MAK 104 – STATICS

WORKING PROBLEMS-4

1. $F_R = ?$ $\quad (M_R)_O = ?$

A(0,0,8) B(0,6,0) C(0,0,6) D(2,-3,0)

$$F_B = F_B \cdot u_{AB} = 5.\left[\frac{(0-0)i + (6-0)j + (0-8)k}{\sqrt{0^2+6^2+8^2}}\right] = [3j - 4k]\ kN$$

$$F_D = F_D \cdot u_{CD} = 7.\left[\frac{(2-0)i + (-3-0)j + (0-6)k}{\sqrt{2^2+(-3)^2+(-6)^2}}\right] = [2i - 3j - 6k]\ kN$$

$F_R = \Sigma F \qquad F_R = F_B + F_D$

$= (3j - 4k) + (2i - 3j - 6k)$

$$\boxed{F_R = (2i - 10k)\ kN}$$

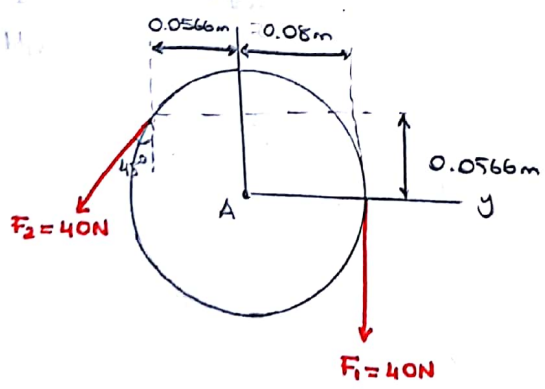
The resultant couple moment about point O can be obtained as

$(M_R)_O = \Sigma M_O \qquad (M_R)_O = r_{OB} \times F_B + r_{OC} \times F_D$

$$= \begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ 0 & 3 & -4 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 0 & 6 \\ 2 & -3 & -6 \end{vmatrix}$$

$$\boxed{(M_R)_O = (6i - 12j)\ kN.m}$$

2.



$F_R = F_1 + F_2$

$= (-40k) + (-40.\cos 45\, j - 40.\sin 45\, k)$

$$\boxed{F_R = (-28.3\, j - 68.3k)\ N}$$

$r_{AF1} = (-0.3i + 0.08j)\ m$

$r_{AF2} = (-0.3i - 0.08 \cdot \sin 45\, j + 0.08 \cdot \cos 45\, k) = (-0.3i - 0.0566j + 0.0566k)\ m$

$M_{RA} = (r_{AF1} \times F_1) + (r_{AF2} \times F_2)$

$$= \begin{vmatrix} i & j & k \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -0.3 & -0.0566 & 0.0566 \\ 0 & -40\cos 45 & -40\sin 45 \end{vmatrix}$$

$$\boxed{M_{RA} = (-20.5j + 8.49k)\ N.m}$$

3. $F_1 = (300k)\ N \quad , \quad F_3 = (100j)\ N$

$F_2 = 200.(\cos 45\, i - \sin 45\, k)\ N = (141.42i - 141.42k)N$

$M_1 = (100k)\ N.m \qquad M_2 = 180\,(\cos 45\, i - \sin 45\, k)\ N.m$

$M_2 = (127.28i - 127.28k)\ N.m$

$\Sigma F = 141.42i + 100j + (300 - 141.42)k$

$$\boxed{F_R = \Sigma F = (141i + 100j + 159k)\ N}$$

The position vectors $r_1 = (0.5j)\ m \quad r_2 = (1.1j)\ m$

$M_{RO} = \Sigma M_O \qquad M_{RO} = r_1 \times F_1 + r_2 \times F_2 + M_1 + M_2$

$$= \begin{vmatrix} i & j & k \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 1.1 & 0 \\ 141.42 & 0 & -141.42 \end{vmatrix}$$

$$+ 100k + 127.28i - 127.28k$$

$$\boxed{M_{RO} = (122i - 183k)\ N.m}$$

4. $F_R = (500i + 300j + 800k)\ N \implies F_R = \sqrt{500^2 + 300^2 + 800^2} = 900N$

$$u_{FR} = \frac{500i + 300j + 800k}{\sqrt{500^2 + 300^2 + 800^2}} = 0.5051i + 0.3030j + 0.8081k$$

