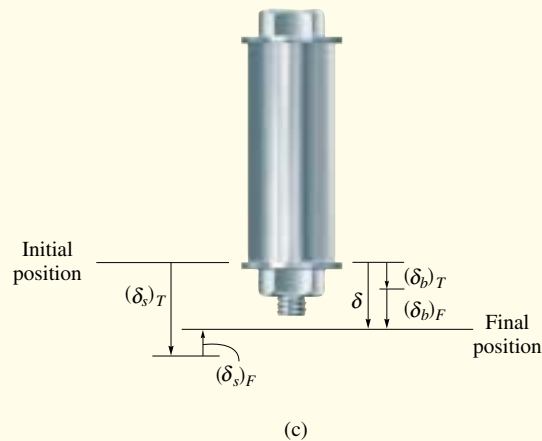
Applying Eqs. 4–2 and 4–4, and using the mechanical properties from the table on the inside back cover, we have

$$[12(10^{-6})/°C](80°C - 15°C)(0.150\ \text{m})$$

$$+ \frac{F_b(0.150\ \text{m})}{(400\ \text{mm}^2)(10^{-6}\ \text{m}^2/\text{mm}^2)[200(10^9)\ \text{N/m}^2]}$$

$$= [23(10^{-6})/°C](80°C - 15°C)(0.150\ \text{m})$$

$$- \frac{F_s(0.150\ \text{m})}{600\ \text{mm}^2\ (10^{-6}\ \text{m}^2/\text{mm}^2)[73.1(10^9)\ \text{N/m}^2]}$$

Using Eq. 1 and solving gives
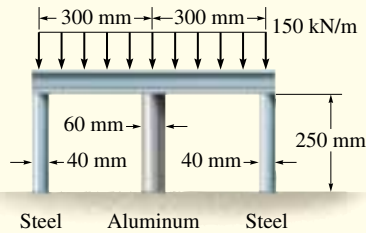
$$F_s = F_b = 20.26\ \text{kN}$$

The average normal stress in the bolt and sleeve is therefore

$$\sigma_b = \frac{20.26\ \text{kN}}{400\ \text{mm}^2\ (10^{-6}\ \text{m}^2/\text{mm}^2)} = 50.6\ \text{MPa} \qquad \textit{Ans.}$$
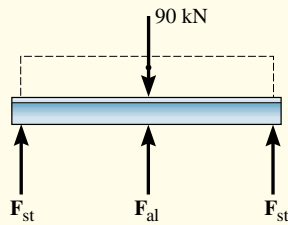
$$\sigma_s = \frac{20.26\ \text{kN}}{600\ \text{mm}^2\ (10^{-6}\ \text{m}^2/\text{mm}^2)} = 33.8\ \text{MPa} \qquad \textit{Ans.}$$

Since linear–elastic material behavior was assumed in this analysis, the calculated stresses should be checked to make sure that they do not exceed the proportional limits for the material.
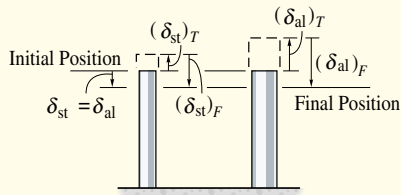
300 mm | 300 mm | 150 kN/m

60 mm →

250 mm

40 mm | 40 mm →

Steel | Aluminum | Steel

(a)

90 kN

F_{st} | F_{al} | F_{st}

(b)

$(\delta_{st})_T$ | $(\delta_{al})_T$

Initial Position | $(\delta_{al})_F$

$\delta_{st} = \delta_{al}$ | $(\delta_{st})_F$ | Final Position

(c)

Fig. 4–20

The rigid bar shown in Fig. 4–20a is fixed to the top of the three posts made of A-36 steel and 2014-T6 aluminum. The posts each have a length of 250 mm when no load is applied to the bar, and the temperature is $T_1 = 20°C$. Determine the force supported by each post if the bar is subjected to a uniform distributed load of 150 kN/m and the temperature is raised to $T_2 = 80°C$.

