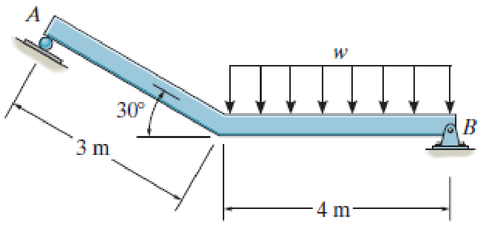
SPRING 2017

MAK104 - WORKING PROBLEMS 5

SOLUTION

1. If the intensity of the distributed load acting on the beam is w = 3 kN/m, determine the reactions at the roller A and pin B.



Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point B by referring to the *FBD* of the beam shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 3(4)(2) - N_A \sin 30^\circ (3 \sin 30^\circ) - N_A \cos 30^\circ (3 \cos 30^\circ + 4) = 0$$

$$N_A = 3.713 \text{ kN} = 3.71 \text{ kN} \qquad \textbf{Ans.}$$

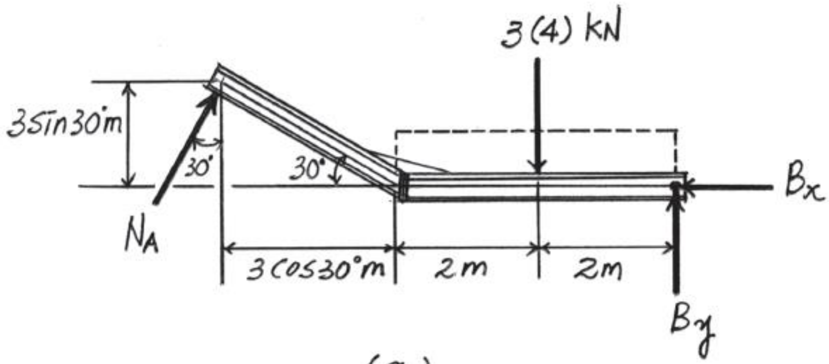
Using this result to write the force equation of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3.713 \sin 30^\circ - B_x = 0$$

$$B_x = 1.856 \text{ kN} = 1.86 \text{ kN} \qquad \textbf{Ans.}$$

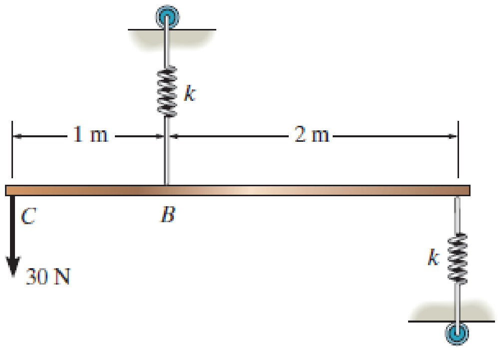
$$+\uparrow \Sigma F_y = 0; \qquad B_y + 3.713 \cos 30^\circ - 3(4) = 0$$

$$B_y = 8.7846 \text{ kN} = 8.78 \text{ kN} \qquad \textbf{Ans.}$$



(a)

2. The bar of negligible weight is supported by two springs, each having a stiffness k = 100 N/m. If the springs are originally unstretched, and the force is vertical as shown, determine the angle θ the bar makes with the horizontal, when the 30-N force is applied to the bar.



Equations of Equilibrium. $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equation of equilibrium about points B and A respectively by referring to the *FBD* of the bar shown in Fig. *a*.

$\circlearrowleft + \Sigma M_B = 0;\qquad 30(1) - F_A(2) = 0 \qquad F_A = 15\text{ N}$

$\circlearrowleft + \Sigma M_A = 0;\qquad 30(3) - F_B(2) = 0 \qquad F_B = 45\text{ N}$

Thus, the stretches of springs A and B can be determined from

$F_A = kx_A;\qquad 15 = 100x_A \qquad x_A = 0.15\text{ m}$

$F_B = kx_B;\qquad 45 = 100x_B \qquad x_B = 0.45\text{ m}$

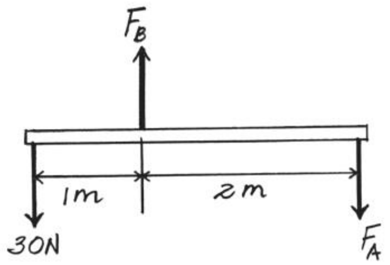
From the geometry shown in Fig. *b*,

$$\frac{d}{0.45} = \frac{2-d}{0.15};\qquad d = 1.5\text{ m}$$

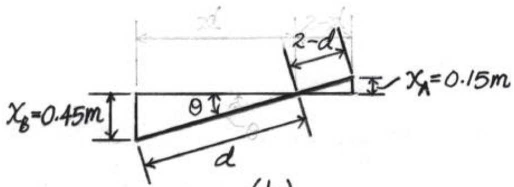
Thus

$$\theta = \sin^{-1}\left(\frac{0.45}{1.5}\right) = 17.46° = 17.5° \qquad \textbf{Ans.}$$