$M_{Rx} = 800.(4 - y). \qquad M_{Ry} = 800x \qquad M_{Rz} = 500.y + 300.(6 - x)$

Since M_R also act in the direction of u_{FR}

$$M_R.(0.5051) = 800.(4-y)$$
$$M_R\,(0.3030) = 800x$$
$$M_R\,(0.8081) = 500y + 300.(6-x)$$

$$M_R = 3.07\ kN.m$$
$$x = 1.16m$$
$$y = 2.06m$$

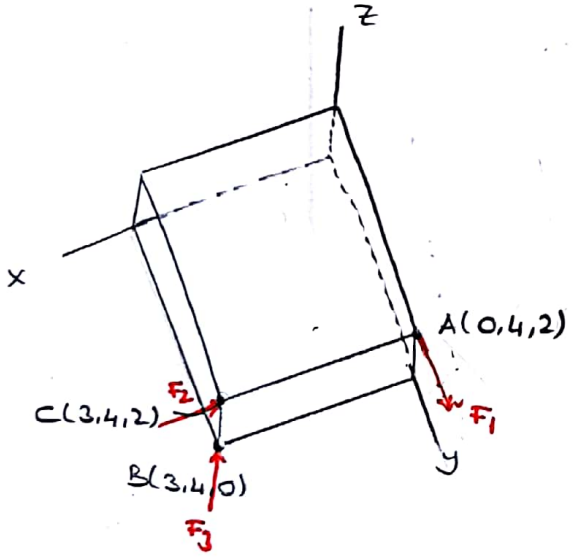
5. $F_1 = (600\bar{j})\,lb \qquad F_2 = (-450i)\,lb \qquad F_3 = (300k)\,lb$

$$F_R = F_1 + F_2 + F_3 = (-450i + 600\bar{j} + 300k)\,lb$$

$$F_R = \sqrt{(-450)^2 + (600)^2 + (300)^2} = 807.77\ lb$$

$$u_{FR} = \frac{-450i + 600\bar{j} + 300k}{\sqrt{(-450)^2 + (600)^2 + (300)^2}} = -0.5571i + 0.7428\bar{j} + 0.3714k$$

$$M_w = M_w.u_{FR} = -0.5571\,M_w\,i + 0.7428\,M_w\,\bar{j} + 0.3714\,M_w\,k$$



The position vectors

$$r_{PA} = (0-x)i + (4-y)\bar{j} + (2-0)k$$
$$= -xi + (4-y)\bar{j} + 2k$$

$$r_{PB} = (3-x)i + (4-y)\bar{j} + (0-0)k$$
$$= (3-x)i + (4-y)\bar{j}$$

$$r_{PC} = (3-x)i + (4-y)\bar{j} + (2-0)k$$
$$= (3-x)i + (4-y)\bar{j} + 2k$$

$$M_w = \Sigma M_P \qquad M_w = r_{PA} \times F_1 + r_{PC} \times F_2 + r_{PB} \times F_3 + M$$

$$-0.5571M_w\,i + 0.7428\,M_w\,\bar{j} + 0.3714M_w k = \begin{vmatrix} i & \bar{j} & k \\ -x & (4-y) & 2 \\ 0 & 600 & 0 \end{vmatrix} + \begin{vmatrix} i & \bar{j} & k \\ (3-x) & (4-y) & 2 \\ -450 & 0 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} i & \bar{j} & k \\ (3-x) & (4-y) & 0 \\ 0 & 0 & 300 \end{vmatrix} + 600i$$

$$-0.5571\,M_w\,i + 0.7428\,M_w\,\bar{j} + 0.3714\,M_w k = (600-300y)i + (300x-1800)\bar{j} + (1800-600x-450y)k$$

$$-0.5571\,M_w = 600 - 300y$$

$$0.7428\,M_w = 300x - 1800$$

$$0.3714\,M_w = 1800 - 600x - 450y$$

$x = 3.52\,ft$

$y = 0.138\,ft$

$M_w = -10031.6\,ft$