Solution

Equilibrium. The free-body diagram of the bar is shown in Fig. 4–20b. Moment equilibrium about the bar's center requires the forces in the steel posts to be equal. Summing forces on the free-body diagram, we have

$$+\uparrow\Sigma F_y = 0; \qquad 2F_{st} + F_{al} - 90(10^3)\text{ N} = 0 \qquad (1)$$

Compatibility. Due to load, geometry, and material symmetry, the top of each post is displaced by an equal amount. Hence,

$$(+\downarrow) \qquad\qquad \delta_{st} = \delta_{al} \qquad (2)$$

The final position of the top of each post is equal to its displacement caused by the temperature increase, plus its displacement caused by the internal axial compressive force, Fig. 4–20c. Thus, for a steel and aluminum post, we have

$$(+\downarrow) \qquad\qquad \delta_{st} = -(\delta_{st})_T + (\delta_{st})_F$$
$$(+\downarrow) \qquad\qquad \delta_{al} = -(\delta_{al})_T + (\delta_{al})_F$$

Applying Eq. 2 gives

$$-(\delta_{st})_T + (\delta_{st})_F = -(\delta_{al})_T + (\delta_{al})_F$$

Using Eqs. 4–2 and 4–4 and the material properties on the inside back cover, we get

$$-[12(10^{-6})/°C](80°C + 20°C)(0.250\text{ m}) + \frac{F_{st}(0.250\text{ m})}{\pi(0.020\text{ m})^2[200(10^9)\text{ N/m}^2]}$$

$$= -[23(10^{-6})/°C](80°C - 20°C)(0.250\text{ m}) + \frac{F_{al}(0.250\text{ m})}{\pi(0.03\text{ m})^2[73.1(10^9)\text{ N/m}^2]}$$

$$F_{st} = 1.216F_{al} - 165.9(10^3) \qquad (3)$$

To be *consistent*, all numerical data has been expressed in terms of newtons, meters, and degrees Celsius. Solving Eqs. 1 and 3 simultaneously yields

$$F_{st} = -16.4\text{ kN} \quad F_{al} = 123\text{ kN} \qquad\qquad \textit{Ans.}$$

The negative value for F_{st} indicates that this force acts opposite to that shown in Fig. 4–20b. In other words, the steel posts are in tension and the aluminum post is in compression.

A steel bar has the dimensions shown in Fig. 4–26. If the allowable stress is $\sigma_{\text{allow}} = 115$ MPa, determine the largest axial force P that the bar can carry.

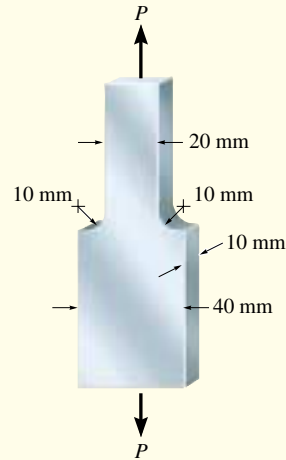

P

20 mm

10 mm   10 mm

10 mm

40 mm

P

Fig. 4–26

Solution

Because there is a shoulder fillet, the stress-concentration factor can be determined using the graph in Fig. 4–24. Calculating the necessary geometric parameters yields

$$\frac{r}{n} = \frac{10 \text{ mm}}{20 \text{ mm}} = 0.50$$

$$\frac{w}{h} = \frac{40 \text{ mm}}{20 \text{ mm}} = 2$$

Thus, from the graph,

$$K = 1.4$$

Computing the average normal stress at the *smallest* cross section, we have

$$\sigma_{\text{avg}} = \frac{P}{(20 \text{ mm})(10 \text{ mm})} = 0.005P \text{ N/mm}^2$$