Note: The moment equations are set up assuming small θ, but even with non-small θ the reactions come out with the same F_A, F_B, and then the rest of the solution goes through as before.

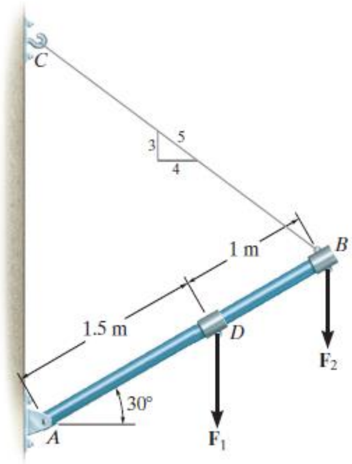


(a)



(b)

3. The boom supports the two vertical loads. Neglect the size of the collars at *D* and *B* and the thickness of the boom, and compute the horizontal and vertical components of force at the pin *A* and the force in cable *CB*. Set $F_1 = 800$ N and $F_2 = 350$ N.



$$\circlearrowleft + \Sigma M_A = 0; \qquad -800(1.5\cos 30^\circ) - 350(2.5\cos 30^\circ)$$

$$+\frac{4}{5}F_{CB}(2.5\sin 30^\circ) + \frac{3}{5}F_{CB}(2.5\cos 30^\circ) = 0$$

$$F_{CB} = 781.6 = 782\text{ N}$$

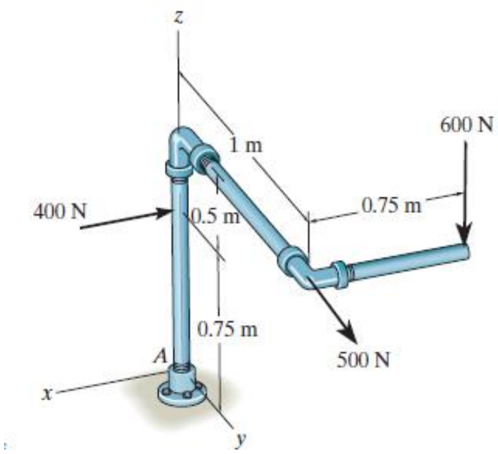
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - \frac{4}{5}(781.6) = 0$$

$$A_x = 625\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 800 - 350 + \frac{3}{5}(781.6) = 0$$

$$A_y = 681\text{ N}$$

4. Determine the components of reaction at the fixed support *A*. The 400 N, 500 N, and 600 N forces are parallel to the *x*, *y*, and *z* axes, respectively.



Equations of Equilibrium. Referring to the *FBD* of the rod shown in Fig. *a*

$\Sigma F_x = 0;\quad A_x - 400 = 0 \quad A_x = 400\ \text{N}$

$\Sigma F_y = 0;\quad 500 - A_y = 0 \quad A_y = 500\ \text{N}$

$\Sigma F_z = 0;\quad A_z - 600 = 0 \quad A_z = 600\ \text{N}$

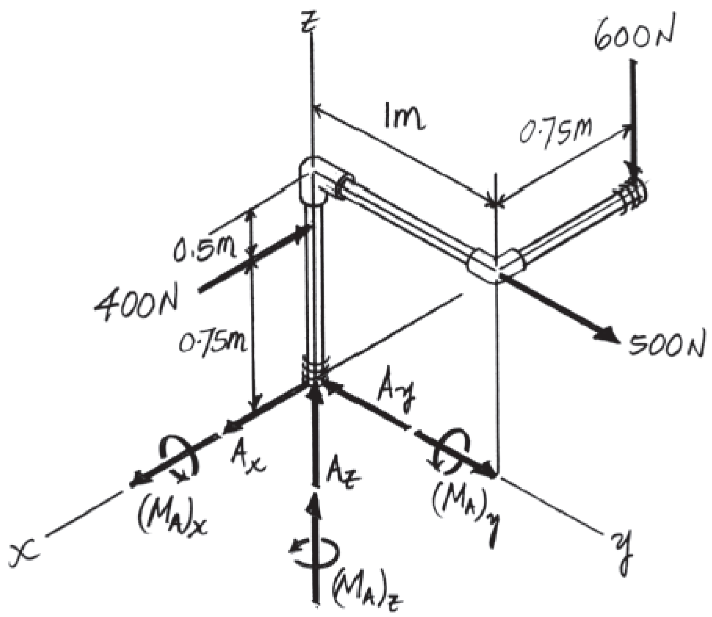
$\Sigma M_x = 0;\quad (M_A)_x - 500(1.25) - 600(1) = 0$