Applying Eq. 4–7 with $\sigma_{\text{allow}} = \sigma_{\text{max}}$ yields

$$\sigma_{\text{allow}} = K\sigma_{\text{avg}}$$
$$115 \text{ N/mm}^2 = 1.4(0.005P)$$
$$P = 16.43(10^3) \text{ N} \qquad \textit{Ans.}$$
$$= 16.43 \text{ kN}$$

The steel strap shown in Fig. 4–27 is subjected to an axial load of 80 kN. Determine the maximum normal stress developed in the strap and the displacement of one end of the strap with respect to the other end. The steel has a yield stress of $\sigma_Y = 700$ MPa, and $E_{st} = 200$ GPa.
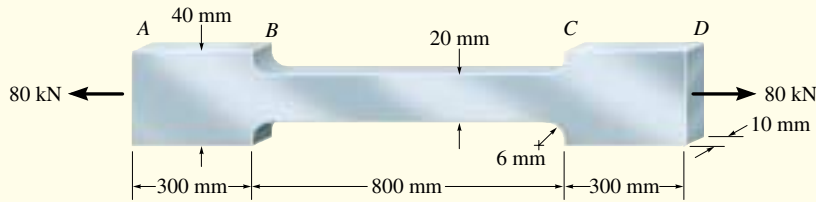

Fig. 4–27

Solution

Maximum Normal Stress. By inspection, the maximum normal stress occurs at the smaller cross section, where the shoulder fillet begins at B or C. The stress-concentration factor is determined from Fig. 4–23. We require

$$\frac{r}{h} = \frac{6\text{ mm}}{20\text{ mm}} = 0.3, \qquad \frac{w}{h} = \frac{40\text{ mm}}{20\text{ mm}} = 2$$

Thus, $K = 1.6$.

The maximum stress is therefore

$$\sigma_{max} = K\frac{P}{A} = 1.6\left[\frac{80(10^3)\text{ N}}{(0.02\text{ m})(0.001\text{ m})}\right] = 640\text{ MPa} \qquad Ans.$$

Notice that the material remains elastic, since 640 MPa $< \sigma_Y = 700$ MPa.

Displacement. Here we will neglect the localized deformations surrounding the applied load and at the sudden change in cross section of the shoulder fillet (Saint-Venant's principle). We have

$$\delta_{A/D} = \sum\frac{PL}{AE} = 2\left\{\frac{80(10^3)\text{ N}(0.3\text{ m})}{(0.04\text{ m})(0.01\text{ m})[200(10^9)\text{ N/m}^2]}\right\}$$

$$+ \left\{\frac{80(10^3)\text{ N}(0.8\text{ m})}{(0.02\text{ m})(0.01\text{ m})[200(10^9)\text{ N/m}^2]}\right\}$$

$$\sigma_{A/D} = 2.20\text{ mm} \qquad Ans.$$

(a)
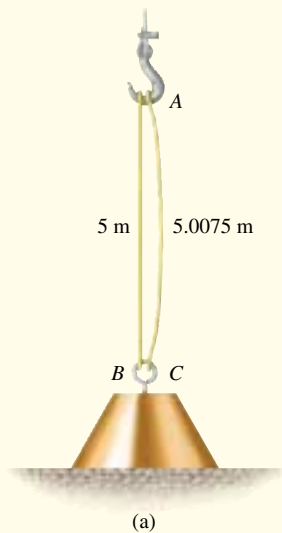
Two steel wires are used to lift the weight of 15 kN ($\approx$ 1.5 kg), Fig. 4–30a. Wire AB has an unstretched length of 5 m and wire AC has an unstretched length of 5.0075 m. If each wire has a cross-sectional area of 30 mm², and the steel can be considered elastic perfectly plastic as shown by the $\sigma - \epsilon$ graph in Fig. 4–30b, determine the force in each wire and its elongation.