$$(M_A)_x = 1225\ \text{N}\cdot\text{m} = 1.225\ \text{kN}\cdot\text{m}$$

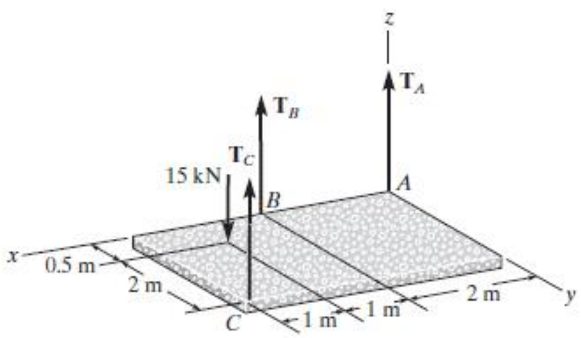
$\Sigma M_y = 0;\quad (M_A)_y - 400(0.75) - 600(0.75) = 0$

$$(M_A)_y = 750\ \text{N}\cdot\text{m}$$

$\Sigma M_z = 0;\quad (M_A)_z = 0$



5. The uniform concrete slab has a mass of 2400 kg. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown.



Equations of Equilibrium. Referring to the *FBD* of the slab shown in Fig. *a*, we notice that T_C can be obtained directly by writing the moment equation of equilibrium about the *x* axis.

$\Sigma M_x = 0; \quad T_C(2.5) - 2400(9.81)(1.25) - 15(10^3)(0.5) = 0$

$T_C = 14{,}772 \text{ N} = 14.8 \text{ kN}$ **Ans.**

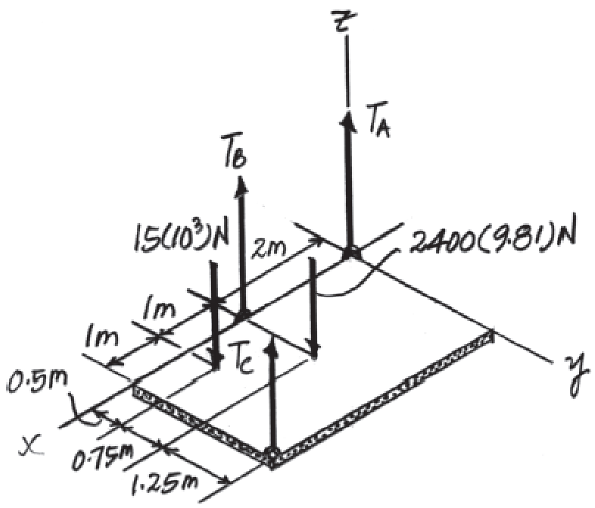
Using this result to write moment equation of equilibrium about *y* axis and force equation of equilibrium along *z* axis,

$\Sigma M_y = 0; \quad T_B(2) + 14{,}772(4) - 2400(9.81)(2) - 15(10^3)(3) = 0$

$T_B = 16{,}500 \text{ N} = 16.5 \text{ kN}$ **Ans.**

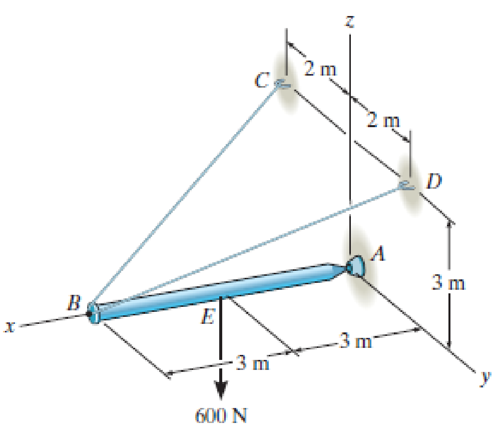
$\Sigma F_z = 0; \quad T_A + 16{,}500 + 14{,}772 - 2400(9.81) - 15(10^3) = 0$

$T_A = 7272 \text{ N} = 7.27 \text{ kN}$ **Ans.**



(a)

6. Determine the components of reaction at the ball-and-socket joint *A* and the tension in each cable necessary for equilibrium of the rod.



Force And Position Vectors. The coordinates of points A, B, C, D and E are $A(0, 0, 0)$, $B(6, 0, 0)$, $C(0, -2, 3)$ m, $D(0, 2, 3)$ m and $E(3, 0, 0)$ m respectively.

$$\mathbf{F}_{BC} = F_{BC}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = F_{BC}\left[\frac{(0-6)\mathbf{i} + (-2-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-2-0)^2 + (3-0)^2}}\right] = -\frac{6}{7}F_{BC}\mathbf{i} - \frac{2}{7}F_{BC}\mathbf{j} + \frac{3}{7}F_{BC}\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = F_{BD}\left[\frac{(0-6)\mathbf{i} + (2-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (2-0)^2 + (3-0)^2}}\right] = -\frac{6}{7}F_{BD}\mathbf{i} + \frac{2}{7}F_{BD}\mathbf{j} + \frac{3}{7}F_{BD}\mathbf{k}$$

$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$

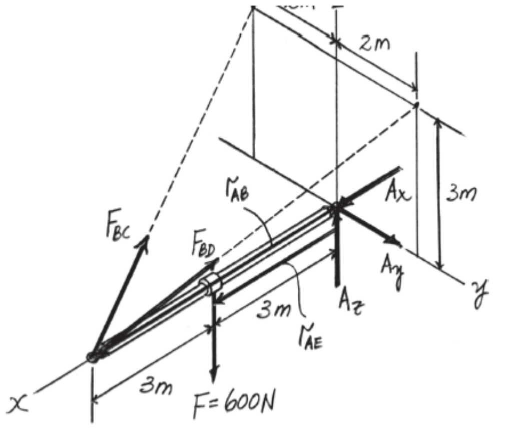
$F = \{-600\mathbf{k}\}$ N

$r_{AB} = \{6\mathbf{i}\}$ m $\qquad r_{AE} = \{3\mathbf{i}\}$ m

Equations of Equilibrium. Referring to the *FBD* of the rod shown in Fig. *a*, the force equation of equilibrium gives

$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_A + \mathbf{F} = 0$

$$\left(-\frac{6}{7}F_{BC} - \frac{6}{7}F_{BD} + A_x\right)\mathbf{i} + \left(\frac{2}{7}F_{BD} - \frac{2}{7}F_{BC} + A_y\right)\mathbf{j} + \left(\frac{3}{7}F_{BC} + \frac{3}{7}F_{BD} + A_z - 600\right)\mathbf{k} = 0$$



(a)

Equating **i**, **j** and **k** components,

$$-\frac{6}{7}F_{BC} - \frac{6}{7}F_{BD} + A_x = 0$$

$$\frac{2}{7}F_{BD} - \frac{2}{7}F_{BC} + A_y = 0$$

$$\frac{3}{7}F_{BC} + \frac{3}{7}F_{BD} + A_z - 600 = 0$$

The moment equation of equilibrium gives

$\Sigma\mathbf{M}_A = 0; \qquad \mathbf{r}_{AE} \times \mathbf{F} + \mathbf{r}_{AB} \times (\mathbf{F}_{BC} + \mathbf{F}_{BD}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ 0 & 0 & -600 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -\frac{6}{7}(F_{BC} + F_{BD}) & \frac{2}{7}(F_{BD} - F_{BC}) & \frac{3}{7}(F_{BC} + F_{BD}) \end{vmatrix} = 0$$

$$\left[1800 - \frac{18}{7}(F_{BC} + F_{BD})\right]\mathbf{j} + \frac{12}{7}(F_{BD} - F_{BC})\mathbf{k} = 0$$

Equating **j** and **k** components,

$$1800 - \frac{18}{7}(F_{BC} + F_{BD}) = 0$$

$$\frac{12}{7}(F_{BD} - F_{BC}) = 0$$

Solving Eqs. (1) to (5),

$F_{BD} = F_{BC} = 350$ N

$A_x = 600$ N

$A_y = 0$

$A_z = 300$ N