Solution

By inspection, wire AB begins to carry the weight when the hook is lifted. However, if this wire stretches more than 0.01 m, the load is then carried by both wires. For this to occur, the strain in wire AB must be

$$\epsilon_{AB} = \frac{0.0075 \text{ m}}{5 \text{ m}} = 0.0015$$

which is less than the maximum elastic strain, $\epsilon_Y = 0.0017$, Fig. 4–30b. Furthermore, the stress in wire AB when this happens can be determined from Fig. 4–30b by proportion; i.e.,

$$\frac{0.0015}{350 \text{ MPa}} = \frac{0.0015}{\sigma_{AB}}$$

$$\sigma_{AB} = 308.82 \text{ MPa}$$

As a result, the force in the wire is thus

$$F_{AB} = \sigma_{AB}A = (308.82 \text{ N/mm}^2)(30 \text{ mm}^2) = 9264.6 \text{ N} = 9.26 \text{ kN}$$

Since the weight to be supported is 15 kN, we can conclude that both wires must be used for support.

Once the weight is supported, the stress in the wires depends on the corresponding strain. There are three possibilities, namely, the strains in both wires are elastic, wire AB is plastically strained while wire AC is elastically strained, or both wires are plastically strained. We will begin by assuming that both wires remain *elastic*. Investigation of the free-body diagram of the suspended weight, Fig. 4–30c, indicates that the problem is statically indeterminate. The equation of equilibrium is
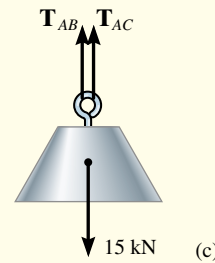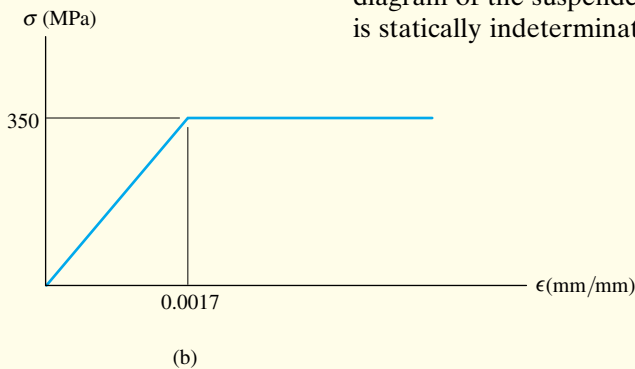

(b)

(c)

Fig. 4–30

$$+\uparrow\Sigma F_y = 0; \qquad\qquad T_{AB} + T_{AC} - 15\text{ kN} = 0 \qquad\qquad (1)$$

Since AC is 0.0075 m longer than AB, then from Fig. 4–30d, compatibility of displacement of the ends B and C requires that

$$\delta_{AB} = 0.0075\text{ m} + \delta_{AC} \qquad\qquad (2)$$

The modulus of elasticity, Fig. 4–30b, is $E_{st} = 350\text{ MPa}/0.0017 = 205.9(10^3)\text{ MPa}$. Since this is a linear–elastic analysis, the load–displacement relationship is $\delta = PL/AE$, and therefore

$$\frac{T_{AB}(5\text{ m})}{30(10^{-6})[205.9(10^6)\text{ kPa}]} = 0.0075\text{ m} + \frac{T_{AC}(5.0075\text{ m})}{30(10^{-6})[205.9(10^6)\text{ kPa}]}$$

$$5T_{AB} = 46.3275 + 5.0075T_{AC} \qquad\qquad (3)$$

Solving Eqs. 1 and 3, we have

$$T_{AB} = 12.135\text{ kN}$$

$$T_{AC} = 2.865\text{ kN}$$

The stress in wire AB is thus

$$\sigma_{AB} = \frac{12.135(10^3)\text{ N}}{30\text{ mm}^2} = 404.5\text{ MPa}$$

This stress is greater than the maximum elastic stress allowed ($\sigma_Y = 350\text{ MPa}$), and therefore wire AB becomes plastically strained and supports its maximum load of

$$T_{AB} = 350\text{ MPa }(30\text{ mm}^2) = 10.5\text{ kN} \qquad\qquad \textit{Ans.}$$

From Eq. 1,

$$T_{AB} = 4.5\text{ kN} \qquad\qquad \textit{Ans.}$$

Note that wire AC remains elastic since the stress in the wire is $\sigma_{AC} = 4.5(10^3)\text{ N}/30\text{ mm}^3 = 150\text{ MPa} < 350\text{ MPa}$. The corresponding elastic strain is determined by proportion, Fig. 4–30b; i.e.,

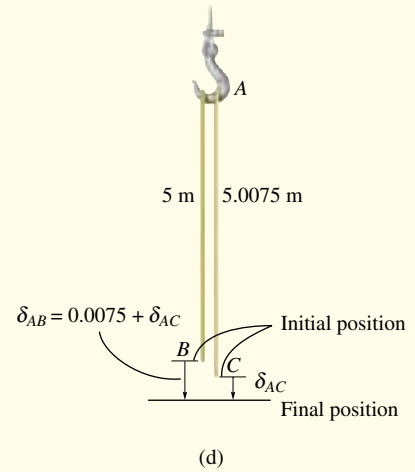$$\frac{\epsilon_{AC}}{150\text{ MPa}} = \frac{0.0017}{350\text{ MPa}}$$

$$\epsilon_{AC} = 0.000729$$

The elongation of AC is thus

$$\delta_{AC} = (0.000729)(5.0075) = 0.00365\text{ m} \qquad\qquad \textit{Ans.}$$

Applying Eq. 2, the elongation of AB is then

$$\delta_{AB} = 0.0075 + 0.00365 = 0.01115\text{ m} \qquad\qquad \textit{Ans.}$$

A

5 m | 5.0075 m

$\delta_{AB} = 0.0075 + \delta_{AC}$ — Initial position

B C

δ_{AC}

Final position

(d)

The bar in Fig. 4–31a is made of steel that is assumed to be elastic perfectly plastic, with $\sigma_Y = 250$ MPa. Determine (a) the maximum value of the applied load P that can be applied without causing the steel to yield and (b) the maximum value of P that the bar can support. Sketch the stress distribution at the critical section for each case.

Solution

Part (a). *When the material behaves* elastically, we must use a stress-concentration factor determined from Fig. 4–23 that is unique for the bar's geometry. Here

(a)
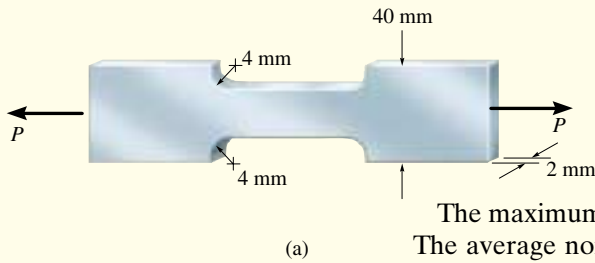
$$\frac{r}{h} = \frac{4 \text{ mm}}{(40 \text{ mm} - 8 \text{ mm})} = 0.125$$

$$\frac{w}{h} = \frac{40 \text{ mm}}{(40 \text{ mm} - 8 \text{ mm})} = 1.25$$

The maximum load, without causing yielding, occurs when $\sigma_{max} = \sigma_Y$. The average normal stress is $\sigma_{avg} = P/A$. Using Eq. 4–7, we have
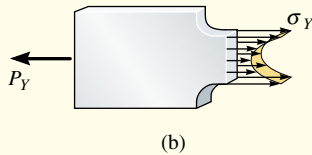
$$\sigma_{max} = K\sigma_{avg}; \qquad \sigma_Y = K\left(\frac{P_Y}{A}\right)$$

$$250(10^6) \text{ Pa} = 1.75\left[\frac{P_Y}{(0.002 \text{ m})(0.032 \text{ m})}\right]$$

$$P_Y = 9.14 \text{ kN} \qquad\qquad Ans.$$

(b)

This load has been calculated using the *smallest* cross section. The resulting stress distribution is shown in Fig. 4–31b. For equilibrium, the "volume" contained within this distribution must equal 9.14 kN.

Part (b). The maximum load sustained by the bar causes *all the material* at the smallest cross section to yield. Therefore, as P is increased to the *plastic load* P_p, it gradually changes the stress distribution from the elastic state shown in Fig. 4–31b to the plastic state shown in Fig. 4–31c. We require

(c)

Fig. 4–31

$$\sigma_Y = \frac{P_p}{A}$$

$$250(10^6) \text{ Pa} = \frac{P_p}{(0.002 \text{ m})(0.032 \text{ m})}$$

$$P_p = 16.0 \text{ kN} \qquad\qquad Ans.$$

Here P_p equals the "volume" contained within the stress distribution, which in this case is $P_p = \sigma_Y A$.
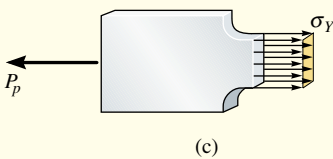
The rod shown in Fig. 4–33a has a radius of 5 mm and is made from an elastic-perfectly plastic material for which σ_Y = 420 MPa, E = 70 GPa, Fig. 4–33b. If a force of P = 60 kN is applied to the rod and then removed, determine the residual stress in the rod and the permanent displacement of the collar at C.

Solution

The free-body diagram of the rod is shown in Fig. 4–33b. By inspection, the rod is statically indeterminate. Application of the load **P** will cause one of three possibilities, namely, both segments AC and CB remain elastic, AC is plastic while CB is elastic, or both AC and CB are plastic.[*]

An *elastic analysis*, similar to that discussed in Sec. 4.4, will produce F_A = 45 kN and F_B = 15 kN at the supports. However, this results in a stress of
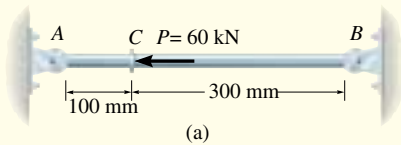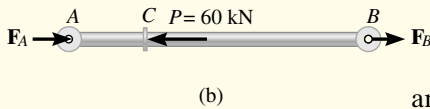
$$\sigma_{AC} = \frac{45 \text{ kN}}{\pi(0.005 \text{ m})^2} = 573 \text{ MPa (compression)} > \sigma_Y = 420 \text{ MPa}$$

in segment AC, and

$$\sigma_{CB} = \frac{15 \text{ kN}}{\pi(0.005 \text{ m})^2} = 191 \text{ MPa (tension)}$$

in segment CB. Since the material in segment AC will yield, we will assume that AC becomes plastic, while CB remains elastic.

For this case, the maximum possible force developed in AC is

$$(F_A)_Y = \sigma_Y A = 420(10^3) \text{ kN/m}^2 \, [\pi(0.005 \text{ m})^2]$$
$$= 33.0 \text{ kN}$$

and from the equilibrium of the rod, Fig. 4–33b,

$$F_B = 60 \text{ kN} - 33.0 \text{ kN} = 27.0 \text{ kN}$$

The stress in each segment of the rod is therefore

$$\sigma_{AC} = \sigma_Y = 420 \text{ MPa (compression)}$$

$$\sigma_{CB} = \frac{27.0 \text{ kN}}{\pi(0.005 \text{ m})^2} = 344 \text{ MPa (tension)} < 420 \text{ MPa (OK)}$$

Residual Stress. In order to obtain the residual stress, it is also necessary to know the strain in each segment due to the loading. Since CB responds elastically,

$$\delta_C = \frac{F_B L_{CB}}{AE} = \frac{(27.0 \text{ kN})(0.300 \text{ m})}{\pi(0.005 \text{ m})^2 \, [70(10^6) \text{ kN/m}^2]} = 0.001474 \text{ m}$$

A   C P= 60 kN   B

100 mm —300 mm—

(a)

F_A → A   C P= 60 kN   B → F_B

(b)

Fig. 4–33

Thus,

$$\epsilon_{CB} = \frac{\delta_C}{L_{CB}} = \frac{0.001474 \text{ m}}{0.300 \text{ m}} = +0.004913$$

Also, since δ_C is known, the strain in AC is

$$\epsilon_{AC} = \frac{\delta_C}{L_{AC}} = -\frac{0.001474 \text{ m}}{0.100 \text{ m}} = -0.01474$$

Therefore, when **P** is *applied*, the stress–strain behavior for the material in segment CB moves from O to A', Fig. 4–33c, and the stress–strain behavior for the material in segment AC moves from O to B'. If the load **P** is applied in the reverse direction, in other words, the load is removed, then an elastic response occurs and a reverse force of $F_A = 45$ kN and $F_B = 15$ kN must be applied to each segment. As calculated previously, these forces produce stresses $\sigma_{AC} = 573$ MPa (tension) and $\sigma_{CB} = 191$ MPa (compression), and as a result the residual stress in each member is

$$(\sigma_{AC})_r = -420 \text{ MPa} + 573 \text{ MPa} = 153 \text{ MPa} \qquad \textit{Ans.}$$

$$(\sigma_{CB})_r = 344 \text{ MPa} - 191 \text{ MPa} = 153 \text{ MPa} \qquad \textit{Ans.}$$

This tensile stress is the *same* for both segments, which is to be expected. Also note that the stress–strain behavior for segment AC moves from B' to D' in Fig. 4–33c, while the stress–strain behavior for the material in segment CB moves from A' to C'.

(c)
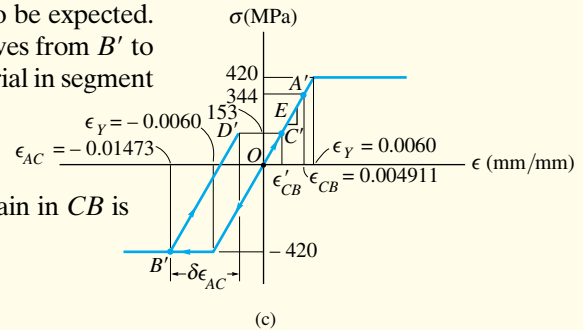
Permanent Displacement. From Fig. 4–33c, the residual strain in CB is

$$\epsilon'_{CB} = \frac{\sigma}{E} = \frac{153(10^6) \text{ Pa}}{70(10^9) \text{ Pa}} = 0.002185$$

so that the permanent displacement of C is

$$\delta_C = \epsilon'_{CB} L_{CB} = 0.002185 \, (300 \text{ mm}) = 0.656 \text{ mm} \leftarrow \qquad \textit{Ans.}$$

We can also obtain this result by determining the residual strain ϵ'_{AC} in AC, Fig. 4–33c. Since line $B'D'$ has a slope of E, then

$$\delta\epsilon_{AC} = \frac{\delta\sigma}{E} = \frac{(420 + 153)10^6 \text{ Pa}}{70(10^9) \text{ Pa}} = 0.008185$$

Therefore

$$\epsilon'_{AC} = \epsilon_{AC} + \delta\epsilon_{AC} = -0.01474 + 0.008185 = -0.006555$$

Finally,

$$\delta_C = \epsilon'_{AC} L_{AC} = -0.006555 \, (100 \text{ mm}) = 0.656 \text{ mm} \leftarrow \qquad \textit{Ans.}$$

*The possibility of CB becoming plastic before AC will not occur because when point C deforms, the *strain* in AC (since it is shorter) will always be larger than the strain in CB.